E_{1+}/M_{1+} and S_{1+}/M_{1+} from an Analysis of $p(e, e'p)\pi^0$ in the Region of the $\Delta(1232)$ Resonance at $Q^2 = 3.2$ (GeV/c)^2

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Abstract

We have analysed exclusive $p(e, e'p)\pi^0$ data to determine the electromagnetic and scalar transition multipoles in the mass region of the $\Delta(1232)$ at the highest Q^2 value where data exist. The ratio of the electric quadrupole and the magnetic dipole transition moments is determined to $E_{1+}/M_{1+} = 0.06 \pm 0.02 \pm 0.03$, and for the scalar quadrupole transition we find $S_{1+}/M_{1+} = 0.07 \pm 0.02 \pm 0.03$. The results rule out that perturbative QCD governs the dynamics of the $\gamma_v p\Delta$ transition at such Q^2. They are more consistent with relativistic quark model calculations using the light cone formalism.

1 Introduction

There is currently a great deal of controversy about which models are valid for describing the baryon excitation in different domains of Q^2.

In recent years there have been numerous discussions about the applicability of perturbative QCD (pQCD) methods in the description of various exclusive reactions at modestly high energy and momentum transfers. Some authors[1] [2] have argued that such a description may be applicable at momentum transfers as low as 3 to 5 $(GeV/c)^2$. Indeed, power falloff behavior consistent with pQCD predictions has been observed for the electromagnetic elastic proton form factors and other exclusive reactions. Others maintain that pQCD may be applied to exclusive reactions at much higher momentum transfers only[3][4].

We focus here on the $\gamma_v p\Delta(1232)$ transition, where dynamical quark models and pQCD make rather distinct predictions, and for which data at reasonably high Q^2 exist. A crucial test of our understanding of the Δ excitation, and the regions of validity of the different models is to determine the electric and scalar quadrupole moments E_{1+} and S_{1+}, and the magnetic dipole moment M_{1+}. The ratios of these multipoles are sensitive to fundamental ingredients of the models. For example, in $SU(6)$ symmetric quark models, the $\gamma N\Delta$ transition is mediated by a single quark spin flip in the nucleon ground state leading to M_{1+} dominance, and $E_{1+} = S_{1+} \cong 0$.

while helicity conservation in pQCD requires $E_{1+} = M_{1+}$ at $Q^2 \to \infty$. Confrontation of experimental data with theoretical models in this kinematical regime is needed and should prove quite fruitful for the development of a more realistic picture of the baryon structure at modestly high Q^2.

Unfortunately, there is a disparity between the model predictions and the analyses of electroproduction data. While models make predictions for the resonant parts of the transition amplitudes only, none of the previous analysis in the Δ region attempted to separate resonant from non-resonant contribution in exclusive pion electroproduction. Information which has been obtained from the abundant inclusive $p(e,e')X$ data collected at DESY or SLAC is very limited, even for states which are relatively separated from others, like the $\Delta(1232)$ resonance, and a determination of individual resonance transition amplitudes is not possible [5].

This motivated us to analyse the existing high Q^2 exclusive π^0 electroproduction data to determine the $\gamma p\Delta(1232)$ multipoles. This data was subject of earlier investigations[2][6]. However, no attempt at separating resonant from non-resonant contributions was made in these studies. In this letter we present the analysis method, discuss the results and compare with model predictions.

2 Method of Analysis

One of the important problems in the analysis of electroproduction data has been, how to parameterize the energy dependence of the transition amplitudes in the absence of sufficiently detailed data that would allow performing an energy-independent analysis. Several methods have been developed for the analysis of pion production data in the $\Delta(1232)$ region [7][8][9]. We have used a generalization of the phenomenological model of Walker [9] which we describe briefly.

