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A Spinor Technique in Symbolic Feynman Diagram Calculation

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Abstract

We present a recursive diagrammatic method for evaluating tree-level Feynman diagrams involving multi-fermions which interact through gauge bosons (gluons or photons). Based on this method, a package called *COMPUTE*, which can generate and calculate all the possible Feynman diagrams for exclusive processes in perturbative QCD, has been developed (available in both *Mathematica* and *Maple*). As an example, a calculation of the nucleon Compton scattering amplitude is given.

I. INTRODUCTION

It can hardly be overemphasized that the evaluation of Feynman diagrams is one of the most common type of calculation that high-energy, nuclear and solid-state physicists

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*address before January 1, 1994.

encounter nowadays. It is therefore very useful to develop methods that can simplify the calculations. Casimir's trick of squaring the amplitude and turning it into a trace has been by far the most commonly used tool, especially when it is required just to find the spin average of the square of the amplitude. However, there are many cases where we need the amplitudes themselves rather than the square of amplitudes, such as in exclusive hadron scattering processes [1], which we will discuss. Furthermore, we need the amplitude for each helicity configuration to calculate the polarized exclusive scattering processes. On the other hand, we often find that in exclusive processes many diagrams contribute already in the lowest order. Thus, our aim is to devise techniques that can simplify the calculation of Feynman diagram amplitudes and can be easily implemented onto a computer.

Bjorken and Chen [2] are the first to evaluate the amplitude itself. Farrar and Neri [3] took the first innovative step to perform the entire calculation analytically by computer. They have developed several codes to evaluate symbolically all possible Feynman amplitudes that contribute to the hard scattering amplitude in QCD exclusive processes. Their most important step was to make use of the Fierz identities in the Weyl representation to perform the contractions of the gamma matrices to simplify the spinor algebra. There are other spinor techniques available [4–8] and most of them employ closely related two-component Weyl-var der Waerden spinor formalism.

In this paper, we present an alternative technique. The basic idea is to rewrite the fermion propagators in terms of spinors, so that a multi-fermion diagram can be expressed as a product of several elementary diagrams of identical type, i.e. the diagram given by two-fermions connected by one gluon exchange, which we call one-gluon-exchange diagram. The plan is to calculate the one-gluon-exchange diagram or the basic building block once and for all and obtain the full amplitude of multi-fermion diagram immediately by multiplying these building blocks.

Compared with other techniques, the one presented here is more transparent and easier to understand and implement, because it is not necessary to do any further manipulation of the fermion-strings such as the evaluation of the strings remained after the contractions of the gamma matrices in Ref. [3].

When it comes to the actual computer implementation, there are generally two approachs: one is to write a specifically tailored program and the other is to develop a program for more general purposes. The *Schoonship* [9] is an example of the first approach with all its necessary algorithms "hard-wired" into assembly code. While the advantage of the first approach is that the performance can be very fast, the disadvantage is that it cannot be easily extended and accessed by wider readers. We therefore follow the second approach and have implemented our spinor techniques onto some standard platform. A package called *COMPUTE* [10,11] had been developed in both *Mathematica* [12] and *Maple* [13], which are two of the most promising symbolic languages capable of providing a powerful environment in which results can be evaluated numerically and manipulated in various ways. Recently, several programs have also been developed using a similar approach, namely, *HIP* [14] with an emphasis on tree-level multiparticle electroweak production processes and *FeynArts* and *FeynCalc* [15,16] to facilitate the calculation of radiative corrections.

Another invaluable asset in *COMPUTE* is the capability of generating all possible Feynman diagrams in a given process. For the multi-fermion diagrams, each fermion is assigned by a number from 1 to n where n is the total number of fermions. Then, the gauge boson interactions among the fermions can be represented by an array of these numbers. For example, the one gluon exchange between ith quark and jth quark can be represented by (i,j) and the tripe gluon vertex connecting three quarks i,j,k can be represented by (i,j,k), etc. Since each Feynman diagram is assigned by a particular combination of these arrays, all possible diagrams can in principle be generated by their permutations and combinations. We have developed such algorithm [11] and implemented in our package to combine with the spinor technique discussed in this paper.

COMPUTE is an on-going project that aims at developing a symbolic package that can

facilitate perturbative QCD calculations for exclusive processes. We employed the present version of the package to generate and calculate all 378 Feynman diagrams for nucleon Compton scattering. On a *SUN Sparc* workstation, this took about two CPU hours and ten MBytes memory. Even though our package does not have any intrinsic limit on the number of external fermions, we were able to calculate the processes which require the number of external fermions only up to five due to the limited CPU hours in our local computers. As the number of external particles increases, there is a rapid proliferation of diagrams. However, we think that the performance of the code could be much more improved by further optimizations and such improvement would be crucial to handle the cases involving more than five external fermions.

Since the color factors for many diagrams are the same and can be calculated easily by hand using the tricks presented in Ref. [17], we will restrict ourselves to the color-independent part of the amplitude. In principle, one can implement the algorithm in Ref. [17] to generate the color factors as well.

We present our spinor techniques in Section II and illustrate our method by calculating the exclusive nucleon compton scattering amplitude in Section III. For definiteness, our discussion of the program will be based on the *Mathematica* version. The conclusion is followed in Section IV. The lists of input and output of our example are given in the Appendix.

