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RELATIVISTIC NUCLEAR HAMILTONIAN AND CURRENTS TO $(v/c)^2$

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Relativistic Nuclear Hamiltonian and Currents to $(v/c)^2$

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ABSTRACT

Relativistic Hamiltonians are defined as the sum of relativistic one-body kinetic energies and many-body interactions and their boost corrections. The calculation of the latter from commutation relations of the Poincaré group is reviewed. It is shown that the most important terms can be understood from classical relativistic mechanics. The constraints of relativistic covariance on the charge and current densities are examined. Nuclear charge and current operators that satisfy them up to order $(1/m)^2$ are derived.

INTRODUCTION

The relativistic dynamics of interacting composite objects, such as nucleons, is non-trivial, and a variety of approaches have been and are presently being developed. These include, for example, various reduction schemes of the Bethe-Salpeter equation, such as the Blankenbecler-Sugar [1] or spectator model [2] equations, and the light front Hamiltonian approach [3].

Many years ago, Bakamjian and Thomas [4], and Foldy and collaborators [5,6] showed that the dynamics of a many-body system could be described in a relativistically covariant form with the Hamiltonian

$$H_R = \sum_i \left(\sqrt{p_i^2+m^2} - m\right) + \sum_{i<j} [\overline{v}_{ij} + \delta v_{ij}(\mathbf{P}_{ij})] + \sum_{i<j<k} [\overline{V}_{ijk} + \delta V_{ijk}(\mathbf{P}_{ijk})] + \dots \ , \tag{1}$$

where $\overline{v}_{ij}$ is the two-body interaction in the rest frame of particles i and j (i.e., the frame in which their total momentum $\mathbf{P}_{ij} = \mathbf{p}_i + \mathbf{p}_j$ vanishes). Similarly, $\overline{V}_{ijk}$ is the three-body interaction in the frame in which $\mathbf{P}_{ijk} = \mathbf{p}_i + \mathbf{p}_j + \mathbf{p}_k = 0$. The $\delta v_{ij}(\mathbf{P}_{ij})$ and $\delta V_{ijk}(\mathbf{P}_{ijk})$ are called "boost interactions" and depend upon the total momentum of the interacting particles. Obviously,

$$\delta v_{ij}(\mathbf{P}_{ij}=0) = \delta V_{ijk}(\mathbf{P}_{ijk}=0) = 0 \ . \tag{2}$$

Only the positive value of $\left(p_i^2+m^2\right)^{1/2}$ is considered in H_R.

The potentials $\overline{v}_{ij}$ and $\overline{V}_{ijk}$ are determined by the fields and the internal structure, if any, associated with the interacting particles. In many cases of physical interest (in particular, that of interacting nucleons), however, the task of deriving these potentials from first principles proves to be too difficult. More simply, the $\overline{v}_{ij}$ and $\overline{V}_{ijk}$ are parametrized within a suitable theoretical framework and fitted to observed data. Once these have been constructed, the boost potentials $\delta v_{ij}(\mathbf{P}_{ij})$ and $\delta V_{ijk}(\mathbf{P}_{ijk})$ are related, in a model independent way, to $\overline{v}_{ij}$ and $\overline{V}_{ijk}$, respectively, by the requirements of relativistic covariance [6,7].

In the present talk, I will briefly review the parametrization of the nucleon-nucleon rest frame interaction and the derivation of the associated boost corrections [8–10]; succintly discuss the methods used to carry out variational calculations with the Hamiltonian (1) [8]; and report results for the binding energies of the ^{3}H and ^{4}He nuclei [8,10]. Finally, I will examine the constraints that relativistic covariance imposes on the nuclear charge and current densities [11], and list charge and current operators that satisfy them up to order $1/m^2$ included.

THE REST FRAME INTERACTION

In the two-nucleon rest frame ($\mathbf{P}_{12}=0$), the Hamiltonian (1) reads

$$\overline{H}_R = 2\left(\sqrt{p^2+m^2} - m\right) + \overline{v}_{12} \ , \tag{3}$$

with $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$. The interaction is written as

$$\overline{v}_{12} = \overline{v}_{12}^{\circ} + \overline{v}_{12}^{\pi} \ , \tag{4}$$

where $\overline{v}_{12}^{\pi}$ is the long-range one-pion-exchange potential (OPEP). In momentum space, it is taken as

$$\overline{v}^{\pi}_{12} = v^{\pi}_{12} + \Delta v^{\pi}_{12} \quad , \tag{5}$$

$$v^{\pi}_{12} = -f^2_{\pi}\, \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{q^2 + m^2_{\pi}} \quad , \tag{6}$$

