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# EUCLIDEAN RESPONSES OF $^4$He AT HIGH MOMENTUM TRANSFER

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## I. ABSTRACT

The Euclidean proton and nucleon responses of $^4$He at $q = 10\ \mathrm{fm}^{-1}$, evaluated with the Correlated Glauber Approximation using a realistic nucleon-nucleon interaction and non-relativistic kinematics, are compared to those obtained from the Green's Function Monte Carlo approach. The results show that final state interactions play a crucial role even at this large value of $q$, and that their effect can be quantitatively accounted for with the Correlated Glauber Approximation.

## II. INTRODUCTION

Assessing the validity of the Plane-Wave-Impulse-Approximation (PWIA) and the relevance of Final-State-Interactions (FSI) in inclusive electron-nucleus scattering at large momentum transfer is critical for the interpretation of the data in the low energy-loss tail of the cross section. The basic assumption underlying the PWIA is that, at large momentum transfer, electron-nucleus scattering reduces to the incoherent sum of scattering processes off single nucleons, with no FSI of the struck particle. According to this picture, the dominant reaction mechanism in the kinematical region corresponding to energy losses $\omega << \omega_{qe} \sim q^2/2m$ ($m$ denotes the nucleon mass and $\mathbf{q}$ is the momentum transfer) is the scattering off strongly correlated nucleons with high initial momentum $\mathbf{k}$. As a consequence, if the PWIA is valid, the $(e, e')$ cross section at low $\omega$ provides a measure of the high momentum components of the nuclear wave function, induced by short range nucleon-nucleon ($NN$) correlations [1]. However, the y-scaling analysis of the data for a variety of nuclear targets, extending up to momentum transfers $q \sim 2$ GeV/c [2–5], shows that the scaling limit, where the PWIA becomes applicable, has not been reached, and significant scaling violations produced by FSI occur [6]. A similar conclusion has been reached on the basis of theoretical calculations of the inclusive cross section, performed within PWIA using realistic nuclear spectral functions. This theory severely underestimates the data at low energy loss, even for the highest values of $q$ ($\sim 2.3$ GeV/c) [7–10].

A treatment of FSI in $(e, e')$ reactions at high momentum transfer, generally referred to as Correlated Glauber Approximation (CGA), has been proposed in ref. [8], and applied to the few-nucleon systems ($^2$H, $^3$He and $^4$He) [9], medium heavy nuclei [10] and infinite nuclear matter [8]. The basic assumptions implied by the CGA have also been widely employed to estimate the effect of FSI in deep-inelastic scattering of neutrons by liquid helium [11,12]. The calculations of refs. [8–10] show that in $^2$H FSI effects are unimportant, and the PWIA predictions are in fair agreement with the observed inclusive scattering cross section (ISCS). In all other nuclei, including $^3$He, the ISCS obtained with the PWIA are below the data at small $\omega$. When FSI effects are estimated using CGA and the observed free

$NN$ cross sections, the theoretical predictions are above the data. However, good agreement with the data is obtained by modifying the free space $NN$ scattering cross section to include the effect of color transparency (CT) [13,14].

The SLAC data addressed in refs. [8–10] is at $q \sim 2$ GeV/c. At these momentum transfers, most of the $eN$ scattering is inelastic, whereas elastic scattering, responsible for the ISCS at small $\omega$, is rare. It is more likely to happen when the struck nucleon is in a compact state of three valence quarks, without any virtual mesons. The FSI of the struck nucleon can therefore be different from those of a free nucleon during a small time interval after the strike. The ISCS at small $\omega$ is very sensitive to FSI in this small time interval, and can probe this difference.

The possible occurrence of CT in electron-nucleus scattering has also been investigated in the coincidence $(e, e'p)$ experiment NE18, recently carried out at SLAC. Although the NE18 data [15,16] shows no clear-cut evidence of the momentum transfer dependence characteristic of CT, the effective $NN$ cross sections extracted from the measured nuclear transparencies assuming classical attenuation of the struck nucleon, are consistently smaller than the free space cross sections. Unlike the ISCS, the nuclear transparency is sensitive to FSI over a rather large time scale, of the order of the nuclear size. It has been computed in ref. [17] using the CGA and the results suggest that, in the kinematical range of the NE18 experiment, the change in the nuclear transparency due to CT effects is only $\sim 10\%$.