II. SPINOR TECHNIQUES

The crux of the method is to observe that amplitude corresponding to a generic gluon exchange diagram such as Fig. 1(a) contains a products of "fermion-strings" of the form

$$\overline{u}_{\mu_1}(l_1)\gamma^\mu u_{\lambda_1}(k_1)\overline{u}_{\mu_2}(l_2)\gamma^\nu(\not p + m)\gamma_\mu u_{\lambda_2}(k_2)\overline{u}_{\mu_3}(l_3)\gamma_\nu u_{\lambda_3}(k_3) \tag{1}$$

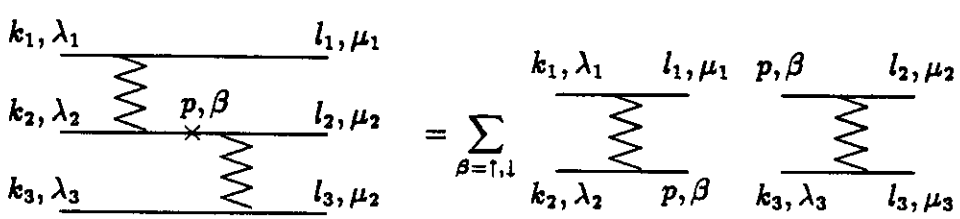
in Feynman gauge. As we will see later, the numerator factor $\not p + m$ from the fermion propagator is given by the sum of the on-shell spinor product and the off-shell spinor product. However, as a starting point for illustration purpose, let's first take only the on-shell spinor product, i.e.

$$\not p + m = \sum_{\beta=\uparrow,\downarrow} u_\beta(p)\overline{u}_\beta(p), \tag{2}$$

then Eq.(1) can be rewritten as

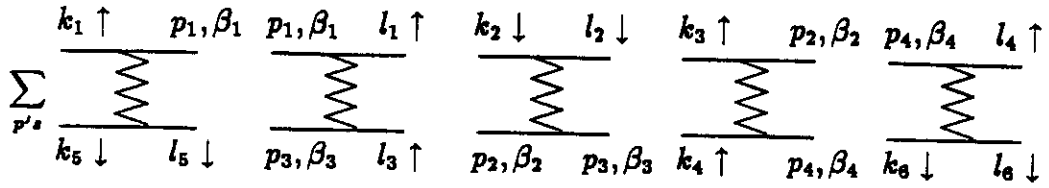
$$\sum_{\beta=\uparrow,\downarrow} \left[\overline{u}_{\mu_1}(l_1)\gamma^\mu u_{\lambda_1}(k_1)\overline{u}_\beta(p)\gamma_\mu u_{\lambda_2}(k_2)\right]\left[\overline{u}_{\mu_2}(l_2)\gamma^\nu u_\beta(p)\overline{u}_{\mu_3}(l_3)\gamma_\nu u_{\lambda_3}(k_3)\right], \tag{3}$$

which can be represented pictorially as a product of two one-gluon-exchange diagrams as shown in Fig. 1(b). Therefore if we have computed the amplitude for a general one-gluon-exchange amplitude, we can write down the answer for a two-gluon-exchange diagram immediately. The above idea can be represented as a pictorial equation



(4)

We have put a "cross", $\times$, to indicate where we "cut" the diagram. It is quite obvious that the same argument can be applied recursively to the many-gluon-exchange cases. For example, a five-gluon-exchange diagram occurring in a nucleon-nucleon scattering processes such as the one shown in Fig. 2, where we denote the propagators by p_1, p_2, p_3 and p_4, can be "cut" into one-gluon-exchange diagrams:



, (5)

where the sum is over the helicity of the $p's$ and we have suppressed the helicity indices. The answer in Eq.(5) may look quite complicated, but it can be handled and simplified quite easily by some standard algebraic program, like *Mathematica*. If one only needs numerical values, the algebraic simplification step can be skipped. Using our trick, it is in fact possible and quite straightforward to write FORTRAN programs to calculate the amplitude numerically.

Our aim is now to compute the one-gluon-exchange diagram. To do this, we need to choose a representation. Most of the other techniques employ two-component Weyl representation. However, in our case, it does not matter which representation we choose and in fact the final answer should be independent of the representations. For convenience, we work with the light-cone representation. For any arbitrary vector v, the light-cone components in terms of the Cartesian components is defined by

$$v^+ = v^0 + v^3,\ v^- = v^0 - v^3,\ v^R = v^1 + iv^2,\ v^L = v^1 - iv^2. \tag{6}$$

The scalar product of any two vectors v, w is then $v^\mu w_\mu = \frac{1}{2}(v^+w^- + v^-w^+ - v^Rw^L - v^Lw^R)$. The Dirac matrices have the form

$$\gamma^+ = \begin{bmatrix} 1 & \sigma^3 \\ -\sigma^3 & -1 \end{bmatrix},\ \gamma^- = \begin{bmatrix} 1 & -\sigma^3 \\ \sigma^3 & -1 \end{bmatrix},\ \gamma^R = \begin{bmatrix} 0 & \sigma^R \\ -\sigma^R & 0 \end{bmatrix}, \gamma^L = \begin{bmatrix} 0 & \sigma^L \\ -\sigma^L & 0 \end{bmatrix} \tag{7}$$

in terms of the 2×2 Pauli matrices $\vec{\sigma}$. The Dirac spinors in this representation is given by