$$\Delta v^{\pi}_{12} = v^{\pi}_{12} \left(\frac{m^2}{\sqrt{p^2+m^2}\sqrt{p'^2+m^2}} - 1 \right) \quad , \tag{7}$$

where $\mathbf{q}$ is the momentum transfer, and $\mathbf{p}' = \mathbf{p}'_1 = -\mathbf{p}'_2$, $\mathbf{p}'_1$ and $\mathbf{p}'_2$ being the final nucleon momenta [9,11]. Note that $\mathbf{p}' = \mathbf{p}+\mathbf{q}$. The leading term is included in all available models of nucleon-nucleon interactions. However, its correction Δv^{π}, generating a momentum-dependent tensor force, has been neglected in most of them, with the exception of the Bonn interactions [12].

The effects of all other subnucleonic degrees of freedom, such as those associated with heavy mesons and nucleonic resonances or quarks and gluons, are absorbed into $\overline{v}^{o}_{12}$. This part of the interaction has a short range, and its operator structure is taken to be the same as that of non-relativistic models [13]. It contains central, spin-spin, tensor, spin-orbit, quadratic spin-orbit and L^2- components ($\mathbf{L}$ being the relative orbital angular momentum). In momentum space, it is expressed as:

$$\overline{v}^{o}_{12} = \sum_p [v^p(q) + v^{p\tau}(q)\tau_1 \cdot \tau_2]\, O^p_{12} \quad . \tag{8}$$

The strength parameters in $v^p(q)$ and $v^{p\tau}(q)$ as well as the short range cutoff of v^{π} (not explicitly included in Eq.(6)) are determined by fitting the nucleon-nucleon phase shifts up to energies of 400 MeV in the laboratory and deuteron properties with the relativistic two-body Hamiltonian (3) [8,11]. However, rather than fitting the phase shifts obtained from the analysis of nucleon-nucleon scattering data, I have chosen to fit the phase shifts calculated with the non-relativistic kinetic energy and a non-relativistic interaction, corresponding to either the Argonne v_{14} model [14] or the charge independent part of the more recent Argonne v_{18} model [15], denoted as v^{ci}_{18}. It is important to point out that in the published version of $\overline{v}_{12}$ [8] the Δv^{π} correction was neglected. However, the full $\overline{v}^{\pi}$ has been included in the more recent $\overline{v}_{12}$ constructed so as $\overline{H}_R$ be phase-equivalent to $p^2/m + v^{ci}_{18}$. Kinetic and potential energy and OPEP expectation values in deuteron obtained with this latter model are listed in Table I [11].

The contribution from Δv^{π} is about 10 % of that from v^{π}. The results in Table I also show that the expansion of Δv^{π} in powers of p/m and p'/m is not accurate. The calculated deuteron binding energy is -2.242 MeV rather than the empirical value -2.225 MeV. This $\simeq$ 1 % difference is accounted for by electromagnetic effects not included here [15].

The effect of Δv^{π} on the binding energy of ^{3}H and ^{4}He has not yet been studied. The results reported in the rest of the present talk have been obtained with the published version of $\overline{v}_{12}$, in which, as already mentioned, this correction is ignored.

TABLE I. Kinetic and potential energy and OPEP contributions to the deuteron binding energy in MeV, obtained with the non-relativistic (NR) and relativistic (R) Hamiltonians. For the relativistic model OPEP is given by Eqs.(5-7). The contribution obtained by expanding Δv^{π} to lowest order in p/m and p'/m, labelled $\Delta v^{\pi}|_0$, is also listed. Note that the calculated binding energy is -2.242 MeV rather than the empirical value -2.225 MeV, see text for an explanation.

	NR	R
t	19.88	18.88
v	-22.12	-21.12
v^{π}	-21.36	-21.39
Δv^{π}		2.59
$\Delta v^{\pi}\|_0$		3.48

THE BOOST CORRECTIONS

Relativistic quantum mechanics

In the original paper [6], the boost interaction $\delta v(\mathbf{P})$ was formally calculated to all orders in $(P/2m)^2$. In the present discussion, however, I will retain only the leading corrections of order $(P/2m)^2$. The methods used in [6] to calculate $\delta v(\mathbf{P})$ can be succintly summarized as follows (the case of two spin one half particles is considered.)