The theoretical aspects of CT have been recently discussed by many authors [18,19], who have studied FSI of the compact three-valence quark state of the struck nucleon, expected to be dominant at very large momentum tranfer. The free $NN$ interaction also has contributions from the meson cloud of the nucleons, which is presumably absent in the struck nucleon. The loss of this cloud can occur at smaller $q$, and contribute to the difference between the free $NN$ interaction and the FSI of the struck nucleon. In this work we will not discuss the physical origin of this difference. Our goal is to study the accuracy of CGA, which could be useful in extracting information on this difference from the observed ISCS and $(e, e'p)$ data.

The Euclidean response is defined as:

$$E(q,\tau) = e^{q^2\tau/2m} \int_0^\infty d\omega \; R(q,\omega) e^{-\tau\omega}. \tag{1}$$

It is very sensitive to the response $R(q,\omega)$ at small $\omega$, and therefore to FSI. It is very difficult to calculate the $E(q,\tau)$ of a nucleus at large $q$ due to the presence of relativistic effects, and $eN$ and $NN$ inelastic processes. However, $E(q,\tau)$ can be computed exactly within the Green's Function Monte Carlo (GFMC) approach [20] for a model of $^4$He, consisting of point nucleons interacting via a non-relativistic Hamiltonian.

In this paper the Euclidean proton and nucleon responses of $^4$He obtained with CGA are compared to the GFMC results, with the aim of quantitatively testing the validity and accuracy of CGA. Both the CGA and GFMC calculations have been carried out using the Argonne $NN$ interaction [21] in the $v8$ form [22] and non-relativistic kinematics. The comparison is carried out at $q = 10$ fm$^{-1} \sim 2$ GeV/c, where the analysis of the SLAC data [8–10] suggests that FSI of the struck nucleon are weaker than indicated by the free $NN$ interaction.

## III. CALCULATION OF THE EUCLIDEAN RESPONSES

### A. GFMC

The proton $(x = p)$ and nucleon $(x = N)$ responses are defined as:

$$R_x(q,\omega) = \frac{1}{C_x} \sum_I |\langle I|\rho_x(\mathbf{q})|0\rangle|^2 \delta(\omega + E_0 - E_I), \tag{2}$$

where

$$\rho_x(\mathbf{q}) = \sum_{i=1}^{A} O_x(i) e^{i\mathbf{q}\cdot\mathbf{r}_i}, \tag{3}$$

with

$$O_p(i) = \frac{1}{2}[1 + \tau_3(i)], \tag{4}$$

and $O_N(i) = 1$. The normalization constants are $C_N = 4$ and $C_p = 2$ for $^4$He.

The non-relativistic Euclidean responses are defined as:

$$E_x(q,\tau) = e^{q^2\tau/2m} \int_0^\infty d\omega R_x(q,\omega) e^{-\tau\omega}, \tag{5}$$

so that for a system of free particles, $E_x(q,\tau) = 1$ at all values of $q$ and $\tau$. The value of the Euclidean response at $\tau = 0$ is given by the sum:

$$S_x(q) = \int_0^\infty d\omega \; R_x(q,\omega) \tag{6}$$

and its slope at $\tau = 0$ is related to the energy-weighted sum

$$W_x^{(1)}(q) = \int_0^\infty d\omega\, \omega R_x(q,\omega). \tag{7}$$

In the GFMC approach the Euclidean responses are computed using the expression:

$$E_x(q,\tau) = \frac{e^{q^2\tau/2m}}{C_x} \frac{\int d\mathbf{R} d\mathbf{R}' \Psi_0^\dagger(\mathbf{R}') \rho_x^\dagger(\mathbf{q}) P(\mathbf{R}',\mathbf{R};\tau) \rho_x(\mathbf{q}) \Psi_0(\mathbf{R})}{\int d\mathbf{R} \Psi_0^\dagger(\mathbf{R}) \Psi_0(\mathbf{R})}, \tag{8}$$

where $\mathbf{R}$ is a $3A$ dimensional vector specifying the positions of all nucleons, and $\Psi_0(\mathbf{R})$ is the ground state wave function obtained with the variational Monte Carlo method [23]. The imaginary time propagator

$$P(\mathbf{R}',\mathbf{R};\tau) = \langle \mathbf{R}' | e^{-(H-E_0)\tau} | \mathbf{R} \rangle, \tag{9}$$

$H$ being the nuclear Hamiltonian, is evaluated by splitting it up into many small steps $\Delta\tau$, choosing an accurate approximation to the short-time propagator, and using stochastic techniques to sample the propagator over many steps.