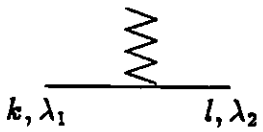
$$u_\uparrow(p) = \frac{1}{\sqrt{2p^+}} \begin{bmatrix} p^+ + m \\ p^R \\ p^+ - m \\ p^R \end{bmatrix},\ u_\downarrow(p) = \frac{1}{\sqrt{2p^+}} \begin{bmatrix} -p^L \\ p^+ + m \\ p^L \\ -p^+ + m \end{bmatrix}, \tag{8}$$

$$v_\downarrow(p) = \frac{1}{\sqrt{2p^+}} \begin{bmatrix} p^+ - m \\ p^R \\ p^+ + m \\ p^R \end{bmatrix},\ v_\uparrow(p) = \frac{1}{\sqrt{2p^+}} \begin{bmatrix} -p^L \\ p^+ - m \\ p^L \\ -(p^+ + m) \end{bmatrix}. \tag{9}$$

with the normalization $\sum_{\lambda=\uparrow,\downarrow} \overline{u}_\lambda(p)u_\lambda(p) = 2m$ [1]. We define a "quark-string" $Q^\mu_{\lambda_1\lambda_2}(k,l)$ by the matrix element

$$Q^\mu_{\lambda_1\lambda_2}(k,l) = \frac{1}{\sqrt{2}}\overline{u}_{\lambda_1}(l)\gamma^\mu u_{\lambda_2}(k), \tag{10}$$

where $\mu = (+,-,R,L)$. Pictorially, we write

$Q^\mu_{\lambda_1,\lambda_2}(k,l) \equiv$ 

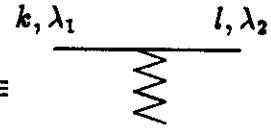
. (11)

Writing $Q^\mu_{\lambda_1\lambda_2}(k,l)$ as a column vector, we find

$$Q_{\uparrow\uparrow}(k,l) = \sqrt{\frac{2}{l^+k^+}}\begin{bmatrix} k^+l^+ \\ k^Rl^L + m^2 \\ k^Rl^+ \\ k^+l^L \end{bmatrix}, \quad Q_{\downarrow\downarrow}(k,l) = \sqrt{\frac{2}{l^+k^+}}\begin{bmatrix} k^+l^+ \\ k^Ll^R + m^2 \\ k^+l^R \\ k^Ll^+ \end{bmatrix}, \tag{12}$$

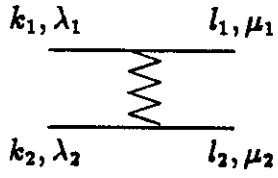
$$Q_{\uparrow\downarrow}(k,l) = \sqrt{\frac{2}{l^+k^+}}\begin{bmatrix} 0 \\ m(k^R - l^R) \\ 0 \\ m(k^+ - l^+) \end{bmatrix}, \quad Q_{\downarrow\uparrow}(k,l) = \sqrt{\frac{2}{l^+k^+}}\begin{bmatrix} 0 \\ m(l^L - k^L) \\ m(l^+ - k^+) \\ 0 \end{bmatrix}. \tag{13}$$

On the other hand, we define $\bar{Q}^\mu$ to be the row vector

$\bar{Q}^\mu_{\lambda_1\lambda_2}(k,l) = (Q^-, Q^+, -Q^L, -Q^R) \equiv$ 

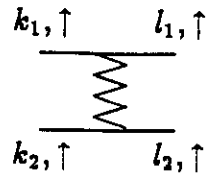
. (14)

Therefore the numerator of the one-gluon-exchange diagram is given by

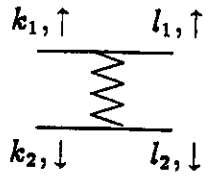


$\equiv \bar{Q}_{\lambda_1\mu_1}(k_1,l_1)Q_{\lambda_2\mu_2}(k_2,l_2).$ (15)

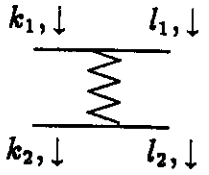
In the case where mass can be neglected, helicity along a quark line is conserved as can be seen from Eq.(13) that $Q^\mu_{\uparrow\downarrow}$ and $Q^\mu_{\downarrow\uparrow}$ vanish. For simplicity, we will restrict ourselves to massless case. For massive fermion case, the only complication is the inclusion of more terms corresponding to helicity-flip contribution. Using Eq.(15), we find



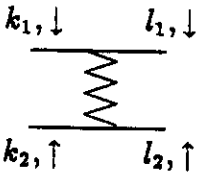
$$= N\left\{k_2^+ l_2^+ k_1^R l_1^L + k_1^+ l_1^+ k_2^R l_2^L - k_1^+ l_2^+ k_2^R l_1^L - k_2^+ l_1^+ k_1^R l_2^L\right\}; \tag{16}$$



$$= N\left\{k_2^+ l_2^+ k_1^R l_1^L + k_1^+ l_1^+ l_2^R k_2^L - k_1^+ k_2^+ l_2^R l_1^L - l_2^+ l_1^+ k_1^R k_2^L\right\}; \tag{17}$$



$$= N\left\{k_2^+ l_2^+ l_1^R k_1^L + k_1^+ l_1^+ l_2^R k_2^L - k_2^+ l_1^+ l_2^R k_1^L - k_1^+ l_2^+ l_1^R k_2^L\right\}; \tag{18}$$



$$= N\left\{k_2^+ l_2^+ l_1^R k_1^L + k_1^+ l_1^+ k_2^R l_2^L - l_1^+ l_2^+ k_2^R k_1^L - k_1^+ k_2^+ l_1^R l_2^L\right\}, \tag{19}$$

where $N = \sqrt{\frac{2}{k_1^+ l_1^+}}\sqrt{\frac{2}{k_2^+ l_2^+}}$.

