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THE INCLUSIVE MUON NEUTRINO REACTION IN ^{12}C

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ABSTRACT: We calculate the charged current inclusive cross section for the reaction, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$, from threshold to 300 MeV. In addition we obtain the average cross section over the spectrum currently being produced at LAMPF. We discuss our results with respect to the current experimental work and consider the possible contributions from higher l states.

I.INTRODUCTION

Recently results have been reported from an experiment[1] being performed at the Los Alamos Meson Physics Facility (LAMPF) for the reaction, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$. The group performing this experiment has obtained the cross section for the above process averaged over a spectrum of muon neutrinos produced by the in-flight decay of muons. We show the spectrum in figure 1. It has a relatively flat peak from approximately 40 to 80 MeV and then falls to about one fifth of its maximal value by 175 MeV and essentially to zero by 300 MeV. The group has reported a spectrum averaged cross section of:

$$< \sigma >= (8.3 \pm\ 0.7\ stat.\ \pm\ 1.6\ sys.) \times\ 10^{-40}\ cm^2. \tag{1}$$

This cross section differs from exisiting calculations[1,2] by a factor of 2 to 3 and so there is an immediate interest for examining this process.

Furthermore this process continues and supplements the work of two other groups[3,4] on the reaction,$\nu_e + {}^{12}C \longrightarrow e^- + X$, over the Michel spectrum. Here, of course, the energy range for the neutrino is substantially higher than for the earlier experiments. This,however, is somewhat counterbalanced by the requirement of producing a muon in the final state which uses 105 MeV of the available energy. Thus there is an overlap of the energy region for these two processes.

In this paper we calculate the cross section for the process, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$, from threshold to 300 MeV. We also obtain a cross section averaged over the LAMPF muon neutrino spectrum. Finally we shall discus possible reasons for the very large discrepancy between the experimental results and some of the current theoretical calculations.

The starting point of this calculation is model which has produced reasonable results[5,6,7] for both large and small nuclei. Descriptions of this model have already appeared in the literature and so we shall give only a brief discussion of its essential features below.

II. MATRIX ELEMENTS

The starting point for the calculation presented here is the first order matrix element:

$$< \mu^- X|H_w|\nu_\mu {}^{12}C >= \frac{G}{\sqrt{2}} \cos\theta_C \overline{u}_\mu \gamma^\lambda (1-\gamma_5) u_\nu < X|J_\lambda(0)|^{12}C > . \tag{2}$$

From this expression we compute a cross section:

$$\sigma_c = \sum_k \frac{m_\nu}{2ME_\nu} \int d^3P_\mu\, |M_{ki}|^2 \frac{m_\mu}{E_\mu(2\pi)^3} \frac{d^3P_k}{2E_k(2\pi)^3} (2\pi)^4 \delta^4(P_k + P_\mu - P_\nu - P_i) \tag{3}$$

where

$$M_{ki} = \frac{G}{\sqrt{2}} \, cos\theta_C \overline{u}_\mu \gamma^\lambda (1-\gamma_5) u_\nu < k|J_\lambda(0)|^{12}C > . \tag{4}$$

We may write:

$$|M_{ki}|^2 = \frac{G^2 \cos\theta_C}{2m_\mu m_\nu} L^{\sigma\lambda} < k|J_\sigma(0)|^{12}C >< k|J_\lambda(0)|^{12}C >^* . \tag{5}$$

Here $L^{\sigma\lambda}$ is the lepton tensor given by:

$$L^{\sigma\lambda} = p_\mu^\sigma p_\nu^\lambda - p_\mu \cdot p_\nu g^{\sigma\lambda} + p_\nu^\sigma p_\mu^\lambda - \epsilon^{\alpha\sigma\beta\lambda} p_{\mu\alpha} p_{\nu\beta}. \tag{6}$$

We assume an average excitation for the nucleus, δ, given by:

$$M_x - M_i = \delta \tag{7}$$

where M_i and M_x are the initial state mass and the excited state mass, respectively. This value of the excitation, δ, must change with neutrino energy. Above the giant dipole resonance of roughly 24 MeV, however, δ can be assumed to only slowly vary because the giant dipole resonance dominates the spectrum of states of the final nucleus. Recently using electron scattering data we have been able to check this assumption. We shall discuss this point later. We also assume from our knowledge of individual states, that most of the interaction tends to be in the forward direction so that we may write:

$$E_\mu = E_\nu - \delta \tag{8a}$$

and

$$< |\overrightarrow{p_\mu}| > \cong ((E_\nu - \delta)^2 - m_\mu^2)^{\frac{1}{2}}. \tag{8b}$$