The 5-fold differential cross section for electroproduction of single pions is given by:

$$\frac{d\sigma}{d\Omega_e dE' d\Omega_\pi} = \Gamma_T \cdot \frac{d\sigma}{d\Omega_\pi}$$

with

$$\Gamma_T = \frac{\alpha}{2\pi^2} \cdot \frac{E'}{E} \cdot \frac{(W^2 - M_p^2)}{2M_p Q^2} \frac{1}{1-\epsilon}$$

where ϵ describes the polarization of the virtual photon, W and M_p are the invariant mass of the hadronic system and the mass of the proton, respectively. The differential cross section for pion production can be written as:

$$\frac{d\sigma}{d\Omega_\pi} = \sigma_T + \epsilon\sigma_L + \epsilon\sigma_{TT} cos2\phi + \sqrt{\epsilon(1+\epsilon)/2}\sigma_{TL} cos\phi$$

where σ_T is the cross section for the absorption of unpolarized transverse photons, σ_L the cross section for the absorption of longitudinal photons, σ_{TT} is a transverse-transverse interference term, and σ_{TL} is a longitudinal-transverse interference term.

The cross section can be expressed in terms of 6 parity conserving helicity amplitudes H_i [9]:

$$\sigma_T = 1/2 \cdot F \cdot (|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2)$$
$$\sigma_{TT} = 1/2 \cdot F \cdot Re(H_2 H_3^* - H_1 H_4^*)$$
$$\sigma_L = F \cdot (|H_5|^2 + |H_6|^2)$$
$$\sigma_{TL} = \sqrt{2} \cdot F \cdot Re[H_5(H_4^* - H_1^*) + H_6(H_3^* + H_2^*)]$$

with

$$F = \frac{2MW|\vec{q}_\pi^*|}{W^2 - M^2}$$

where $\vec{q}_\pi^*$ is the pion 3-momentum in the hadronic cms frame. We define $\bar{H}_i = H_i - H_i^{Born}$, where H_i^{Born} are the helicity amplitudes of the familiar electric Born terms. The Born terms are subtracted as well known contributions, and also to avoid unreasonably high partial waves from the non-resonant terms in the partial wave expansion. The $\bar{H}_i$ are then expanded in terms of Legendre Polynomials for the pion orbital angular momentum $l_\pi = l, l+1$, and the total resonance spin $J = l_\pi \pm 1/2$:

$$\bar{H}_1 = \frac{1}{2}\sqrt{2} sin\theta cos\frac{\theta}{2} \sum_{l=1}^{\infty}(B_{l+} - B_{(l+1)-}) \cdot (P_l'' - P_{l+1}'')$$

$$\bar{H}_2 = \sqrt{2} cos\frac{\theta}{2} \sum_{l=0}^{\infty}(A_{l+} - A_{(l+1)-}) \cdot (P_l' - P_{l+1}')$$

$$\bar{H}_3 = \frac{1}{2}\sqrt{2} sin\theta sin\frac{\theta}{2} \sum_{l=1}^{\infty}(B_{l+} + B_{(l+1)-}) \cdot (P_l'' + P_{l+1}'')$$

$$\bar{H}_4 = \sqrt{2} sin\frac{\theta}{2} \sum_{l=0}^{\infty}(A_{l+} + A_{(l+1)-}) \cdot (P_l' + P_{l+1}')$$

$$\bar{H}_5 = \sqrt{2} cos\frac{\theta}{2} \sum_{l=0}^{\infty}(C_{l+} - C_{(l+1)-}) \cdot (P_l' - P_{l+1}')$$

$$\bar{H}_6 = 2 sin\frac{\theta}{2} \sum_{l=0}^{\infty}(C_{l+} + C_{(l+1)-}) \cdot (P_l' + P_{l+1}')$$

The partial wave helicity elements A_{l+}, B_{l+}, C_{l+}, $A_{(l+1)-}$, $B_{(l+1)-}$, and $C_{(l+1)-}$ are linear combinations of the electromagnetic multipoles for the transition from the ground state into the final πN state. For the $\Delta(1232)$, the multipoles E_{1+}, M_{1+}, S_{1+} are related to the partial wave helicity elements like:

$$2A_{1+} = M_{1+} + 3E_{1+}$$

$$B_{1+} = E_{1+} - M_{1+}$$

$$C_{1+} = 2\frac{\sqrt{Q^2}}{|\vec{Q}^*|} S_{1+}$$

where $\vec{Q}^*$ is the photon 3-momentum in the cms frame. We use the following ansatz for the partial wave helicity elements, e.g.:

$$A_{l+} = A_{l+}^{BW} + A_{l+}^{BG}, \quad etc.$$

The helicity elements are described by sums of relativistic Breit-Wigner amplitudes with energy-dependent widths, and a phenomenological background contribution. Similar to the Born terms, the background terms are assumed as real amplitudes. The energy dependence of the resonant terms is parameterized as a relativistic Breit-Wigner amplitude with momentum-dependent widths, e.g.:

$$A_{l+}^{BW}(W) = A_{l+}(W_R) \cdot \sqrt{\frac{|\vec{Q}_R^*||\vec{q}_\pi^{*R}|}{|\vec{Q}^*||\vec{q}_\pi^*|}} \cdot \frac{\sqrt{\Gamma_\gamma \cdot \Gamma}}{W_R^2 - W^2 - iW_R\Gamma},$$

where the index R denotes that the value be taken at the resonance mass. The amplitudes for a specific resonance transition are assumed to have the same phases. However, due to the non-resonant contributions, the helicity amplitudes for $p\pi^0$ production of a given orbital angular momentum will in general have different phases. This is allowed by Watson's theorem which constrains the phases of amplitudes for eigenstates of isospin. The $p\pi^0$ channel by itself is not an eigenstate of isospin, and since there are no $n\pi^+$ data available, this constraint could not be implemented. Such constraints have been used in the analysis of photoproduction data [8].

The momentum-dependence of Γ and Γ_γ is parameterized according to Walker[9]. The background parameterization gives the correct $|\vec{q}_\pi^*|^l$ threshold behavior[10], and a dipole-like form factor fall-off with $\vec{q}_\pi^2$, e.g.

$$A_{l+}^{BG} = \frac{|\vec{q}_\pi^*|^l}{(|\vec{q}_\pi^*|^2 + (0.35GeV/c)^2)^{l/2}} \cdot \frac{a_{l+}}{(1 + |\vec{q}_\pi^*|^2/0.71)^2}$$

where a_{l+} is a fit parameter. A monopole-like form factor, which has the correct asymptotic behavior, yields results which are not significantly different but have a slightly larger χ^2. In particular, the ratios $R_{EM} = E_{1+}/M_{1+}$ and $R_{SM} = S_{1+}/M_{1+}$ show a weak sensitivity to the specific background parameterization, which gives us confidence in the stability of the results. Each resonant and each background helicity element is described by one amplitude which is determined by a fit to the data. The $\gamma_v p\Delta(1232)$ multipoles are found to be rather insensitive to the specific choice of the scale factor $(0.35GeV/c)^2$, which we fixed at the empirical value found by Walker for the higher mass resonant amplitudes.

In order to account for contributions from higher mass resonances, partial waves up to $l_\pi = 3$ and $J = 5/2$ were used in fitting the data.

3 Discussion

The data used in the analysis are differential cross sections from DESY [6] on π^0 and η production off protons, at fixed $Q^2 = 3.2\ (GeV/c)^2$. The data extend over an invariant mass range of 1.145 GeV/c^2 to 1.715 GeV/c^2. The only clear structure in the $p\pi^0$ invariant mass is the $\Delta(1232)$ peak. Consequently, it is difficult to extract resonant contributions of any of the higher mass resonances from the $p\pi^0$ data alone. To obtain some sensitivity to the mass region around 1.5 GeV/c^2, $p(e,e'p)\eta$ data were analyzed in a two channel fit. Inclusion of the η data in the analysis made it possible to account for η threshold effects (cusp) in the $p\pi^0$ channel. This is accomplished by analytical continuation of the $p\eta$ amplitude into the unphysical region below threshold[11]. The enhancement seen in the data and in the fit near $W \simeq 1.49$ GeV/c^2 is due to the η threshold. The large error bars and the low sensitivity of the $p(e,e'p)\pi^0$ process to isospin 1/2 states (note that most of the prominent resonances in the mass region from 1.4 to 1.7 GeV/c^2 are I = 1/2 states) prevented the extraction of the higher mass resonance contributions with acceptable accuracy. Since we are primarily interested in determining the $\gamma_v p\Delta(1232)$ transition amplitudes, the final fit was performed with only four resonant states included, the $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$, and $N^*(1650)$. The A_{0+} amplitude for the $\gamma_v pN^*(1650)$ was parameterized using symmetry relationships in the single quark transition model[12], which allows excitation of the $N^*(1650)$ from protons only due to configuration mixing between the two quark model states $[^2 8]$ and $[^4 8]$. An empirical mixing angle of $\theta_m = 38^o$ was used. With the exception of the $\Delta(1232)$ mass, the resonance masses and widths were fixed using the values compiled by the Particle Data Group[13]. Only the mass of the $\Delta(1232)$ was allowed to vary. The best fit was obtained with an approximately 20 MeV/c^2 higher mass than the PDG value. Although the extracted helicity elements for the Δ depended somewhat on the mass, the ratios R_{EM} and R_{SM} did not change significantly when the mass of the $\Delta(1232)$ was fixed at its nominal value.