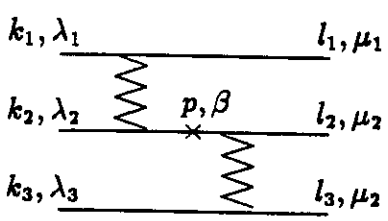
Up to now, we have ignored the off-shell spinor product. However, introducing a parameter $\epsilon^2(p) = p^2 - m^2$ to account for the off-shellness and explicitly expanding of the 4×4 matrix $\not{p} + m$, it is not difficult to show that

$$\not{p} + m = \sum_\lambda u_\lambda(p)\overline{u}_\lambda(p) + w_\lambda(p)\overline{w}_\lambda(p), \tag{20}$$

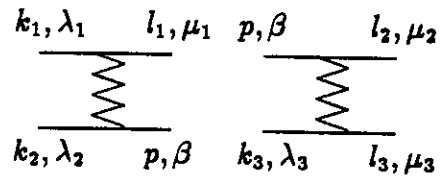
where

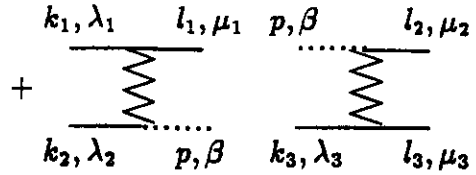
$$w_\uparrow(p) = \frac{\epsilon(p)}{\sqrt{2p^+}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}, \quad w_\downarrow(p) = \frac{\epsilon(p)}{\sqrt{2p^+}}\begin{bmatrix}-1\\0\\1\\0\end{bmatrix}. \tag{21}$$

Therefore, on substituting Eq.(21) into Eq.(1), we get one more contribution from the "w" spinors, or off-shell spinors. Representing it by a dot line, Eq.(1) becomes



$$= \sum_{\beta=\uparrow,\downarrow}$$

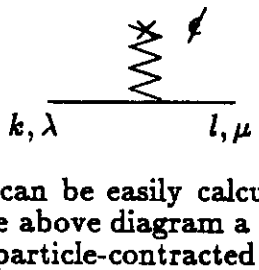




(22)

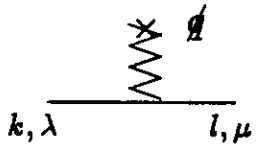
Comparing Eqs.(21) and (8), we see that the w spinor, apart from the normalization factor, can be obtained from the u spinor by setting $m = 0$, $p^+ = 0$ and $p^R = p^L = 1$. Therefore, we have the prescription for computing the off-shell diagram (i.e. one-gluon-exchange diagram involving w spinors), namely, replacing the corresponding momentum p by the momentum $\omega = (0, 0, 1, 1)$ in $(+, -, R, L)$ components and changing the normalization from $1/\sqrt{2p^+}$ to $\epsilon(p)/\sqrt{2p^+}$in the corresponding on-shell one-gluon-exchange diagram. In short, after we have cut the propagators, we have to sum up both the off-shell and on-shell diagrams where the off-shell diagrams can be obtained from the on-shell diagram by the above prescription. It is useful to note that any off-shell diagram with three or more w-spinors vanishes (see Eqs.(16−19)).

The basic on-shell one-gluon-exchange diagrams are summarized in Eqs.(16−19). However, there are other types of diagrams that, along with the one-gluon-exchange diagram, form the basics building blocks of more complex Feynman diagrams. For example, in processes that involve a real photon such as Compton scattering, after the cutting the fermion propagators, there appears the diagram



$$\equiv \bar{u}_\mu(l)\, \not\epsilon u_\lambda(k) = \frac{1}{\sqrt{2}} \left[\epsilon^-, \epsilon^+, -\epsilon^L, -\epsilon^R\right] Q_{\lambda,\mu}(k, l)\,, \tag{23}$$

which can be easily calculated once the polarization vector $\epsilon(\lambda), \lambda = \uparrow, \downarrow$is chosen. If we call the above diagram a "photon-contracted diagram", then we may have more generally a one-particle-contracted diagram or a gluon-contracted diagram

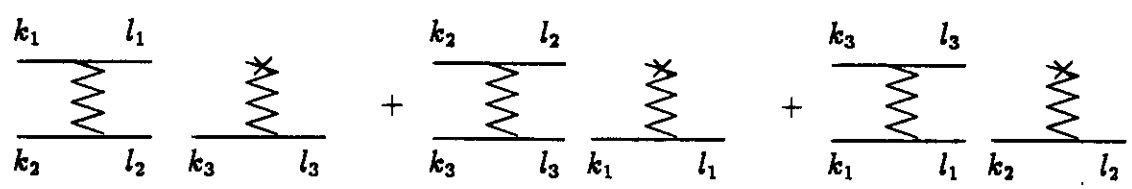


$$\equiv \bar{u}_\mu(l)\, \not q u_\lambda(k) = \frac{1}{\sqrt{2}} \left[q^-, q^+, -q^L, -q^R\right] Q_{\lambda,\mu}(k, l)\,, \tag{24}$$

where q is the momentum of the particle. These diagrams also appear when we try to break down three-gluons or four-gluons vertices. For instances, the three-gluon vertices in Fig.(2) gives a factor

$$[(p_1 - p_2)_\nu g_{\lambda\mu} + (p_2 - p_3)_\lambda g_{\mu\nu} + (p_3 - p_1)_\mu g_{\nu\lambda}]\,. \tag{25}$$

Using the same idea of breaking down more complex Feynman diagrams into basic ones, it is not difficult to see that the amplitude represented by Fig.(3) can be pictorially rewritten as



(26)

Figuratively, this can be thought as cutting one of the gluon line and connecting the remaining two together and summing over different permutations. Similarly, a four-gluon

vertex can be handled by cutting the vertex, then connecting two pairs of gluon lines together and summing over different possibilities of choosing gluon lines pairs.