Among the commutation relations satisfied by the generators of the Poincaré group, those involving the Hamiltonian H, the three components of the total momentum $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$, and the three components of the boost operator $\mathbf{K}$ are relevant for the derivation of $\delta v(\mathbf{P})$:

$$[K_{\alpha}\, ,\, P_{\beta}] = \mathrm{i}\, H \delta_{\alpha\beta} \quad , \tag{9}$$

$$[\mathbf{K}\, ,\, H] = \mathrm{i}\, \mathbf{P} \quad . \tag{10}$$

The generators H and $\mathbf{K}$ have interaction terms:

$$H = H_0 + H_I \quad , \tag{11}$$

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I \quad . \tag{12}$$

The non-interacting terms H_0 and $\mathbf{K}_0$ can easily be shown to satisfy relations (9) and (10). Therefore it follows that:

$$[K_{I,\alpha}\, ,\, P_\beta] = \mathrm{i}\, H_I\, \delta_{\alpha\beta} \quad , \tag{13}$$

$$[\mathbf{K}_0\, ,\, H_I] = [H_0 + H_I\, ,\, \mathbf{K}_I] \quad . \tag{14}$$

With the focus on the lowest order corrections, it is convenient to expand H and $\mathbf{K}$ in powers of $1/m$. The non-interacting H_0 and $\mathbf{K}_0$ have the well known expressions:

$$H_0 = 2m + \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + O(1/m^3) \quad , \tag{15}$$

$$\mathbf{K}_0 = -t\mathbf{P} + 2m\mathbf{R} + \frac{1}{4m}\left([\mathbf{r}_1\, ,\, p_1^2]_+ - \sigma_1 \times \mathbf{p}_1 + 1 \rightleftharpoons 2\right) + O(1/m^3) \quad , \tag{16}$$

where $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$. The first two terms in $\mathbf{K}_0$ are of the same order m. The interaction Hamiltonian is also expanded as

$$H_I = \overline{v} + \delta v(\mathbf{P}) \quad , \tag{17}$$

where the terms $\overline{v}$ and $\delta v(\mathbf{P})$ are assumed to be of order $1/m$ (or higher) and $1/m^3$ (or higher), respectively. It is further assumed that $\overline{v}$ is independent of $\mathbf{P}$.

The commutation relation (13) is then satisfied by taking

$$\mathbf{K}_I = \overline{v}\,\mathbf{R} + \mathbf{w} \quad , \tag{18}$$

where $\mathbf{w}$ is a translationally invariant vector function. It is not too hard to show that [7,9]

$$\delta v(\mathbf{P}) = -\frac{P^2}{8m^2}\overline{v} - \mathrm{i}\,[\chi_0\, ,\, \overline{v}] - \mathrm{i}\,[\chi_v\, ,\, H_0 + \overline{v}] \quad , \tag{19}$$

$$\chi_0 = -\frac{1}{16m^2}\left(\mathbf{P}\cdot\mathbf{r}\,\mathbf{P}\cdot\mathbf{p} + \mathrm{h.c.}\right) - \frac{1}{8m^2}\left(\sigma_1 - \sigma_2\right)\times\mathbf{P}\cdot\mathbf{p} \quad , \tag{20}$$

$$\chi_v = -\frac{1}{4m}\int_0^{\mathbf{P}} \mathbf{w}\cdot d\mathbf{P}' + \mathrm{h.c.} \quad , \tag{21}$$

where the line integral in (21) is independent of the path (this fact follows from $\nabla_P \times \mathbf{w} = 0$, which can be deduced from the commutation relations satisfied by the components of $\mathbf{K}$ [6,7].) The term dependent on χ_v has the form of a unitary transformation of the Hamiltonian $H_0 + \overline{v}$ [7]. It will not contribute in first order perturbation theory to the eigen-energies of $H_0 + \overline{v}$. However, special attention must be paid to it when studying reactions [16].

Relativistic classical mechanics

By evaluating the commutators in Eq.(19), the boost corrections δv to a central interaction $\overline{v}$ are found to be:

$$\delta v(\mathbf{P},\mathbf{r}) = -\frac{P^2}{8m^2}\overline{v}(r) + \frac{\mathbf{P}\cdot\mathbf{r}\,\mathbf{P}\cdot\nabla\overline{v}(r)}{8m^2} + \frac{(\sigma_1 - \sigma_2)\times\mathbf{P}\cdot\nabla\overline{v}(r)}{8m^2} \quad , \tag{22}$$

where the choice $\mathbf{w} = 0$ has been made, exploiting the unitary equivalence. The three terms above can be obtained in relativistic classical mechanics, and are attributed to the relativistic energy-momentum relation, Lorentz contraction and Thomas precession, respectively [7–9].