The four momentum of the nuclear state, k, may therefore be written as:

$$P_k^\mu = P_i^\mu + p_\nu^\mu - < p_\mu^\mu > . \tag{9}$$

We also define $< q^\mu > = p_\nu^\mu - < p_\mu^\mu >$ which we shall use shortly. We note that the subscripts identify the particles in Eq.(9) and are not four-vector indices. We are now able to write the cross section as:

$$\sigma_c = \frac{G^2 cos^2\theta_C}{2ME_\nu} \int d\Omega_\mu \sum_k < k|J_\sigma(0)|^{12}C >< k|J_\lambda(0)|^{12}C >^* L^{\sigma\lambda}$$
$$\times \frac{< |\overrightarrow{p_\mu}| >}{2M - 2E_\mu + 2E_\mu \cos\theta\_mu \frac{<E_\mu>}{<|\overrightarrow{p_\mu}|>}}. \tag{10}$$

All parts of σ_c are therefore known except for the quantity $< k|J_\sigma(0)|^{12}C >< k|J_\lambda(0)|^{12}C >^*$. We replace this quantity by a hadron tensor given by:

$$\sum_k < k|J_\sigma(0)|^{12}C >< k|J_\lambda(0)|^{12}C >^* \equiv Q_{\lambda\sigma}(P_i, < q >). \tag{11}$$

The most general form of this tensor may be written as:

$$\begin{aligned} Q^{\mu\nu} =& \alpha g^{\mu\nu} + \frac{\beta}{M^2} P_i^\mu P_i^\nu + \frac{\gamma}{M^2} P_i{}^\mu < q^\nu > \\ &+ \frac{\delta}{M^2} < q^\mu > P_i^\nu + \frac{\rho}{M^2} < q^\mu >< q^\nu > + \frac{\eta}{M^2} \epsilon^{\mu\nu\lambda\sigma} P_\lambda < q_\sigma > . \end{aligned} \tag{12}$$

where the coefficients α, β, γ, δ, ρ and η must be determined. At present it is not possible to completely determine these coefficients, which are functions of the average q^2. However if the individual nuclear states, k_i, which can contribute[8] are examined, one finds that the functions δ, γ and η receive their contributions from individual states in the form:

$$\sum_{j,k} F_{ij} F_{ik}^* .$$

In cases for which there are many states contributing, these terms should have random phases and average to zero. In addition, the terms contributing to these coefficients are all cross terms which in the cases that have been calculated are never dominant and are usually quite small. This is consistent with an impulse approximation treatment[8,9]. Furthermore,because the interaction is in the forward direction, $< q^\mu >$ is small and so we can drop terms proportional to $< q^\sigma >< q^\lambda >$. Thus the hadron tensor becomes:

$$Q^{\mu\nu} = \alpha g^{\mu\nu} + \frac{\beta}{M^2} P_i^\mu P_i^\nu . \tag{13}$$

The cross section for the reaction, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$, is immediately obtained as:

$$\sigma_c = \frac{G^2 \cos^2(\theta_C)}{4\pi} \frac{< |\vec{p_\mu}| >< E_\mu > D}{M(M + E_\nu)} \tag{14a}$$

where

$$D = \beta - 2\alpha. \tag{14b}$$

We have thus obtained a simple result for the cross section which depends only on the quantity $\beta - 2\alpha$. Thus we have only to find α and β or at least the quantity $\beta - 2\alpha$. The quantity $\beta - 2\alpha$ as a function of $< q^2 >$ has been previously obtained[5,6,8] by the use of total muon capture data , pion electroproduction data and electron scattering data. In particular the capture rate which may be written in terms of D as:

$$\Gamma_{TOT} = \frac{C|\Phi(0)|^2 G^2 \cos^2\theta_C < E_\nu >^2 D}{8\pi M_i(M_i + m_\mu)} \tag{15}$$

plays a very important role. Here $\Phi(0)$ is the momentum space ground state wave function of the muon, C is a correction factor for the charge spread of the initial nucleus, and $q^2 \simeq (0.75 m_\mu)^2$.