Although we have restricted ourselves to strong interaction and Feynman gauge, all the necessary ingredients have been laid down by now. It is not difficult to see how to generalize our technique to other field theories. For example, the vertices changes from γ^μ to $\gamma^\mu(1-\gamma^5)$ in weak interaction. Then we can define another ferimon–string

$$Q^\mu_{\lambda_1\lambda_2}(k,l) = \overline{u}_{\lambda_2}(l)\gamma^\mu(1-\gamma^5)u_{\lambda_1}(k)$$

and similarly calculate the corresponding one-gluon-exchange diagram which can then be used to build up more complex diagrams. When antiparticles are involved, one can also consider $\overline{u}.....v$ or $\overline{v}.....u$ types of fermion-strings. The actual implementation should be straightforward because one can just modify the one-gluon-exchange function declared in the program.

III. SAMPLE CALCULATION

We apply our technique to a particular diagram as an example. The diagram of our example along with the momentum assignment is shown in Fig. 4. It represents a typical Feynman diagram one has to evaluate in nucleon Compton scattering in the framework of perturbative QCD. We choose this diagram because it has been worked out by hand in detail in Appendix C of Ref. [18]. Following Ref. [18], the momenta in the center-of-mass frame have Cartesian components

$$p = E(1,0,0,1), \tag{27}$$
$$k = E(1,0,0,-1), \tag{28}$$
$$p' = E(1,\sin\theta,0,\cos\theta), \tag{29}$$
$$k' = E(1,-\sin\theta,0,-\cos\theta), \tag{30}$$

and the photon polarization vectors are

$$\epsilon_i(\uparrow) = \tfrac{1}{\sqrt{2}}(0,1,-i,0), \epsilon_i(\downarrow) = -\tfrac{1}{\sqrt{2}}(0,1,i,0), \tag{31}$$
$$\epsilon_f(\uparrow) = \frac{1}{\sqrt{2}}(0,\cos\theta,-i,-\sin\theta), \tag{32}$$
$$\epsilon_f(\downarrow) = -\frac{1}{\sqrt{2}}(0,\cos\theta,i,-\sin\theta), \tag{33}$$

Their light-cone components $(+,-,R,L)$ are

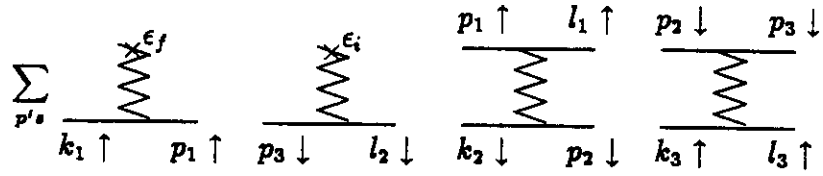
$$p = 2E(1,0,0,0), \qquad k = 2E(0,1,0,0), \tag{34}$$
$$p' = 2E(c^2,s^2,sc,sc), \qquad k' = 2E(s^2,c^2,-sc,-sc), \tag{35}$$
$$\epsilon_i(\uparrow) = \sqrt{2}(0,0,1,0), \epsilon_i(\downarrow) = \sqrt{2}(0,0,0,-1), \tag{36}$$
$$\epsilon_f(\uparrow) = \sqrt{2}(-sc,sc,c^2,-s^2), \tag{37}$$
$$\epsilon_f(\downarrow) = \sqrt{2}(sc,-sc,s^2,-c^2). \tag{38}$$

where $s = \sin(\frac{1}{2}\theta)$ and $c = \cos(\frac{1}{2}\theta)$. Here, we denote the center-of-mass energy Mandelstam invariants by $S = (p+k)^2 = 4E^2$. It is convenient to consider general linear combination of polarization vectors

$$\epsilon_i = \alpha\epsilon_i(\uparrow) + \beta\epsilon_i(\downarrow), \quad \epsilon_f = \gamma\epsilon_f(\uparrow) + \delta\epsilon_f(\downarrow) \tag{39}$$

so that all four amplitudes with different combination of photon helicities can be computed at the same time.

According to the algorithm in described in Section II, we cut the diagram in Fig. 4 along the quark propagators. Thus, the numerator of the color- and flavor-independent part of the amplitude is given by



(40)

where the sum over $p's$ include both on-shell and off-shell diagrams.

To evaluate this expression, one has to specify the above expression by a list

```
gdiag = {{k1,p1},{p3,l2},{p1,l1,k2,p2},{p2,p3,k3,l3}}
```

with an obvious convention when it is compared with Eq.(40). Similarly, the spin configuration is specified by the list

```
spindiag = {{f,u},{i,d},{u,d},{d,u}}
```

where `u` and `d` denote spin up and spin down respectively and `f` and `i` for final and initial photon states. The propagator is collected into a list

```
propagatorlist = {p1, p2, p3, p4, p5}
```

To evaluate the numerator of the original diagram, one calls the function `CalNumDiag` by the statement:

```
numera = CalNumDiag[nquarks,nphotons, gdiag, spindiag, propagatorlist]
```

where nquarks = 3 is the number of quarks and nphotons = 2 is the number of photons.