A total of 47 $p(e,e'p)\eta$ data points, and 636 points from $p(e,e'p)\pi^0$ were included in the fit. The normalized χ^2 varied typically from 0.93 to 1.1, depending on which higher resonant contributions were included, or which background parameterization was used. The χ^2 value reflects the large statistical error bars. Because of the large statistical uncertainties, no attempt was made to extract amplitudes for weak resonances such as the $N^*(1440)$. The obtained resonant A_{1-} and C_{1-} amplitudes for the $N^*(1440)$, were very sensitive to the specific ingredients in the fit. Figure 1 and figure 2 show examples of the π^0 differential cross section data together with our fit.

Our results for R_{EM} and R_{SM} are summarized in table I together with the prediction of various models. For the M_{1+}, which describes the dominant magnetic dipole transition to the $\Delta(1232)$, we find the value $|M_{1+}| = 0.488 \pm 0.013 \pm 0.017\ \sqrt{\mu b}$ at $Q^2 = 3.2 (GeV/c)^2$. The systematic uncertainties were estimated using various parametrizations of the non-resonant amplitudes and by including or omitting other resonant states. The fit results for R_{EM} and R_{SM} are stable within the given systematic uncertainties.

It has been asserted[2] that the $p(e,e'p)\pi^0$ data at $Q^2 = 3.2$ $(GeV/c)^2$ in the $\Delta(1232)$ region are consistent with the hypothesis that perturbative QCD governs the $\gamma_v p\Delta(1232)$ transition. In order to verify this result we made fits with the

	R_{EM}	R_{SM}
Fit results	0.06±0.02±0.03	0.07±0.02±0.03
LCQM[14]	0.11	
NRQM[15]	-0.046	
RQM[15]	-0.004	
RQM[16]	-0.02	$\simeq$ 0.00
Diquark[17]	0.063	0.66
VDM[18]	0.026	-0.050

Table 1: Fit results at $Q^2 = 3.2$ $(GeV/c^2$ with fit errors (first term) and systematic uncertainties. Note that in the pQCD limit R_{EM}=1 and R_{SM}=0.

constraint $B_{1+} << A_{1+}$ for the resonant helicity elements, which is the pQCD prediction for asymptotic values of Q^2. These fits resulted in a large χ^2 in the region of the Δ and failed to reproduce the angular dependence or the shape of the W dependence of the data. Our analysis does clearly not support the interpretation of this transition in terms of pQCD.

4 Summary

Our analysis of the $p(e, e'p)\pi^0$ data at $Q^2 = 3.2$ (GeV/c)^2 gives small positive value for R_{EM} and R_{SM}. At $Q^2 = 0$, R_{EM} was found to have a small negative value[8]. Our results therefore indicate a zero crossing of R_{EM} at $Q^2 < 3$ $(GeV/c)^2$. The small value of R_{EM} shows that leading order pQCD contributions are still small at such Q^2. R_{EM} is in reasonable agreement with relativistic calculations on the light cone[14] and relativised version of of the non-relativistic quark model[15], while extrapolations of the non-relativistic quark model to such high Q^2 values are not supported by our analysis. R_{SM} also remains small, which is qualitatively in agreement with quark models. Our value for R_{EM} is in agreement with predictions of a quark-diquark model[17], which was constructed to explain the inclusive Δ transition form factor at high Q^2, however the predicted large value of R_{SM} is not compatible with our result. This demonstrates the importance of using separated multipoles for discriminating between models.

More complete data sets, both in the $p(e, e'p)\pi^0$, as well as in the $p(e, e'\pi^+)n$ channel are needed to reduce the sizeable statistical and systematic errors. In particular, measurement of different isospin channels would allow implementation of unitarity constraints using the Fermi-Watson theorem. It will be important to study the smaller Q^2 range, as well as to improve on the quality of the high Q^2 data. Also, data extending to higher Q^2 values are needed for a systematic study of the Q^2 evolution of R_{EM} and R_{SM}.