From experiment[12] $\Gamma_{TOT} = (3.97 \pm 0.01) \times 10^4 sec^{-1}$. This yields a value of D given by:

$$D = 3.76 \times 10^9 MeV^2. \tag{16}$$

With the known fixed value for $< q^2 >$ this provides an important constraint on D. With the additional electroproduction and electron scattering data we find[5,6,8]:

$$D = 3.61 \times 10^9 (1 + \frac{0.074 q^2}{m_\mu^2}). \tag{17}$$

The other quantity which must be determined is the average excitation, $\delta(E_\nu)$. Above the giant dipole resonance we had assumed that we could set the average excitation to its value at the giant dipole resonance, i.e. $\delta(E_\nu) = 24 MeV$. This seemed adequate for the Michel spectrum energy range for the neutrino but might not be adequate for the present situation. Recently, using electron scattering data[11,12] for incoming electrons of 60, 71, 82 and 100 MeV at 155 degrees, and 178 MeV at 140 degress,we have summed the excited states and found the average excitations. These results are shown in figure 2 and are well described by a straight line given by:

$$\delta(E_\nu) \simeq .051\ E_\nu + 21.96\ MeV. \tag{18}$$

Although we would have preferred smaller angle data, or total cross section data, it was not available at suitable lepton energy. Our experience with electron scattering processes[13], however leads us to believe that this data should provide us with the general trend for δ. Thus our result is a pleasing one because the slope is small as expected, and we incorporate this equation for $\delta(E_\nu)$ into our cross section. Below the giant dipole resonance we use a linear approximation previously used[5], namely:

$$\delta(E_\nu) = \frac{(E_\nu + 14.97)}{2}. \tag{19}$$

One consideration still remains and that is the final state electromagnetic interaction of the muon with the resulting nucleus. In the case of the electron which is relativistic at only a few MeV over threshold this is not a large contribution. However for the muon it is much more important. To incorporate this correction we use a result due to Tzara[14], namely that;

$$C_F \simeq \frac{2\pi\alpha Z m_\mu}{p_\mu}\left(1 - e^{-\left(\frac{2\pi\alpha\ Z m_\mu}{p_\mu}\right)}\right). \tag{20}$$

This result is incorporated into the equation for σ, Eq.(14). We are now ready to obtain the cross sections of interest.

III. RESULTS

From Eq.(14) the cross section for the reaction, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$, may be obtained and the results are shown in figure 3. Here the solid curve includes the final state interaction and the dotted curve omits it. We also calculate a spectrum averaged value for the cross section which yields:

$$< \sigma >= 10.8\ \times\ 10^{-40}\ cm^2 \tag{21a}$$

without the final state interaction and:

$$< \sigma >= 13.1\ \times\ 10^{-40}\ cm^2 \tag{21b}$$

with the final state interaction. We estimate our error in the 25 percent to 30 percent range, with about 9 percent due to the uncertainty in the q^2 behavior of D,approximately 10 percent due to the uncertainty in the form of $\delta(E_\nu)$ and approximately 7 percent due to uncertainty in the final state interaction.

We have also calculated a spectrum averaged cross section for this process using a model developed earlier by Kim and Mintz[10,11] which has been previously described. This model is a non-relativistic,closure approximation based result which uses the total muon capture rate as an imput so that it is semi-phenomenological. The result of this model for the spectrum averaged cross section is:

$$< \sigma >= 11.0\ \times\ 10^{-40}\ cm^2. \tag{22}$$

IV. DISCUSSION

As we stated earlier, there is an experimental result for this process[1], namely:

$$< \sigma >= (8.3 \ \pm \ 0.7 \ stat \ \pm \ 1.6 \ sys.) \ \times \ 10^{-40} \ cm^2.$$

This result overlaps well with the results of Eq.(21a),Eq.(21b), and also with the results of Eq.(22). We should note that the model of Kim and Mintz which produced the result given by Eq.(22) did not include a final state electromagnetic interaction between the muon and the final state nucleus. If this interaction is included the value for $< \sigma >$ increases by about 20 percent. Because the model of Kim and Mintz is a closure approximation model, it cannot be used below the giant dipole resonance and is not accurate below an excitation of approximately 40 MeV. However for this spectrum the important contributions are all above this region and so the model should give a reasonable result for the spectrum used here.