In classical relativistic mechanics, the energy of two particles at rest with interparticle distance r_0 is given by

$$E_0 = 2m + \overline{v}(r_0) \quad . \tag{23}$$

Consider a frame in which the two particles are moving with velocity β. In this frame the particles have momenta $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{P}/2$, energy $E_P = \left(P^2 + E_0^2\right)^{1/2}$ and $\beta = \mathbf{P}/E_P$. If the distance in the moving frame is denoted with r, then the energy E_P can also be expressed as

$$E_P = 2\sqrt{P^2/4 + m^2} + v(\mathbf{P},\mathbf{r}) \quad , \tag{24}$$

where $v(\mathbf{P},\mathbf{r})$ being the interparticle potential in the moving frame,

$$v(\mathbf{P},\mathbf{r}) = \sqrt{[2m + \overline{v}(r_0)]^2 + P^2} - 2\sqrt{P^2/4 + m^2} \quad . \tag{25}$$

Since the primary interest is in momenta $P/2m << 1$, the square roots above can be safely expanded in powers of $P/2m$. Furthermore, as shown in the next section, almost all contribution to δv comes from the region $r \gtrsim 0.7$ fm, in which $\overline{v}/2m \lesssim 5$ %. Therefore,

$$v(\mathbf{P},\mathbf{r}) = \overline{v}(r_0) - \frac{P^2}{8m^2}\overline{v}(r_0) \quad . \tag{26}$$

The relation between $\mathbf{r}_0$ and $\mathbf{r}$ is given by

$$\begin{aligned} \mathbf{r}_0 &= \mathbf{r} + \frac{(\beta\cdot\mathbf{r})\,\beta}{\beta^2}\left[\left(1-\beta^2\right)^{-1/2} - 1\right] \\ &\simeq \mathbf{r} + \frac{(\mathbf{P}\cdot\mathbf{r})\,\mathbf{P}}{8m^2} \quad , \end{aligned} \tag{27}$$

and hence to lowest order in $P/2m$ it is found that

$$v(\mathbf{P},\mathbf{r}) - \overline{v}(r) = -\frac{P^2}{8m^2}\overline{v}(r) + \frac{\mathbf{P}\cdot\mathbf{r}\,\mathbf{P}\cdot\nabla\overline{v}(r)}{8m^2} \quad . \tag{28}$$

The two terms above coincide with the first two in Eq.(22).

The last term in Eq.(22) has its origin in the Thomas precession [7,9]. The precession in a frame moving with velocity $\beta \simeq \mathbf{P}/2m$ is given to lowest order by

$$\frac{(d\beta/dt)\times\beta}{2} = -\frac{\nabla\overline{v}(r)\times\mathbf{P}}{4m^2} \quad . \tag{29}$$

Thus, the Thomas precession potential for the first particle is given by

$$-\frac{\nabla\overline{v}(r)\times\mathbf{P}}{4m^2}\cdot\frac{\sigma_1}{2} \quad . \tag{30}$$

Both particles have the same velocity due to their center of mass motion, but their accelerations due to $\overline{v}$ are equal but opposite. Therefore the Thomas precession potential for the second particle is

$$\frac{\nabla\overline{v}(r)\times\mathbf{P}}{4m^2}\cdot\frac{\sigma_2}{2} \quad . \tag{31}$$

The sum of the two makes up the last term in Eq.(22).

VMC CALCULATION OF ^{3}H AND ^{3}HE BINDING ENERGIES

The Hamiltonian in Eq.(1) is written as

$$H_R = \overline{H}_R + \sum_{i<j}\delta v(\mathbf{P}_{ij}) \quad . \tag{32}$$

Three-body boost interactions are neglected, as their contribution is expected to be very small [8]. The variational Monte Carlo calculations with $\overline{H}_R$ are analogous to those carried out with non-relativistic quantum mechanics. The variational wave functions are of the form

$$\psi = \left[S\prod_{i<j}F_{ij}\right]\phi \quad , \tag{33}$$

having a symmetrized product of correlation operators operating on an anti-symmetric uncorrelated wave function. The pair correlation operator has the structure

$$F_{ij} = \sum_p\left[f^p(r_{ij}) + f^{p\tau}(r_{ij})\tau_i\cdot\tau_j\right]O^p_{ij} \quad . \tag{34}$$

Only central, spin-spin and tensor correlations are included in the sum above. The correlation functions f^p and $f^{p\tau}$ are obtained from solutions of the relativistic two-body equation:

$$\left[2\sqrt{p^2+m^2} - 2m + \overline{v}_{12} + \lambda_{12}\right]F_{12} = 0 \tag{35}$$

in the $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$ and $^3P_2$-$^3F_2$ channels [8]. The λ's represent the modification of the bare interaction due to the presence of the other particles, and depend upon a set of parameters which are determined by minimizing the Hamiltonian $\overline{H}_R$.