High precision data over a large kinematical regime are expected to come from the new continuous wave electron accelerators CEBAF in Newport News, and MAMI-B in Mainz[19].

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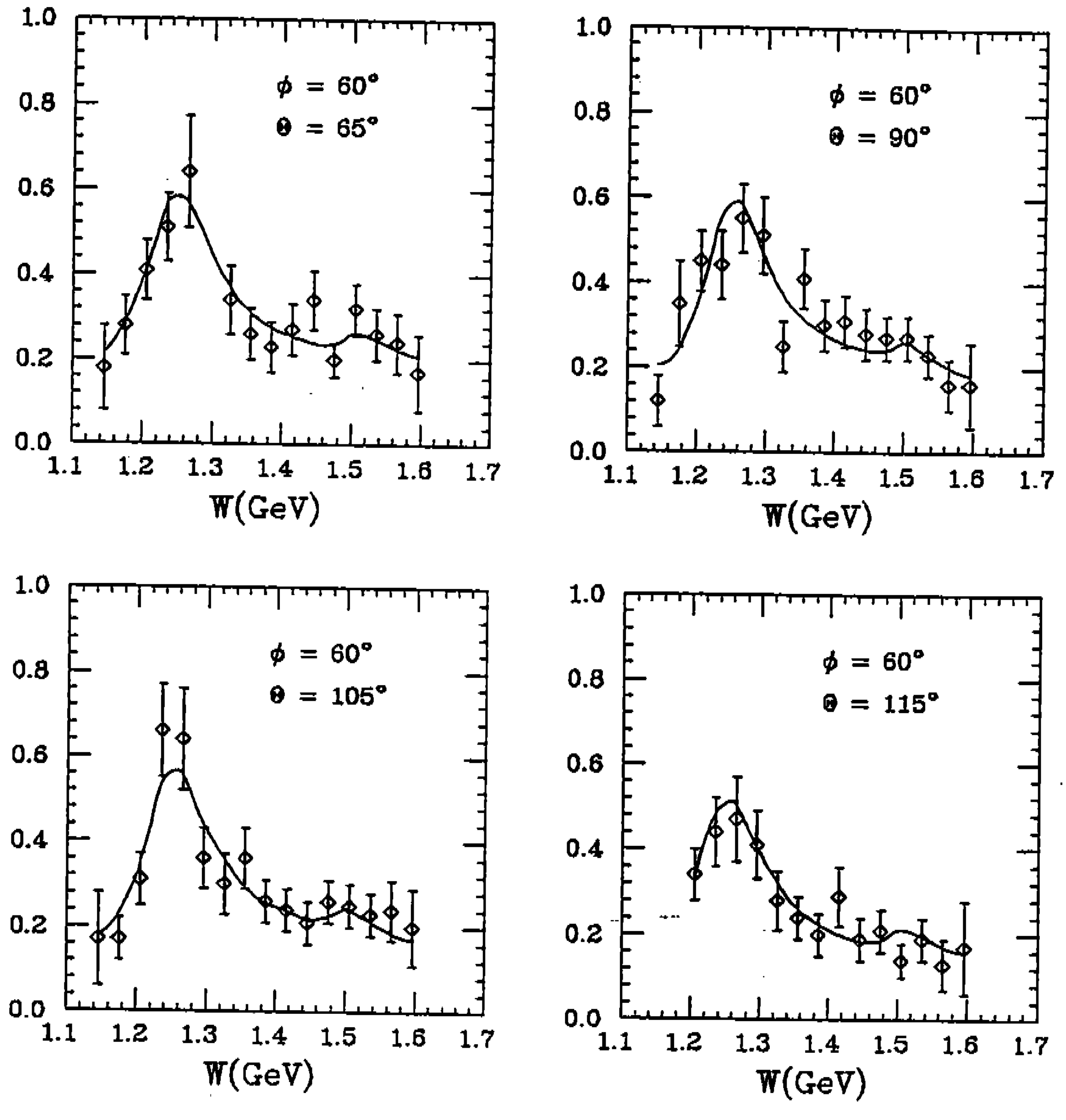
Figure Captions

Figure 1: Examples of our fit to the π^0 electroproduction data in the W plane ($\mu barn/sterad$) for different θ and ϕ bins.