There are two other calculations for $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$. The first is a Fermi gas model calculation[1] which yielded:

$$< \sigma >= 24 \ \times \ 10^{-40} \ cm^2 \tag{23}$$

and the second is a random phase approximation calculation[2] which yielded:

$$< \sigma >= 20 \ \times \ 10^{-40} \ cm^2. \tag{24}$$

The above results are from 2.5 to 3 times the measured experimental value and are roughly twice as large as the the calculations described here and so it is reasonable to ask what might cause these differences in the results among these theoretical calculations. The first difference among these calculations is that those leading to Eq.(21a),Eq.(22b) have phenomenolgical imput in that the total muon capture rate is used to determine D in Eq.(21a) and Eq.(21b), and is directly inserted into the cross section for the method leading to Eq.(22). This is not done for the calculations leading to the results Eq.(23) and Eq.(24). Because, as we have noted earlier for the ${}^{12}C$ case[5,6,8], the q^2 dependence of D was small and the behavior of D was dominated by its value for muon capture. The question remains for what energy range this may be viewed as reasonable. We note that in the present case if we choose a neutrino energy of 175 MeV, the spectrum has already fallen to less than twenty percent of the threshold value. If we assume a reasonably generous outgoing lepton angle of 20 degrees, then $q^2 = -3166.13 \ MeV^2/c^2$. This is only about half of the $q^2 = 6280 \ MeV^2/c^2$ appropriate to the muon capture process and so we believe that it is reasonable that the muon capture result dominates in the energy range of interest here.

Another question which is not unrelated to that just considered is the contribution of higher l states to this reaction. In the calculation for the results Eq.(23) and Eq.(24) the claim is that fifty percent of this result is due to higher l states. We do not believe it to be so high in ours. We note that in a phenomenological model such as ours, the l state contributions enter through the phenomenological data used. In the case of the calculation presented here, the l state contributions enter through the coefficients α and β and therefore D. We do have some evidence[5] of an increase in D with q^2 at a faster rate than that given by Eq.(17). For example our electron scattering data indicated at $q^2 = -8,046 Mev^2/c^2$ that $D = 4.01 \times 10^9$ whereas the D of Eq.(17) would have been 3.8×10^9 but this is only about a 5 percent correction indicating only a modest increase in l state contributions in this energy range.

Finally an important factor in the calculation of any spectrum averaged quantity is obviously the accuracy to which the spectrum is known. Members[15] of the group believe that the spectrum is accurate to about 20 percent. Should the unccertainties be greater, particularly in the higher energy part, this could lead to large changes in the results given by Eq.(23) and Eq.(24). Particularly because these calculations have almost half of their contributions from higher l states, a reduction in the assumed spectrum at higher energies would significantly reduce the spectrum averaged cross section for these calculations. However even a fifty percent reduction in the spectrum above 175 MeV, where the spectrum is most uncertain is not likely to lead to agreement between these calculations and experiment. In our calculation, this reduction in the spectrum would lead to a decrease in the spectrum averaged cross section of approximately 17.6 percent. Even a decrease double this would still leave the results given here by Eq.(23) and Eq.(24) too large.

We may concluded by remarking that clearly much work needs to be done for this reaction. It would be very useful if the experimental group could isolate the transition to the ^{12}N ground state. Calculations exist for this reaction which have proved reliable[16,17,18] for the corresponding electron neutrino reaction over the Michel spectrum. An agreement between theory and experiment for this transition would confirm the expected spectrum. In any event, it is clearly premature to make any claim for neutrino oscillations based on present experimental and theoretical work for this reaction. The results surrounding the present reaction show that a complete understanding of the field of neutrino reactions in nuclei is still quite far away which ,of course, adds to the interest of the field. The present efforts on $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$ are very important however as they are serving to open a very important higher energy region for neutrino reactions in nuclei. It is to be hoped that the group will continue to accumulate data which may help in obtaining a better understanding of this reaction.

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FIGURE CAPTIONS

Figure 1. Plot of the neutrino flux versus the neutrino energy for the neutrinos produced by the in flight decay of muons. This is the spectrum used by the Los Alamos Meson Physics Facility experiment for the reaction, $\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$.