The calculations are carried out in coordinate space. Contrary to naive expectations, it is rather simple to evaluate the expectation value of square-root kinetic energy operators in coordinate space with the Monte Carlo method. In ref. [8] it is shown that

$$\begin{aligned} \langle\psi|\sum_i\left(\sqrt{p_i^2+m^2} - m\right)|\psi\rangle &= (m^2/2\pi^2)\int d\mathbf{R}\sum_i\int d\mathbf{r}'_i \\ &\times\left[\psi^\dagger(\mathbf{R}) - \psi^\dagger(\mathbf{R}+\mathbf{r}'_i)\right]\frac{1}{r_i'^2}K_2(mr'_i)\psi(\mathbf{R}) \quad , \end{aligned} \tag{36}$$

where $\mathbf{R}$ is the 3A-dimensional vector $\mathbf{r}_1, \ldots, \mathbf{r}_A$ denoting the positions of all the nucleons in the nucleus, and K_2 is the modified Bessel function of order 2. The

TABLE II. Kinetic and potential energy contributions in MeV to the ^{3}H and ^{4}He binding energies obtained with non-relativistic (NR) and relativistic (R) Hamiltonians. The Coulomb potential contribution is not included in the ^{4}He results.

	^{3}H		^{4}He	
	NR	R	NR	R
t	48.7 (2)	48.7 (2)	105.2 (7)	105.0 (6)
v	-56.0 (2)	-55.9 (2)	-128.5 (7)	-127.4 (5)
V	-1.12 (2)	-1.21 (2)	-5.43 (15)	-5.89 (10)
$\overline{H}_R$		-8.41 (2)		-28.38 (10)
δv		0.34 (2)		1.76 (3)
H	-8.42 (2)	-8.07 (3)	-28.73 (8)	-26.62 (8)

two-body boost interaction contributions are evaluated in first order perturbation theory after the energy has been minimized with $\overline{H}_R$.

The results obtained with H_R are compared with those of non-relativistic calculations for the Argonne v_{14} interaction in Table II [8,10]. The Urbana model-VII three-nucleon interaction model [17] is included in both H_R and H_{NR}. The difference $\langle H_R \rangle - \langle H_{NR} \rangle$ is found to be 0.35 (2.11) MeV in ^{3}H (^{4}He), and is mostly due to the boost interaction contributions.

The individual contributions to δv are listed in Table III, where δv_{RE} denotes the first term in Eq.(19), while δv_{LC} and δv_{TP} denote those associated with χ_0, Eq.(20). The leading correction is that due to the $-P^2\overline{v}/8m^2$ term. As expected, the contribution of δv_{TP} is much smaller than those of δv_{RE} and δv_{LC}. In fact, this contribution would vanish if there were no two-nucleon P-waves in these nuclei.

TABLE III. Contributions to δv in MeV.

	^{3}H	^{4}He
δv_{RE}	0.23 (2)	1.17 (3)
δv_{LC}	0.10 (1)	0.53 (1)
δv_{TP}	0.012 (3)	0.060 (6)

It is interesting to calculate the expectation values

$$\sum_{i<j} -\frac{P_{ij}^2}{8m^2}\overline{v}_{ij}\theta(r_m - r_{ij}) \tag{37}$$

and

$$\sum_{i<j} \frac{1}{8m^2}\mathbf{P}_{ij}\cdot\mathbf{r}_{ij}\theta(r_m - r_{ij})\mathbf{P}_{ij}\cdot\nabla_{ij}\overline{v}_{ij} \tag{38}$$

as functions of the cutoff distance r_m. Their values for $r_m = 0.5, 0.7, 1$ and 2 fm are shown in Fig. 1. They indicate that most of the δv contribution comes from the region $r_{ij} = 1 - 2$ fm, in which $\overline{v}_{ij} << 2m$, thus justifying *a posteriori* the validity of the expansion in powers of $\overline{v}/2m$ used in the derivation of δv in the previous section.