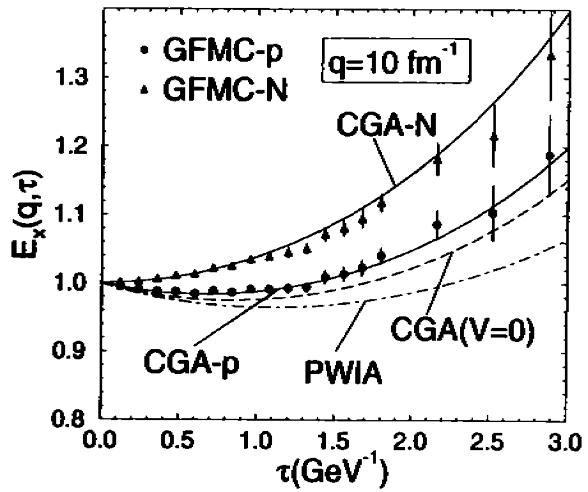


FIG. 1. The GFMC Euclidean proton (filled circles) and nucleon (filled triangles) responses are compared to the corresponding CGA results (solid lines). The CGA response obtained by neglecting the real part of the optical potential (dashed line) and the PWIA response (dash-dotted line) are also displayed.

The $^4$He proton and nucleon responses obtained with the GFMC approach at $q = 10$ fm$^{-1}$ are shown in Fig. 1. The strength in the proton response is pushed out to higher energies than in the nucleon response by charge exchange $NN$ interactions, causing $E_N(q,\tau)$ to be larger than $E_p(q,\tau)$ at $\tau > 0$. Monte Carlo calculations of the energy-weighted sums [24] also give $W_p^{(1)}(q) > W_N^{(1)}(q)$ for the same reason. The PWIA $E_p(q,\tau)$ and $E_N(q,\tau)$ are identical in $^4$He. Hence, the differences displayed by the corresponding GFMC results indicate that FSI effects are sizeable even at $q$=10 fm$^{-1}$.

In order to exhibit the sensitivity of $E_x(q,\tau)$ to the $R_x(q,\omega)$, in Fig. 2 we show the $R(q,\omega;\text{PWIA})$, and the $R_N(q,\omega)$ and $R_p(q,\omega)$ obtained with CGA. At large $\tau$ the $E_x(q,\tau)$ is more sensitive to the $R_x(q,\omega)$ at small $\omega$, where $R$(PWIA), $R_N$ and $R_p$ differ most. For example, at $\tau = 3$ GeV$^{-1}$ the contribution of the response in the tail region ($\omega \sim 1$ GeV) to the $E_x(q,\tau)$ is enhanced by a factor $e^{3.2} \sim 25$.

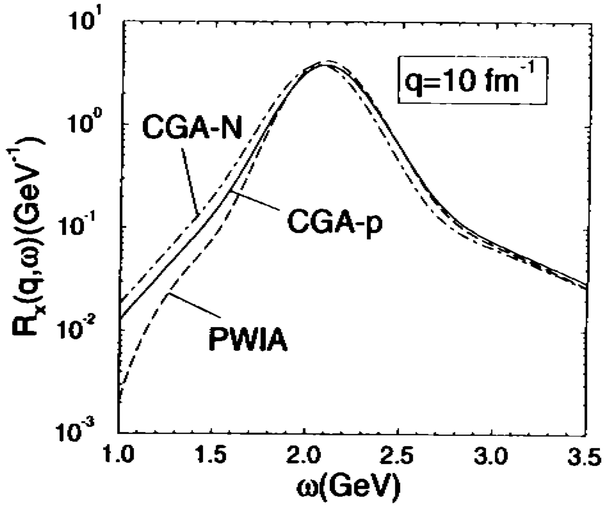


FIG. 2. The nucleon (dash-dotted line) and proton (solid line) CGA responses are compared to the PWIA predictions (dashed line).

### B. CGA

The basic assumptions underlying CGA can be better understood with the nucleon response re-written in the form:

$$R_N(q,\omega) = \frac{1}{C_N\pi} Re \int_0^\infty dt e^{i(\omega+E_0)t} \langle 0|\rho_N^\dagger(\mathbf{q}) e^{-iHt} \rho_N(\mathbf{q})|0\rangle. \tag{10}$$

Keeping only the contributions of incoherent scattering processes, which are known to be dominant at large $q$, eq.(10) becomes

$$R_N(q,\omega) = \frac{1}{\pi} Re \int_0^\infty dt e^{i(\omega+E_0)t} A(t), \tag{11}$$

with

$$A(t) = \langle 0|e^{-i\mathbf{q}\cdot\mathbf{r}_1} e^{-iHt} e^{i\mathbf{q}\cdot\mathbf{r}_1'}|0\rangle, \tag{12}$$

where the struck nucleon has been labelled with the index 1.