Figure 2. Plot of $\delta(E)$ as a function of electron energy. The diamonds represent points calculated from electron scattering data.

Figure 3. Plot of the charged current neutrino cross section, σ as a function of neutrino energy for the reaction,$\nu_\mu + {}^{12}C \longrightarrow \mu^- + X$. The solid curve includes the final state electromagnetic interaction of the muon and nucleus and the dotted curve excludes this final state interaction.

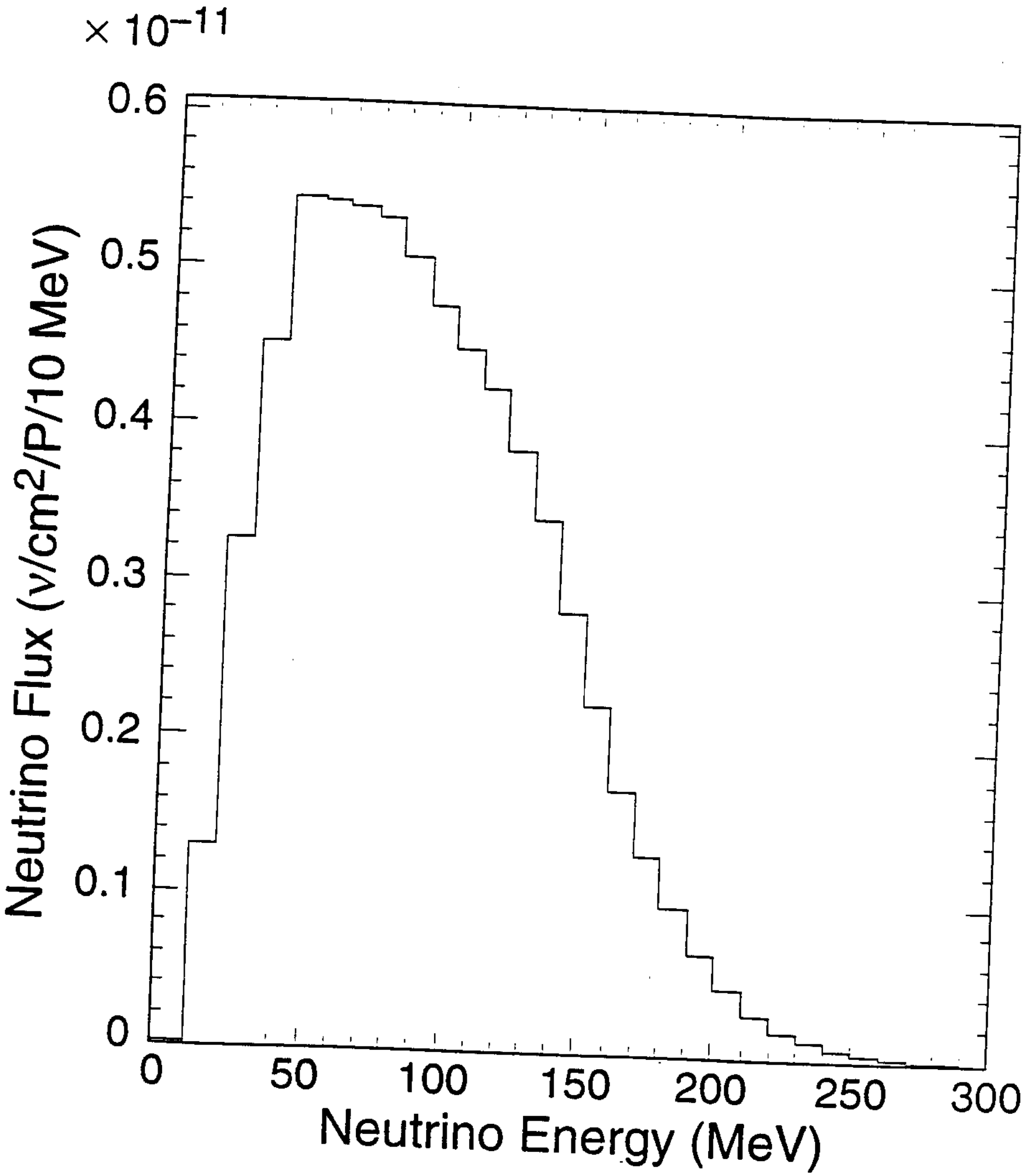
× 10^-11
0.6
0.5
0.4
0.3
0.2
0.1
0
Neutrino Flux (ν/cm²/P/10 MeV)
0
50
100
150
200
250
300
Neutrino Energy (MeV)