Figure 2a,b: Examples of our fit to the pion center-of-mass angular distribution at fixed values of W and ϕ.



ϕ = 60°
θ = 65°
ϕ = 60°
θ = 90°
ϕ = 60°
θ = 105°
ϕ = 60°
θ = 115°
W(GeV)
W(GeV)
W(GeV)
W(GeV)

Figure 1

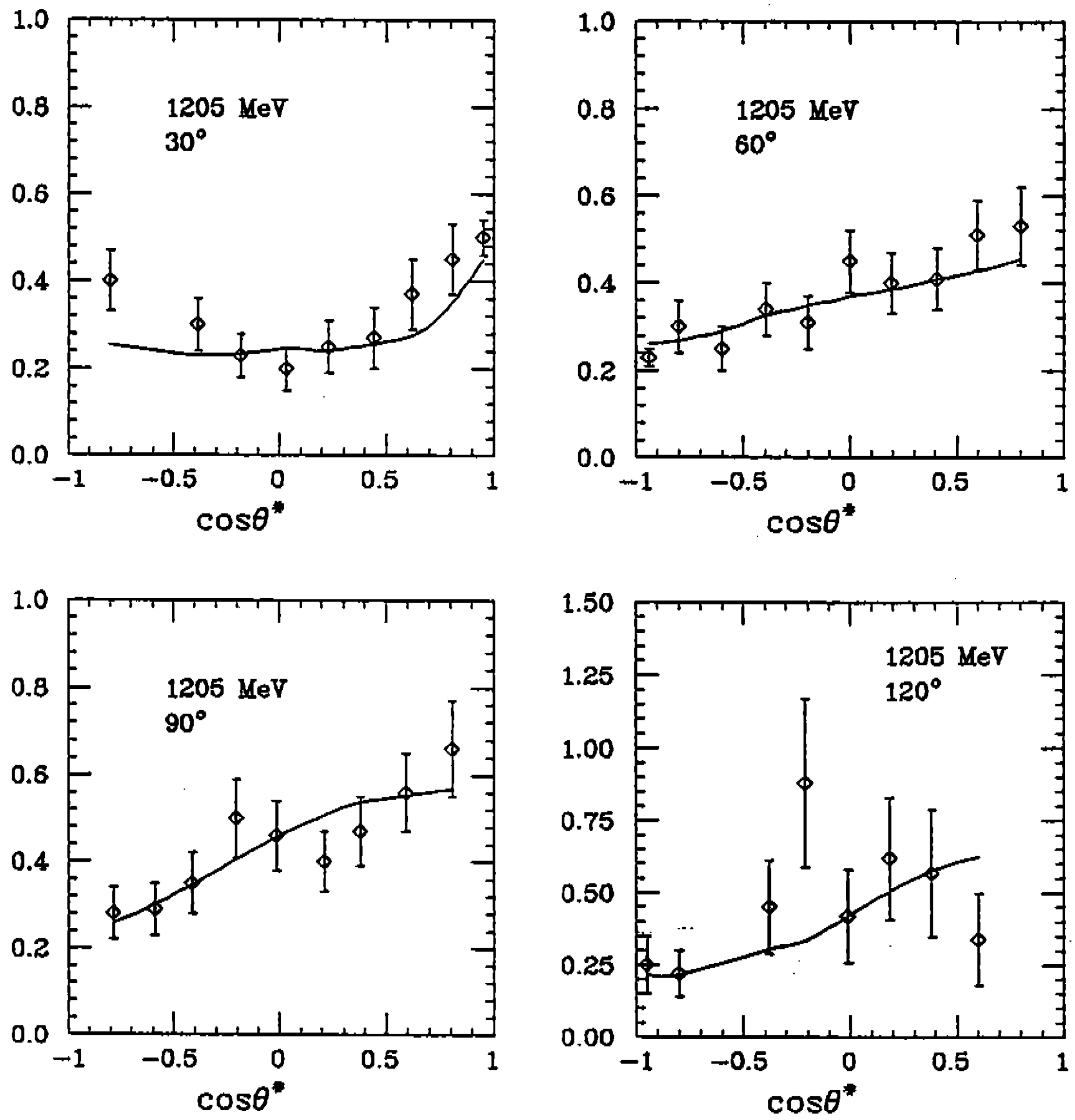

1205 MeV
30°
1205 MeV
60°
1205 MeV
90°
1205 MeV
120°
cosθ*
cosθ*
cosθ*
cosθ*

Figure 2a

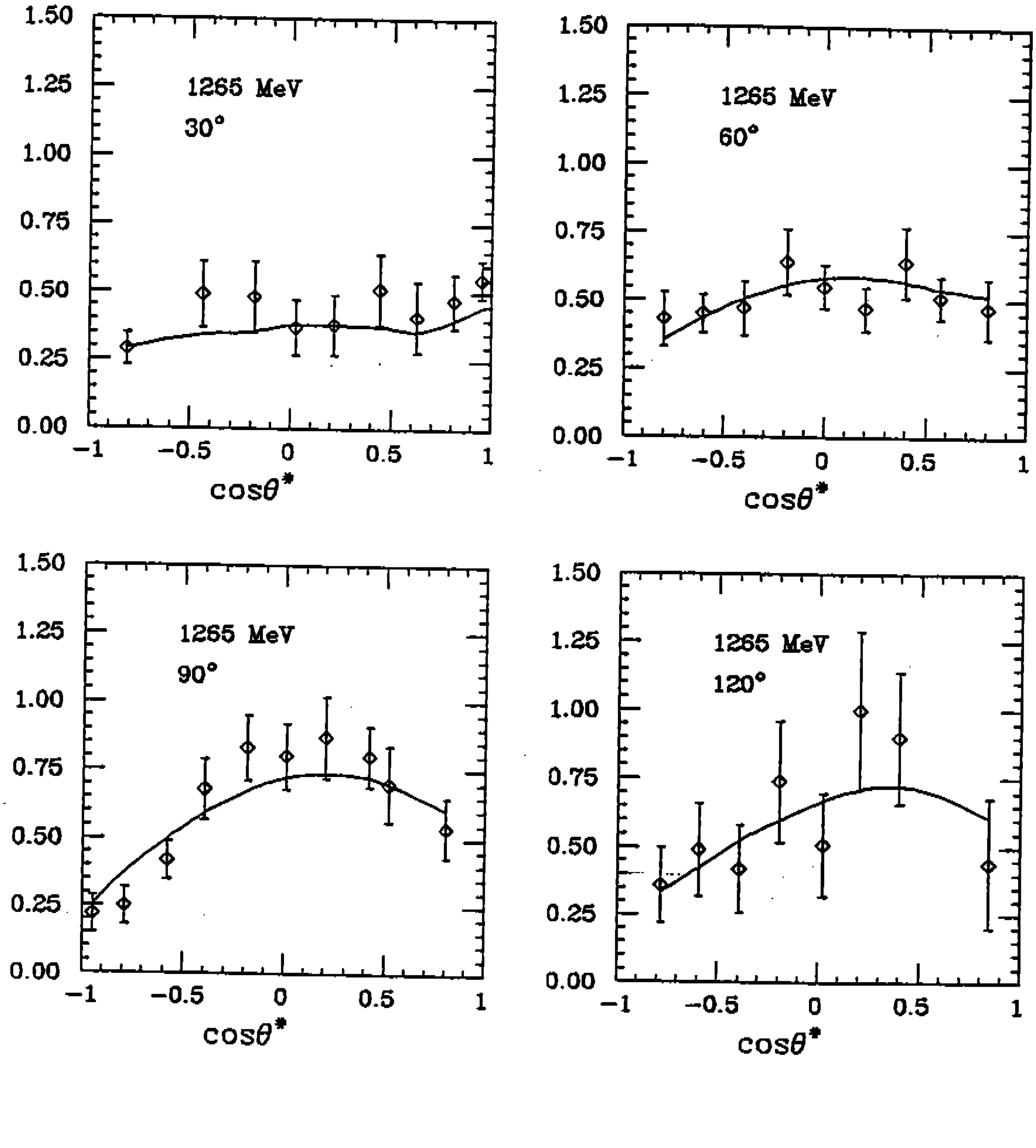


Figure 2b