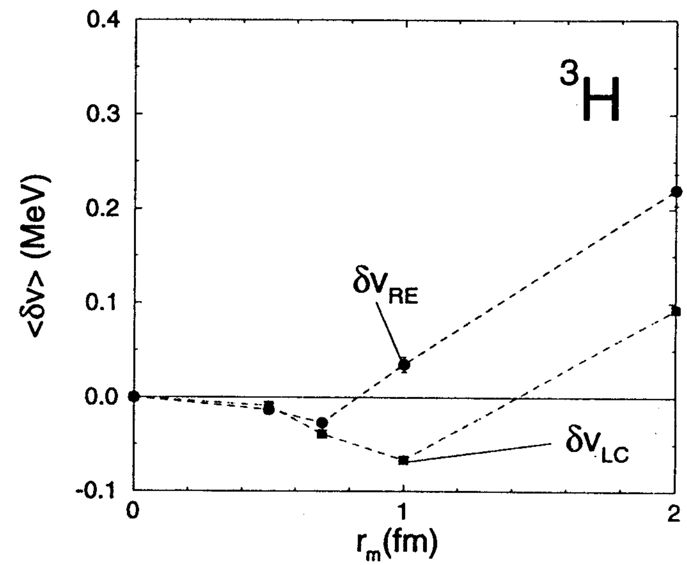

FIG. 1. Contributions of the cutoff interactions given by Eqs. (37) (circles) and (38) (squares) as function of the radius r_m. The dashed lines joining the data are purely for convenience.

Stadler and Gross [18] have estimated the boost corrections for ^{3}H within the spectator model approach, and have obtained similar results. However, Rupp and Tjon [19,20] have reported ^{3}H calculations based on the Bethe-Salpeter (BS) equation, which indicate that relativistic effects contribute about 300 keV attraction to the triton binding energy, quite different from the 350 keV repulsion found in the present work. There are two major differences between the BS and present approach. The free particle propagator used in the BS equation allows the propagation with both positive and negative energies. In contrast, the free particle propagator of the present relativistic Hamiltonian has propagation with only positive energies. Secondly, the methods used to make the interaction covariant are different. The approach discussed here may prove to be useful if the compositeness of nucleons plays an important role in suppressing the antinucleon degrees of freedom, and if classical relativistic corrections dominate.

ELECTROMAGNETIC CURRENTS AND RELATIVISTIC COVARIANCE

Relativistic covariance requires that the Fourier transforms of the charge and current densities $\rho(\mathbf{q})$ and $\mathbf{j}(\mathbf{q})$ obey the commutation relations [21,22]:

$$[\mathbf{K}(0)\,,\,\rho(\mathbf{q})] = \mathrm{i}\,\mathbf{j}(\mathbf{q}) - \mathrm{i}\,\nabla_q\,[H\,,\,\rho(\mathbf{q})] \quad , \tag{39}$$

$$[K_\alpha(0)\,,\,j_\beta(\mathbf{q})] = \mathrm{i}\,\delta_{\alpha\beta} - \mathrm{i}\,\nabla_{q,\alpha}\,[H\,,\,j_\beta(\mathbf{q})] \quad , \tag{40}$$

where $\mathbf{K}(0) \equiv \mathbf{K}(t=0)$. In addition to these relations, ρ and $\mathbf{j}$ must also satisfy the continuity equation:

$$[H\,,\,\rho(\mathbf{q})] = \mathbf{q}\cdot\mathbf{j}(\mathbf{q}) \quad . \tag{41}$$

In a relativistic theory, the interactions modify both generators H and $\mathbf{K}$, and therefore the relativistic covariance relations (39) and (40) as well as the continuity equation (41) imply the presence of interaction terms in both ρ and $\mathbf{j}$:

$$\rho(\mathbf{q}) = \rho_0(\mathbf{q}) + \rho_I(\mathbf{q}) \quad , \tag{42}$$

$$\mathbf{j}(\mathbf{q}) = \mathbf{j}_0(\mathbf{q}) + \mathbf{j}_I(\mathbf{q}) \quad , \tag{43}$$

where the subscripts denote the interaction-independent (0) and interaction-dependent (I) terms. Only the long-range pion-exchange contribution is included in ρ_I and $\mathbf{j}_I$ in the following discussion.