fig 1

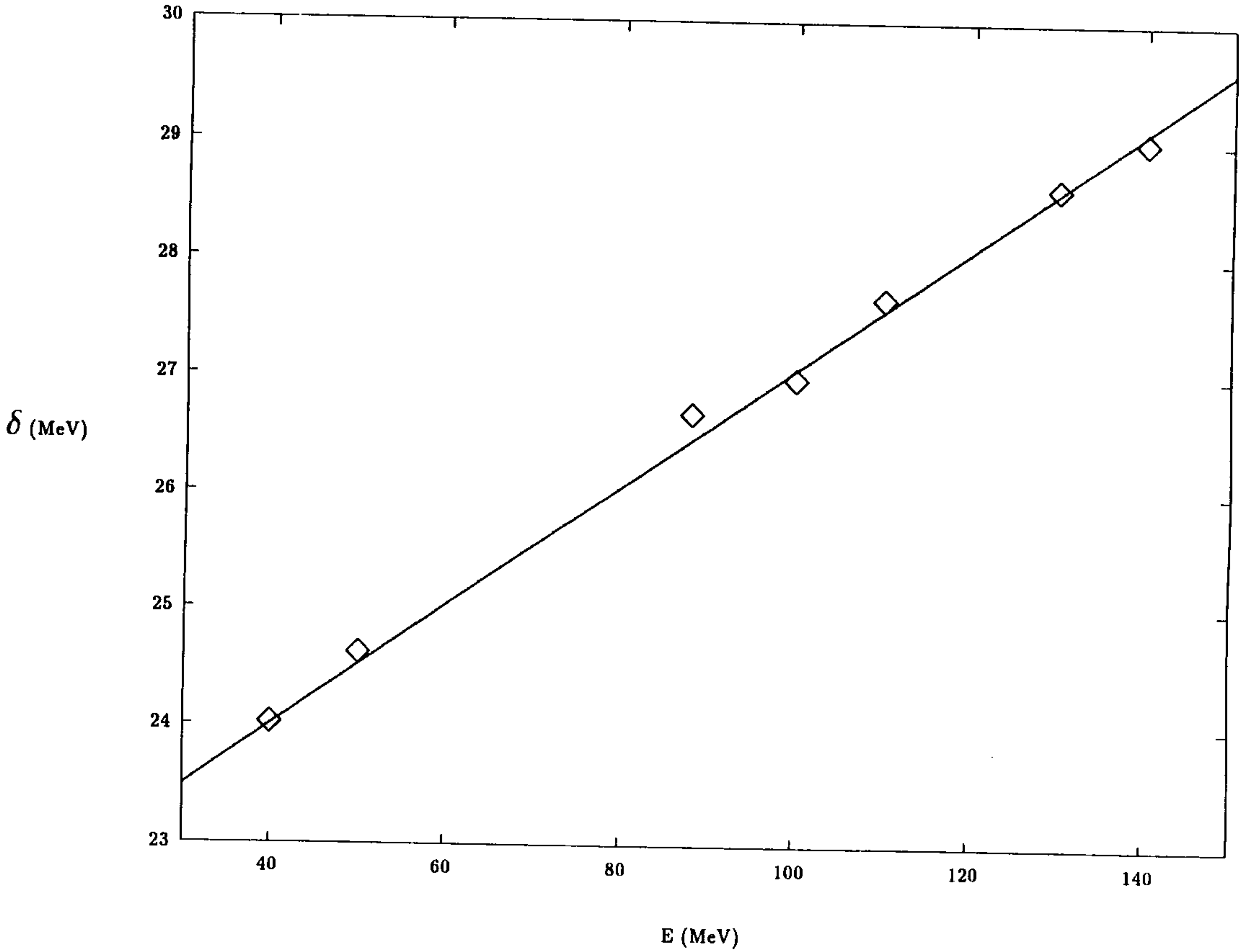

fig 2.