This gives the result

$$8S^3c^3s^2\left[\alpha(1-y_2s^2)+\beta y_2s^2\right](\gamma x_1+\delta x_2),$$

where x_i and y_i are the momentum fractions of the ith quark in the initial and final states respectively. It is the same as that obtained in Ref. [18].

To evaluate the denominator of the diagram, one calls the function `CalDenDiag` by the statement:

```
den = CalDenDiag[nquarks,nphotons, propagatorlist]
```

On the other hand, one can calculate both the numerator and denominator of the diagram at the same time by calling the function `CalDiag` :

```
answer = CalDiag[nquarks,nphotons,gdiag,spindiag,propagatorlist]
```

We applied our method to other diagrams in Ref. [18] and verified the equivalence. We applied the function which generates all the possible diagrams to nucleon Compton scattering to find the total 378 diagrams. Among 378 diagrams, 42 diagrams are coming from the triple gluon vertex and the color factors of these diagrams are zero due to the symmetry. An more extensive descriptions of the functions available in our package can be found in Ref. [11].

IV. CONCLUSION

In summary, we have presented a technique that breaks complex diagrams down into simple building blocks. In our approach, the building blocks consist of pairs of fermion lines connected by gluons or simple fermion lines that radiate an external gluon or photon. These building blocks are formed by replacing the numerators of the fermion propagators by spinor outer products, utilizing the completeness relations. Summing both on-shell and off-shell spinor products are necessary in this method and thus the final step of summing all the terms and simplifying the result would be a tedious task if it is done by hand. Therefore, we have developed a symbolic program to utilize this technique and generate the final result automatically.

Although there are other techniques available that can also simplify Feynman diagram calculations, we believe that our method is worth presentation due to its elegance and straightforward implementation. A traditional construction of the Feynman diagrams is the collection of the basic elements such as Dirac spinors, vertices and propagators involving non-abelian gamma matrices. On the other hand, in our picture the basic elements of the Feynman diagrams are the one-particle-exchange and one-particle-contracted diagrams which are just ordinary functions of the kinematical variables. Thus, we have avoided dealing with the non-abelian algebra of the gamma matrices and greatly simplified the calculation. We have developed a package called *COMPUTE* which is available in both *Mathematica* and *Maple*. *COMPUTE* is able to generate all the possible Feynman diagrams for a given processes and perform the subsequent evaluation of each diagram. With the help of such computer symbolic program, we expect to be able to do calculations which were almost impossible before.

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FIGURES

FIG. 1. (a) A generic two-gluon-exchange diagram. (b) Product of two one-gluon-exchange diagram corresponding to Eq.(3). We use $k's, l's$ and $p's$ to denote four-momenta and Greek's letters $\lambda's, \mu's, \beta's$ for helicities. The zigzag lines represent photons or gluons.

FIG. 2. A typical Born diagrams contributing to pp scattering.

FIG. 3. A diagram containing a three-gluon vertex.

FIG. 4. A typical diagram in nucleon Compton scattering processes.

APPENDIX A: LISTING OF INPUT FILE

```
Print["\t Sample Calculation of Compton scattering \n
\t Calculating the diagram A51 in Kronfeld and Nizic paper \n
\t Please wait .... \n" ]

(* this file calculate the diagram A51 for Compton Scattering
   discussed in the paper
    Phys. Rev. D44, 3445 (1991) by A.S. Kronfeld and B. Nizic,
   where they have calculated the diagram A51 by hand
*)
(* One has to specify the momenta of both the incoming quarks
   and the outgoing quarks.
*)

p = Sqrt[S] {1,0,0,0};     k = Sqrt[S] {0,1,0,0};
pprime = Sqrt[S] {c^2,s^2,s c,s c};
kprime = Sqrt[S] {s^2,c^2,-s c,-s c};
k1 = x1 p;          k2 = x2 p;                    k3 = x3 p;
l1 = y1 pprime;     l2 = y2 pprime;               l3 = y3 pprime;

(* the following specify the quarks and gluon propagator *)
p1 = k1 - kprime;  p2 = k1 - kprime - l1 + k2;  p3= l2 - k;
p4 = p2 - p1;  p5 = l3 - k3;

x3 = 1 - x1 - x2
y3 = 1 - y1 - y2

(* the following specify the configuration of the Feynman as
   well as the helicity (ie. the spin) configuration *)

spindiag={{f,u},{i,d},{u,d},{d,u}}
gdiag ={{k1,p1},{p3,l2},{p1,l1,k2,p2},{p2,p3,k3,l3}}
propagatorlist ={p1,p2,p3,p4,p5}

(* the following calculates the Feynman diagram. The Hard Scattering
   Amplitude is defined with the extra factor of Sqrt[x1 x2 x3 y1 y2
   y3 ] *)
(* we can calculate just the numerator or denominator separately *)

num=CalNumDiag[3,2,gdiag,spindiag,propagatorlist]

num = num /Sqrt[x1]/Sqrt[x2]/Sqrt[x3]/Sqrt[y1]/Sqrt[y2]/Sqrt[y3]

num = Cancel[num] /. c^2 + s^2 -> 1

Print["\n numerator ="]; Print[num]

(* num gives the same result as given by the reference *)

den=CalDenDiag[3,2,propagatorlist]

Print["\n denominator ="]; Print[den]
```