The Hamiltonian of the fully interacting $A$-nucleon system can be separated into two parts:

$$H = H_0 + \sum_{i=2}^{A} v_{1i}, \tag{13}$$

where the two-body interactions $v_{1i}$ are responsible for the FSI of the struck nucleon, whereas $H_0$ is the PWIA Hamiltonian:

$$H_0 = H_{A-1} + \frac{p_1^2}{2m}. \tag{14}$$

The amplitude obtained by approximating $H$ with $H_0$ in eq.(12) is denoted by $A_0(t)$. Inserting a complete set of intermediate states $|\mathbf{k}, f\rangle$, where $\mathbf{k}$ is the momentum of nucleon 1 and the label $f$ specifies the state of the $(A-1)$-nucleon spectator system, $A_0(t)$ can be written as:

$$\begin{aligned} A_0(t) &= \langle 0|e^{-i\mathbf{q}\cdot\mathbf{r}_1} e^{-iH_0t} e^{i\mathbf{q}\cdot\mathbf{r}_1'}|0\rangle \\ &= \sum_f \int d^3k |\langle \mathbf{k}, f|0\rangle|^2 e^{-i[E_f+(\mathbf{k}+\mathbf{q})^2/2m]t}, \end{aligned} \tag{15}$$

where $E_f$ includes the kinetic energy of the recoiling $(A-1)$-particle system. The nucleon spectral function is defined as:

$$P(k,E) = \sum_f |\langle \mathbf{k}, f|0\rangle|^2 \delta(E + E_0 - E_f). \tag{16}$$

Substitution of eq.(16) into eq.(15) yields

$$A_0(t) = \int d^3k dE \, P(k,E) e^{-i[E+E_0+(\mathbf{k}+\mathbf{q})^2/2m]t}, \tag{17}$$

and the PWIA nucleon response can finally be written as:

$$\begin{aligned} R(q,\omega;\mathrm{PWIA}) &= \frac{1}{\pi} Re \int_0^\infty dt A_0(t) e^{i(\omega+E_0)t} \\ &= \int d^3k dE \, P(k,E) \delta\left[\omega - E - \frac{(\mathbf{k}+\mathbf{q})^2}{2m}\right]. \end{aligned} \tag{18}$$

It can be easily verified that, in this approximation, the $R_p$ of an isospin $T=0$ nucleus like $^4$He also reduces to $R$(PWIA).

Let us now consider the structure of $A(t)$ for the fully interacting $A$-body system. The physical process described by the matrix element of eq.(12) consists of three steps: i) at time $t=0$ nucleon 1 is struck and acquires a large momentum $\mathbf{q}$; ii) the $A$-body system propagates for a time $t$ after the strike; iii) at time $t$ nucleon 1 is given a momentum $-\mathbf{q}$. The function $A(t)$ is the amplitude for the system to go back to its ground state at time $t$.

The two-body interactions $v_{1i}$ give rise to scattering processes in which nucleon 1 can exchange a momentum of order $\mathbf{q}$ with the spectator nucleons. These processes obviously reduce the probability that the $A$-nucleon system be brought back to its ground state by transferring a momentum $-\mathbf{q}$ to nucleon 1 after a time $t$. Therefore, the amplitude $A(t)$ decreases due to FSI.

The attenuation of $A(t)$ is reminiscent of the loss of incident flux in the elastic channel in nucleon-nucleus scattering. This scattering can be described in terms of a complex optical potential, whose imaginary part produces an exponential attenuation of the incident wave. It is therefore useful to extend the notation of optical model to describe FSI. The real and imaginary potentials $V_N$ and $W_N$ are defined such that:

$$A(t) = \langle 0|e^{-i\mathbf{q}\cdot\mathbf{r}_1(t)} e^{-iH_0t} e^{-i\int_0^t [V_N(t')-iW_N(t')]dt'} e^{i\mathbf{q}\cdot\mathbf{r}_1(t=0)}|0\rangle. \tag{19}$$