To lowest and next to lowest order in $1/m$, ρ_0 and $\mathbf{j}_0$ have the standard expressions in coordinate space for a Dirac nucleon:

$$\rho_0(\mathbf{q}) \simeq \rho_0^{(0)}(\mathbf{q}) + \rho_0^{(2)}(\mathbf{q}) \quad , \tag{44}$$

$$\rho_0^{(0)}(\mathbf{q}) = \exp\left(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1\right)\frac{1+\tau_{1,z}}{2} + 1 \rightleftharpoons 2 \quad , \tag{45}$$

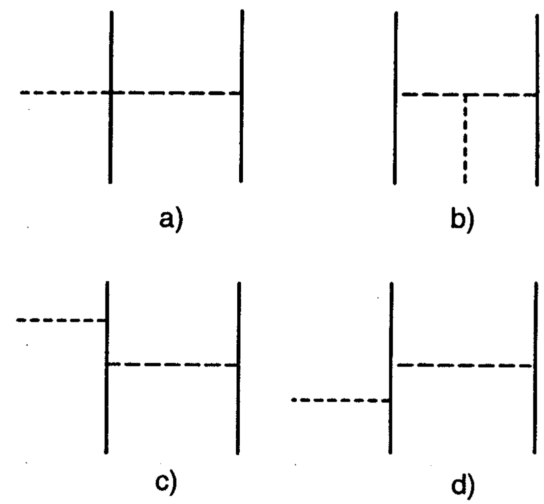

FIG. 2. Feynman amplitudes for virtual pion photo-production. Nucleons, pions and photons are denoted by solid, long-dashed and short-dashed lines, respectively.

$$\rho_0^{(2)}(\mathbf{q}) = \exp\left(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1\right)\left[-\frac{q^2}{8m^2} - \mathrm{i}\frac{\mathbf{q}\cdot(\sigma_1\times\mathbf{p}_1)}{4m^2}\right]\frac{1+\tau_{1,z}}{2} + 1 \rightleftharpoons 2 \quad , \tag{46}$$

and

$$\mathbf{j}_0(\mathbf{q}) \simeq \mathbf{j}_0^{(1)}(\mathbf{q}) \quad , \tag{47}$$

$$\mathbf{j}_0^{(1)}(\mathbf{q}) = \left[\frac{[\mathbf{p}_1\,,\,\exp\left(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1\right)]_+}{2m} - \mathrm{i}\exp\left(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1\right)\frac{\mathbf{q}\times\sigma_1}{2m}\right]\frac{1+\tau_{1,z}}{2} + 1 \rightleftharpoons 2 \quad , \tag{48}$$

where the superscript denotes the order in $1/m$. These operators satisfy relations (39-41) with the non-interacting terms H_0 and $\mathbf{K}_0$ given in Eqs.(15-16).

The pion-exchange contributions to ρ_I and $\mathbf{j}_I$ can be obtained from a nonrelativistic reduction of the virtual pion photo-production amplitudes given in Fig. 2. Assuming pseudo-vector pion-nucleon coupling, I find in coordinate space:

$$\rho_I^{(2)}(\mathbf{q}) = \rho_{I,1}^{(2)}(\mathbf{q}) + \rho_{I,2}^{(2)}(\mathbf{q}) + \rho_{I,3}^{(2)}(\mathbf{q}) \quad , \tag{49}$$

$$\rho^{(2)}_{I,1}(\mathbf{q}) = \mathrm{i}\ (f_\pi^2/2m)\,(\tau_1\cdot\tau_2+\tau_{2,z})\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1)\,(\sigma_1\cdot\mathbf{q})\,(\sigma_2\cdot\nabla)\,h_\pi(r) + 1\rightleftharpoons 2\ , \tag{50}$$

$$\rho^{(2)}_{I,2}(\mathbf{q}) = (f_\pi^2/4m)\,(\tau_1\times\tau_2)_z\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1)\,(\sigma_1\cdot\nabla)\,(\sigma_2\cdot\nabla)\,[\mathbf{p}_2\,,\,\cdot\mathbf{r}h_\pi(r)]_+ + 1\rightleftharpoons 2\ , \tag{51}$$

$$\rho^{(2)}_{I,3}(\mathbf{q}) = -\,(f_\pi^2/2m)\,(\tau_1\times\tau_2)_z\,(\sigma_1\cdot\nabla_1)\,(\sigma_2\cdot\nabla_2) \times\nabla_1\cdot\left[\mathbf{p}_1, \int d\mathbf{x}\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{x}}h_\pi(|\mathbf{x}-\mathbf{r}_1|)\,h_\pi(|\mathbf{x}-\mathbf{r}_2|)\right]_+ + 1\rightleftharpoons 2\ , \tag{52}$$

and

$$\mathbf{j}^{(1)}_I(\mathbf{q}) = \mathbf{j}^{(1)}_{I,1}(\mathbf{q}) + \mathbf{j}^{(1)}_{I,2}(\mathbf{q})\ , \tag{53}$$