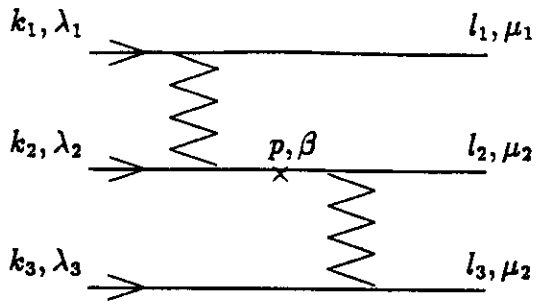
```
(* or we can calculate both numerator and denominator at the same
   time *)

ans=CalDiag[3,2,gdiag,spindiag,propagatorlist]
ans=ans/Sqrt[x1]/Sqrt[x2]/Sqrt[x3]/Sqrt[y1]/Sqrt[y2]/Sqrt[y3]
ans=Cancel[ans] /. c^2 + s^2 -> 1

Print["\n hard scattering amplitude ="];
Print[ans]
```

APPENDIX B: EXAMPLE RUN OUTPUT

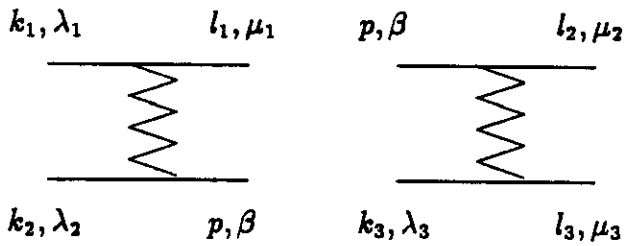
The following is an output from an actual run of the our example.

```
In[1]:= <<COMPUTE.m
        ********* Welcome to COMPUTE 1.0 ***********

         COMPUTE is a package to calculate the amplitude
         for exclusive processes in perturbative QCD.

         Copyright by Alex Pang and Chueng-Ryong Ji

         For more information, please type ?ComputeInfo.

In[2]:= <<Compton.m
         Sample Calculation of Compton scattering
         Calculating the diagram A51 in Kronfeld and Nizic paper
         Please wait ....

numerator =
   3  2  3                                          2            2
8 c  s  S  (gamma x1 + delta x2) (alpha - alpha s  y2 + beta s  y2)

denominator =
 4  4  5
c  s  S  x1 x2 (-1 + x1 + x2) y1

      2       2       4         2  2       4       2          2
>   (c  x1 + c  x2 - c  y1 - 2 c  s  y1 - s  y1 + s  x1 y1 + s  x2 y1)

>   (1 - y1 - y2) y2

hard scattering amplitude =
                                                2            2
(8 (gamma x1 + delta x2) (alpha - alpha s  y2 + beta s  y2)) /

        2  2
>   (c s  S  x1 x2 (-1 + x1 + x2) y1

      2       2       4         2  2       4       2          2
>   (c  x1 + c  x2 - c  y1 - 2 c  s  y1 - s  y1 + s  x1 y1 + s  x2 y1)

>   (1 - y1 - y2) y2

hard scattering amplitude =
                                                2            2
(8 (gamma x1 + delta x2) (alpha - alpha s  y2 + beta s  y2)) /

        2  2
>   (c s  S  x1 x2 (-1 + x1 + x2) y1

          2        2       4         2  2       4       2          2
>     (-(c  x1) - c  x2 + c  y1 + 2 c  s  y1 + s  y1 - s  x1 y1 - s  x2
y1)
```

```
>      y2 (-1 + y1 + y2))

In[3]:= Quit
```


k_1, λ_1
l_1, μ_1
k_2, λ_2
p, β
l_2, μ_2
k_3, λ_3
l_3, μ_2

(a)


k_1, λ_1
l_1, μ_1
p, β
l_2, μ_2
k_2, λ_2
p, β
k_3, λ_3
l_3, μ_3

(b)

Fig. 1

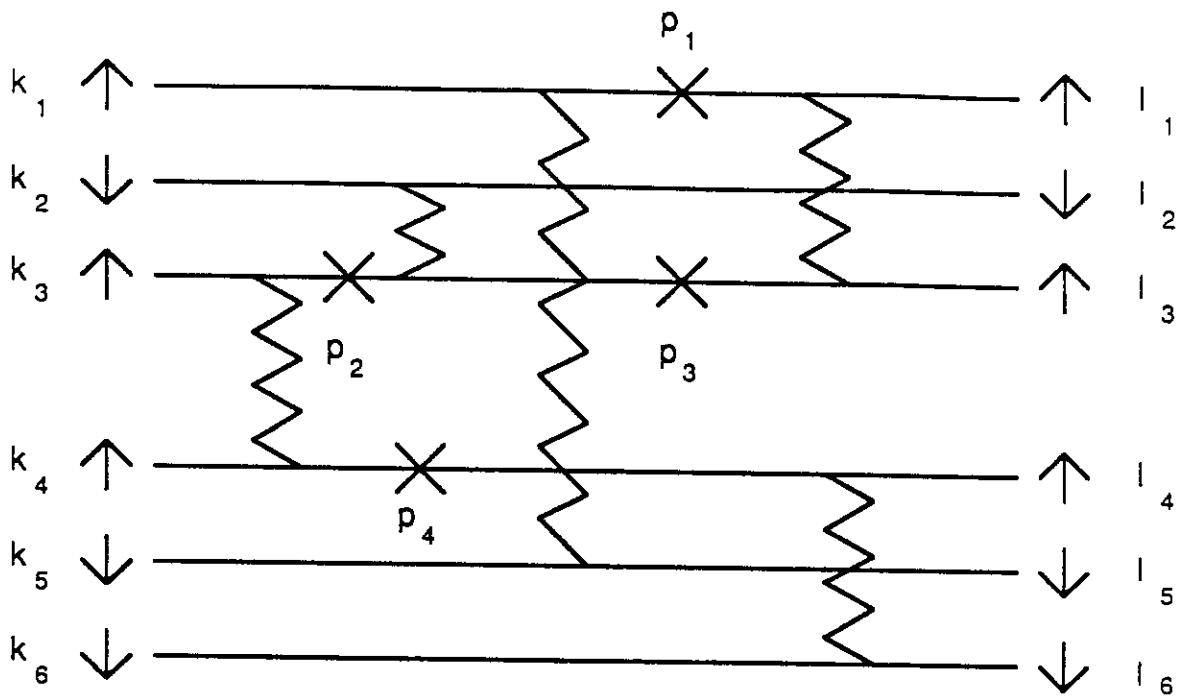


Fig. 2.