There are important differences between the optical potential introduced in eq.(19) and the one describing nucleon-nucleus scattering. In nucleon-nucleus collisions, the projectile particle is initially decoupled from the target, and scatters off a collection of $A$ nucleons distributed according to the nuclear density $\rho(\mathbf{r})$. In the case of the nuclear response, on the other hand, the struck nucleon scatters off the $(A-1)$-spectators, to which it was bound at $t=0$. As a consequence, denoting by $\mathbf{r}_1(t=0)$ the position of the struck nucleon at time $t=0$, the density of scatterers at position $\mathbf{r}$ is given by:

$$\xi[\mathbf{r}_1(t=0), \mathbf{r}] = \rho_2[\mathbf{r}_1(t=0), \mathbf{r}]/\rho[\mathbf{r}_1(t=0)], \tag{20}$$

where the two-nucleon density $\rho_2[\mathbf{r}_1(t=0),\mathbf{r}]$ gives the probability of finding two nucleons, one at $\mathbf{r}_1(t=0)$ and the other at $\mathbf{r}$ in the nuclear ground state. The short-range repulsive core of the $NN$ interaction makes $\xi[\mathbf{r}_1(t=0),\mathbf{r}]$ vanishingly small at small $|\mathbf{r}_1(t=0)-\mathbf{r}|$, leading to a strong suppression of FSI at $t<1$ fm.

At high $q$ one can use the eikonal approximation, according to which the struck particle moves along a straight-line trajectory with constant velocity $\mathbf{v}=\mathbf{q}/m$, to obtain the position of the struck nucleon at time $t$ as

$$\mathbf{r}_1(t)=\mathbf{r}_1(t=0)+\mathbf{v}t. \tag{21}$$

The validity of the eikonal approximation at momenta $q>1$ GeV/c has been recently shown in ref. [25], where the response of a non-relativistic model of $^4$He, obtained using the eikonal approximation, has been compared to that obtained from the exact proton propagator calculated with the path integral Monte Carlo method. In CGA it is assumed that the positions of the spectator nucleons do not change over the relevant time scale.

It should also be pointed out that the attenuation of $A(t)$ at finite $t$, induced by FSI, does not affect the total strength $S_N(q)$ of the nuclear response, given by $A(t=0)=1$. Replacing $A_0(t)$ with $A(t)$ only leads to a redistribution of strength as a function of $\omega$.

For any fixed momentum $\mathbf{q}$, the optical potential $V_N-iW_N$ depends upon the position, spin-isospin and energy of the struck nucleon. However, its time dependence is most important in evaluating $A(t)$. Eq.(19) can be greatly simplified by approximating $V_N$ and $W_N$ with average quantities $\overline{V}_N$ and $\overline{W}_N$, which depend only upon $t$. In ref. [9] $\overline{V}_N$ and $\overline{W}_N$ have been calculated by approximating the energy of the struck nucleon with $\omega_{qe}$, and averaging over the initial position $\mathbf{r}_1(t=0)$. In uniform nuclear matter $\overline{V}_N$ and $\overline{W}_N$ do not depend upon $\mathbf{r}_1(t=0)$, and there is no need to average over their values [8]. With the time dependent $\overline{V}_N$ and $\overline{W}_N$ we obtain:

$$R_N(q,\omega)=\frac{1}{\pi}Re\int_0^\infty dte^{i(\omega+E_0)t}A_0(t)e^{-i\int_0^t[\overline{V}_N(t')-i\overline{W}_N(t')]dt'}. \tag{22}$$

Using the relationship

$$\begin{aligned} e^{iE_0t}A_0(t) &= \int d\omega\ e^{-i\omega t}R(q,\omega;\mathrm{PWIA}) \\ &= R(q,t;\mathrm{PWIA}), \end{aligned} \tag{23}$$

one can re-write the nucleon response in the form

$$R_N(q,\omega)=\frac{1}{\pi}Re\int_0^\infty dte^{i\omega t}R(q,t;\mathrm{PWIA})e^{-i\int_0^t[\overline{V}_N(t')-i\overline{W}_N(t')]dt'}, \tag{24}$$

leading to the convolution expression:

$$R_N(q,\omega)=\int d\omega'\ R(q,\omega';\mathrm{PWIA})F_N(\omega-\omega')\ , \tag{25}$$

where the folding function $F_N(\omega)$ is given by

$$F_N(\omega)=\frac{1}{\pi}Re\int_0^\infty dte^{i\omega t}e^{-i\int_0^t[\overline{V}_N(t')-i\overline{W}_N(t')]dt'}. \tag{26}$$

In absence of FSI ($\overline{V}_N=\overline{W}_N=0$), $F_N(\omega)=\delta(\omega)$, and PWIA is recovered. In presence of FSI, $F_N(\omega)$ has a width due to the imaginary part $\overline{W}_N$, whereas the real part $\overline{V}_N$ produces a shift in the position of its peak, otherwise at $\omega=0$. These features can be easily seen in the case of time independent $\overline{V}_N=V_0$ and $\overline{W}_N=W_0$. The resulting $F_N(\omega)$ is a lorentzian centered at $\omega=V_0$, whose width at half maximum is given by $W_0$.