$$\mathbf{j}^{(1)}_{I,1}(\mathbf{q}) = f_\pi^2\,(\tau_1\times\tau_2)_z\exp(\mathrm{i}\mathbf{q}\cdot\mathbf{r}_1)\,\sigma_1\,(\sigma_2\cdot\nabla)\,h_\pi(r) + 1\rightleftharpoons 2\ , \tag{54}$$

$$\mathbf{j}^{(1)}_{I,2}(\mathbf{q}) = -2\,f_\pi^2\,(\sigma_1\cdot\nabla_1)\,(\sigma_2\cdot\nabla_2)\,\nabla\int d\mathbf{x}\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{x}}h_\pi(|\mathbf{x}-\mathbf{r}_1|)\,h_\pi(|\mathbf{x}-\mathbf{r}_2|)\ , \tag{55}$$

where $h_\pi(r) = \exp(-m_\pi r)/4\pi$. It can be easily shown that the operators above satisfy relations (39-41) with $H_I = v^\pi_{12}$ and

$$\mathbf{K}_I = v^\pi_{12}\,\mathbf{R} + \mathbf{w}^\pi_{12}\ , \tag{56}$$

$$\mathbf{w}^\pi_{12} = \frac{f_\pi^2}{2}\tau_1\cdot\tau_2\,\sigma_1\,(\sigma_2\cdot\nabla)\,h_\pi(r) + 1\rightleftharpoons 2\ . \tag{57}$$

The charge and current seagull operators $\rho^{(2)}_{I,1}$ and $\mathbf{j}^{(1)}_{I,1}$ as well as the "pion in flight" current operator $\mathbf{j}^{(1)}_{I,2}$ have been commonly included in calculations of the elastic form factors of the A=2–4 nuclei, and deuteron electrodisintegration at threshold [23–27]. The charge operator $\rho^{(2)}_{I,2}$ arises from taking into account, in the non-relativistic reduction of the Born amplitudes *c*) and *d*) of Fig. 2, the energy dependence of the pion propagator, while the "pion in flight" charge operator $\rho^{(2)}_{I,3}$ is due to the direct coupling of the photon to the exchanged pion, diagram *b*) of Fig. 2. The effect of these operators on the isovector charge form factor of the trinucleons has not yet been studied.

The expressions for $\rho^{(2)}_I$ in Eqs.(50-52) are in agreement with those derived by Friar and collaborators [16,28], if the "equivalence-transformation-fixing" parameters μ and ν in their expressions are set equal to -1 and 1, respectively. Note that $\rho^{(2)}_I(\mathbf{q}=0) = 0$, as it should because of charge conservation, and that the $\mathbf{w}$-term in the interaction-dependent boost $\mathbf{K}_I$ is required if Eq.(39) is to satisfied.

The wave function of a nucleus with total momentum $\mathbf{P}$ is given to order $1/m^2$ by [6,7]

$$|\mathbf{P}\rangle = [1 - \mathrm{i}\,\chi_0 - \mathrm{i}\,\chi_v]\,|0,\mathbf{P}\rangle\ , \tag{58}$$

$$|0,\mathbf{P}\rangle = \exp(\mathrm{i}\mathbf{P}\cdot\mathbf{R})\,|0\rangle\ , \tag{59}$$

where $|0\rangle$ is the wave function in the rest frame of the nucleus. Neglecting the χ-terms the usual non-relativistic result is recovered. The χ_0 terms produce the Lorentz contraction of the internal wave function and the Thomas precession of the spins of the moving system. The origin of the interaction-dependent terms χ_v is less obvious [7].

It is now a simple matter to write down matrix elements of operators connecting states with initial and final momenta $\mathbf{P}_i$ and $\mathbf{P}_f$, respectively. For example, the charge form factor of a nucleus in the Breit frame is obtained as:

$$\langle\mathbf{q}/2|\rho(\mathbf{q})|-\mathbf{q}/2\rangle \simeq \langle 0,\mathbf{q}/2|\rho^{(0)}_0(\mathbf{q}) + \rho^{(2)}_0(\mathbf{q}) + \rho^{(2)}_I(\mathbf{q}) + \mathrm{i}\left[\chi_0+\chi_v\,,\,\rho^{(0)}_0(\mathbf{q})\right]|0,-\mathbf{q}/2\rangle\ . \tag{60}$$

These $1/m^2$ boost corrections need to be studied. Of course, the recent calculations of the deuteron elastic [29,30] and inelastic [29] form factors based on quasi-potential reductions of the Bethe-Salpeter equation do include boost effects to all orders. Indeed, it will be interesting to see whether these are accurately represented in the present theory, which satisfies the requirements of relativistic covariance to order $1/m^2$. Such studies are being vigorously pursued.

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