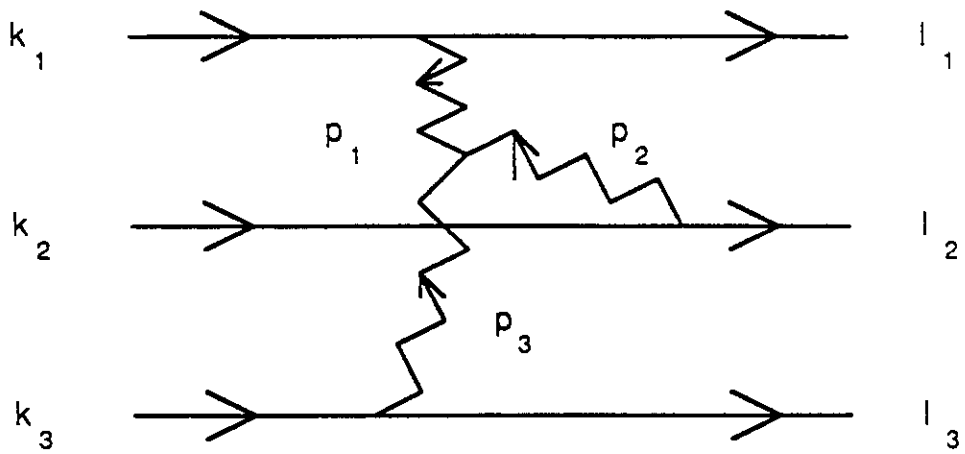


Fig. 3.

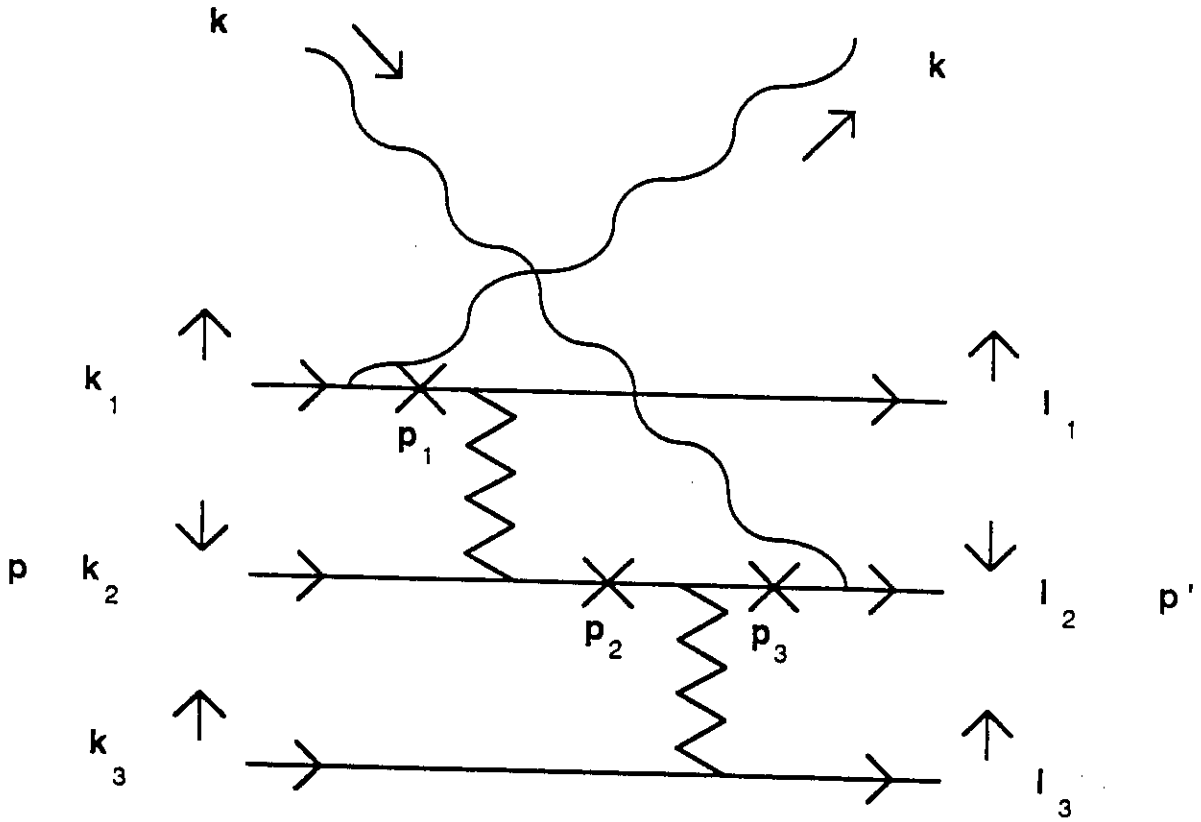


Fig. 4.