In previous applications of CGA [8-10], $\overline{W}_N(t)$ has been computed from the imaginary part of the measured $NN$ scattering amplitude [26] at incident momentum $q$ and momentum transfer $p$, $f_q(p)$, using the expression:

$$\overline{W}_N(t)=\frac{1}{A}\sum_{i=1}^{A}\sum_{j\neq i}\langle 0|w(|\mathbf{r}_i+\mathbf{v}t-\mathbf{r}_j|)|0\rangle, \tag{27}$$

where

$$w(r)=-\frac{2\pi}{m}\int\frac{d^3p}{(2\pi)^3}e^{i\mathbf{p}\cdot\mathbf{r}}Imf_q(p). \tag{28}$$

The inclusion of the imaginary part of the optical potential produces a substantial increase of the $(e,e')$ cross section at low energy loss, with respect to PWIA predictions.

In order to consistently compare the CGA to GFMC results, in the present work we compute $\overline{W}_N$ from eqs.(27) and (28) using the imaginary part of the amplitude calculated with non-relativistic quantum mechanics using the Argonne $v8$ interaction.

The proton-proton and neutron-proton cross sections at lab momentum of 2 GeV/c are, respectively, 42.8 and 39.4 mb in this non-relativistic model. They determine the volume integral of $w(r)$. The range of $w(r)$ is obtained by fitting the calculated $Imf_q(p)$ at small $p$ to the gaussian form

$$Imf_q(p)=\frac{q}{4\pi}\ \sigma\ \exp[-(\beta p)^2]\ , \tag{29}$$

the resulting value of $\beta$ being 0.3 fm. We note that the responses are most sensitive to the value of the cross section, whereas their dependence on $\beta$ is relatively weak. The neutron-proton and proton-proton $w(r)$ are not very different at 2 GeV/c lab momentum, and an average is used in the calculations. In this case, it can be easily verified that the $\overline{W}_p$ appropriate in the calculation of the proton response is identical to $\overline{W}_N$.

In ref. [8], the nuclear matter $\overline{V}_N$ has been assumed to be independent of $t$. The non-relativistic nucleon optical potential evaluated in ref. [27] has been used for nucleon momenta up to 3.5 fm$^{-1}$, whereas the results of the phenomenological Dirac model analysis of proton-nucleus scattering data [28] have been employed at larger momenta. The shift produced by $\overline{V}_N$ in the $(e,e')$ cross section for nuclear matter is small ($\sim$ 20 MeV) on the energy loss scale relevant at $q \sim 2$ GeV/c, and does not affect significantly the comparison to data. For this reason, the effect of the real part of the optical potential has been disregarded in the applications to the few-nucleon systems [9]. In this paper we will discuss the real part of the optical potential in a more detailed fashion, to point out its effects on the Euclidean responses.

In the ground state of $^4$He the spins and isospins of the four nucleons are arranged to gain maximum attraction from nuclear forces. This spin-isospin arrangement is not disturbed by the nucleon probe $e^{i\mathbf{q}\cdot\mathbf{r}_i}$, and therefore $\overline{V}_N(t)$ is large and negative at small $t$. In contrast, the proton probe $O_p(i)e^{i\mathbf{q}\cdot\mathbf{r}_i}$ partly converts the total isospin $T=0$ ground state to a $T=1$ four nucleon state, in which the nuclear forces are not as attractive. Hence, the real part $\overline{V}_x(t)$ has to be calculated separately for $x=N$ and $p$, and is responsible for the difference between $R_N$ and $R_p$ in CGA.

In the Born approximation, the real part $\overline{V}_x(t)$ is given by:

$$\overline{V}_x^B(t) = \frac{1}{C_x}\sum_{i=1}^{A}\sum_{j\neq i}\langle 0|O_x(i)v_{ij}(|\mathbf{r}_i+\mathbf{v}t-\mathbf{r}_j|)O_x(i)|0\rangle \ , \tag{30}$$

where $v_{ij}$ is the bare $NN$ interaction. The above expectation value has large contributions for $t>0$, from the repulsive core in the $NN$ interaction, whenever the struck particle closely approaches one of the spectators. On the other hand, these processes will lead to large momentum transfers between the interacting nucleons, and therefore their contributions are expected to be strongly suppressed by the imaginary part. In order to approximately include this suppression we modify the definition of $\overline{V}_x(t)$ in the following way:

$$\overline{V}_x^C(t) = \frac{1}{C_x}\sum_{i=1}^{A}\sum_{j\neq i}\langle 0|\left[\prod_{j\neq i}\frac{f_c(|\mathbf{r}_i+\mathbf{v}t-\mathbf{r}_j|)}{f_c(r_{ij})}\right]O_x(i)$$

$$\times v_{ij}(|\mathbf{r}_i+\mathbf{v}t-\mathbf{r}_j|)O_x(i)\left[\prod_{j\neq i}\frac{f_c(|\mathbf{r}_i+\mathbf{v}t-\mathbf{r}_j|)}{f_c(r_{ij})}\right]|0\rangle \ . \tag{31}$$

Here, $f_c(r)$ is the central correlation function in the ground state wave function, whose main effect is to suppress the wave function at short interparticle distances. The $\overline{V}_x^B(t)$ and $\overline{V}_x^C(t)$ obtained from eqs.(30) and (31) are displayed in Fig. 3. Note that these two estimates of $\overline{V}_x$ coincide at $t=0$. The response is most sensitive to $\overline{V}_x$ at small $t$, and is not too different for $\overline{V}_x^B(t)$ and $\overline{V}_x^C(t)$ (Fig. 4).

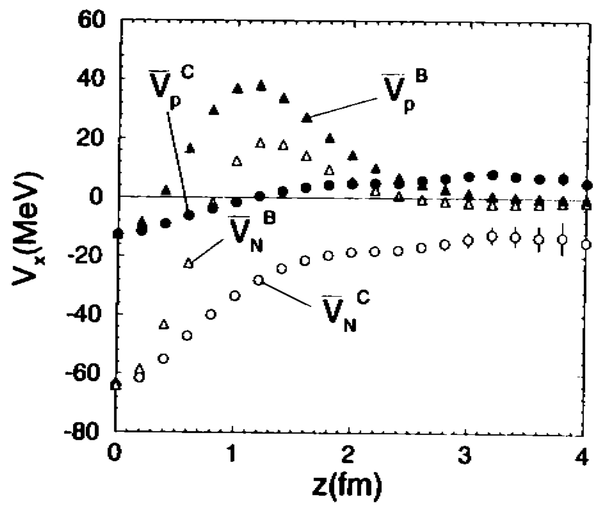


FIG. 3. The real part of the proton and nucleon optical potentials obtained from eq.(30) (filled and open triangles, respectively) are compared to the corresponding quantities obtained from eq.(31) (filled and open circles).

The energy-weighted sums of the proton and nucleon CGA responses can be easily shown to satisfy

$$\int d\omega\, \omega\, [R_p(q,\omega;\mathrm{CGA}) - R_N(q,\omega;\mathrm{CGA})] = \overline{V}_p(t=0) - \overline{V}_N(t=0) \ , \tag{32}$$

where the quantity in the rhs is a constant independent of $q$ (with the Argonne $v8$ interaction $\overline{V}_p(t=0)-\overline{V}_N(t=0) = 51.1$ MeV). It should be pointed out that the exact calculation of the energy-weighted sums $W_p^{(1)}(q)$ and $W_N^{(1)}(q)$ from

$$W_x^{(1)}(q) = \frac{1}{2C_x} \langle 0 | \left[\rho_x^\dagger, [H, \rho_x]\right] | 0 \rangle \tag{33}$$

gives $W_p^{(1)} - W_N^{(1)}$=49.6 MeV at $q$=10 fm$^{-1}$, providing further evidence for the applicability of CGA in this kinematical regime.

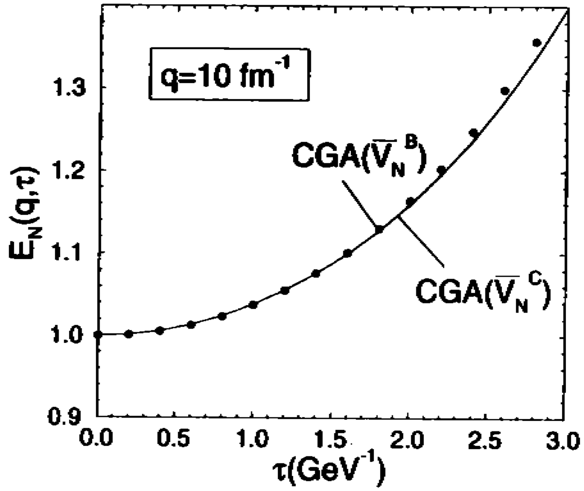


FIG. 4. The CGA Euclidean nucleon responses obtained with the real part of the optical potential evaluated from eqs.(30) (filled circles) and (31) (solid line) are compared.

## IV. RESULTS

The results reported in the present work have been obtained using the Argonne $v8$ $NN$ interaction [22]. The details of the GFMC calculation of the Euclidean responses can be found in ref. [20]. The PWIA response of $^4$He has been calculated with the spectral function of ref. [9], where $P(k,E)$ has been estimated using the momentum distributions of nucleons, deuterons, $^3$H and $^3$He in $^4$He, available from variational Monte Carlo calculations [29].

The folding function has been computed according to eq.(26). The integrations involved in eqs.(27), (30), and (31) have been carried out with the Monte Carlo method, from configurations distributed with probablity $|\Psi_0(\mathbf{R})|^2$ [29].

In Fig. 1 the CGA Euclidean responses are compared to the PWIA (dashed-dotted) and GFMC results. The curve labelled CGA(V=0) (dashed) has been obtained by setting to zero the real part of the optical potential. We note that $E_p$[CGA(V = 0)] = $E_N$[CGA(V = 0)]. The full CGA calculations (solid curves) are obtained by including the $\overline{V}_x(t)$ calculated from eq.(31). The close agreement between the CGA and GFMC results suggests that the difference between the nucleon and proton Euclidean responses is due to differences between $\overline{V}_N$ and $\overline{V}_p$. It should be noted that the effect of $\overline{V}_x$ is much more visible in $E_x(q,\tau)$ than in $R_x(q,\omega)$.

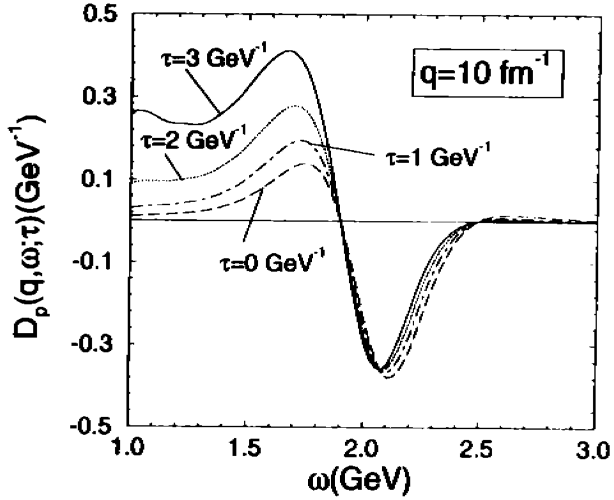


FIG. 5. The integrand given by eq.(35) at various values of $\tau$.

These results indicate that the Laplace transforms of the response calculated with the CGA are much more accurate than those obtained with PWIA. They show that

$$\int [R_x(q,\omega;\mathrm{CGA}) - R_x(q,\omega;\mathrm{PWIA})]\, e^{(q^2/2m-\omega)\tau} d\omega$$
$$\sim \int [R_x(q,\omega;\mathrm{EXACT}) - R_x(q,\omega;\mathrm{PWIA})]\, e^{(q^2/2m-\omega)\tau}\, d\omega. \tag{34}$$

The integrand of the above integral:

$$D_p(q,\omega;\tau) = [R_p(q,\omega;\mathrm{CGA}) - R_p(q,\omega;\mathrm{PWIA})]\, e^{(q^2/2m-\omega)\tau}\ , \tag{35}$$

is shown in Fig. 5 for various values of $\tau$. At large $\tau$ the integral receives its main contribution from the response at $\omega < 1.8$ GeV, suggesting that the CGA response at small $\omega$ may be fairly accurate. This region contains interesting information on both the high momentum components in the nuclear ground state and the possible modifications of the struck nucleon.

The present calculations do not test for effects of spectator correlations [31] and medium modifications of the $NN$ cross section [32], neglected in CGA. These effects are expected to be small in a nucleus of the size of $^4$He. However, recent studies [33] show that they are small even in uniform nuclear matter, where they should be more visible.

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