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Elastic Charge Form Factors for K Mesons

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The elastic charge form factors for the charged K^+ and neutral K^0 are presented. The model employed makes use of solutions of the coupled Bethe-Salpeter and Schwinger-Dyson equations. This covariant model, which provides good predictions for the pion's elastic charge and transition form factor, has been presented elsewhere and will be briefly outlined here. The results for the kaon are in good agreement with the available data and can serve as an eventual guide in the interpretation of the approved CEBAF experiment E-93-018. The Q^2 dependent K^0 results suggest a new area for experimental exploration.

The set of pseudoscalar mesons can be thought of as the spectroscopy of a single atom. The spectroscopic levels (meson masses) are accessible through flavor (weak) transitions and radial (electro-strong) excitations. Verification of this perspective is realized in the solution of a wave equation having a specific interaction that reproduces the energy levels and at the same time provides a good description of the charge distributions. The quark model does a reasonable job of the former but there is no reasonable self consistent description of the latter.

The focus of this report is to provide some insight into the problem of a self consistent description of the charge form factors. The starting point is near the threshold of such an interaction, namely the pion. A brief description of a covariant model that employs both the Bethe-Salpeter and Schwinger-Dyson equations is presented. Combining these two equations is not new, but solving them in this context provides new insights as will be shown below. The next step is to examine the mesons made up of unequal mass quarks. The smallest such meson is the kaon. Only predictions for the kaon's charge and neutral elastic form factors are provided here. Work continues toward inclusion of the D and B mesons, for example, with our formalism (but will not be reported on here); however, the lack of a confining mechanism in our approach appears to make the calculations of the D and B form factors problematic.

The covariant model description employed was introduced by Ito, Buck and Gross (IGB) [1] and is a generalization of the Nambu-Jona-Lasinio model. The IBG model makes use of a separable interaction between the quark-antiquark ($q\bar{q}$) pair inside the meson with a single mass scale (Λ) [2]. Such a separable interaction is frequently employed to treat the pairing problem in the BCS theory [3]. The dynamical quark mass which emerges from the IBG model and the $q\bar{q}$ momentum distribution in the meson wave function are both related to the same parameters. Whereas the IGB work addressed the pion only, the present work addresses the pion and the kaon. The separable interaction is

$$V_{\alpha\beta:\delta\gamma}(k',k) = g\, f(k'^2)\, f(k^2)\, [I_{\alpha\beta} I_{\delta\gamma} - (\gamma^5\tau)_{\alpha\beta}(\gamma^5\tau)_{\delta\gamma}] \ , \qquad (1)$$

where the function $f(k^2)$ ($f(k'^2)$) depends on the relative momentum, k (k'), of the $q\bar{q}$ pairs in the initial (final) state. We choose a monopole function $f(k^2) = 1/(k^2 - \Lambda^2)$ with a scale parameter Λ. The relativistic meson vertex function will be proportional to f and if the energy of the $q\bar{q}$ pair of quarks in the meson is (on the average) shared equally, the four vector k^2 reduces to $-\mathbf{k}^2$ and the function f becomes the familiar momentum space Yukawa function for an S-state. The solution of the wave equation with a more preferred interaction is not performed but rather the solution is inferred from the data. This inference results in a good

explanation of the pion and kaon data. It is concluded that any more detailed description using a more realistic interaction must reproduce the same qualitative features presented here.

The Bethe-Salpeter equation for the two quark bound state vertex function is

$$\Gamma_{\alpha\beta}(k',p) = i \int \frac{d^4k}{(2\pi)^4}\, V_{\alpha\beta:\delta\gamma}(k',k)\, S_{\gamma\gamma'}(k+\frac{p}{2})\, \Gamma_{\gamma',\delta'}(k;p)\, S_{\delta'\delta}(k-\frac{p}{2}) \ , \qquad (2)$$

where the quark propagators are $S(p) = [\not{p} - m_q - \Sigma(p^2) + i\epsilon]^{-1}$, with m_q the current quark mass of various flavors (u or d for the pion, for example), and $\Sigma(p^2)$ the quark self energy. For unequal quark masses, there is a set of three coupled equations; two for the self energy and one wave equation. Thus, for the pion there are only two coupled equations and it is the pure γ^5 term in the vertex that is kept. The quark self energy is a solution of the Schwinger-Dyson (SD) equation, namely

$$\begin{aligned}\Sigma(k'^2) &= i \int \frac{d^4k}{(2\pi)^4} V_{\alpha\beta:\delta\gamma}(k',k)\, S_{\gamma\delta}(k) \\ &= 4 i n_f g\, f(k'^2) \int \frac{d^4k}{(2\pi)^4} f(k^2) \frac{m_0 + \Sigma(k^2)}{k^2 - [m_0 + \Sigma(k^2)]^2} \ , \end{aligned} \qquad (3)$$

where n_f is the number of quark flavors (always equal to 2 in our $q\bar{q}$ system). The eigenvalue equation generated by equation (2) for the $q\bar{q}$ mesons is

$$1 = i n_f g \int \frac{d^4k}{(2\pi)^4}\, f^2(k^2) \frac{4[m_0 + \Sigma(k^2)]^2 + p^2 - 4k^2}{[(k-\frac{p}{2})^2 - [m_0 + \Sigma(k^2)]^2]\, [(k+\frac{p}{2})^2 - [m_0 + \Sigma(k^2)]^2]} \qquad (4)$$

In the Chiral Limit (CL) when the meson mass and all $m_0 = 0$, this equation is identical to equation (3) for the quark self energy, provided $\Sigma(k^2) = C f(k^2)$, where C is a constant. For non-zero self energies, constraints are placed on the coupling strength, g, such that $g >> g_c$ (CL). However, instead of using the full momentum dependent self-energy, the size of which depends on the parameter C, the self-energy is replaced by its mean value $< \Sigma > \equiv M = m_q - m_0$. This leads to replacing C by m_q. This definition of the self energy is realized in eqn. (4) via

$$1 = -i n_f g \int \frac{d^4k}{(2\pi)^4}\, f^2(k^2) \frac{4m_q^2 + p^2 - 4k^2}{[(k-\frac{p}{2})^2 - m_q^2]\, [(k+\frac{p}{2})^2 - m_q^2]} \ , \qquad (5)$$

where m_q is our dynamical quark mass. For the purposes of this present work, another major assumption is made about the self energy of the heavier quark: the

self energy of the heavier quark is chosen to be equal to the strange quark mass. In some regards this could be considered a rather limiting assumption; however, the mass of this quark is taken to be a free parameter and, fortunately, the form factor results emerging from this assumption are in quite good agreement with the data. This means that the interpretation of the value of the strange quark mass, in the case of the kaon, should not be taken too seriously. On the other hand, the resulting value, 430 MeV, is consistent with other researchers' results and we note that our results (below) give $m_s - m_{u,d} \sim 180\ MeV$ (about the same as the difference between current quark masses). Again, a more detailed calculation employing a more realistic interaction should, in principle, provide the same qualitative results.

In the case of very heavy-light quark mesons, this assumption about the heavy quark self energy should become better as the heavy quark mass approaches the meson's mass. A good discussion of the pion case can be found in ref [1]. The work on heavier quarks is in progress [4]. At any rate, the strange quark mass (taken to be a parameter), in the case of the kaon, enters in the quark propagator only.

To determine the parameters m_q (light quark mass) and Λ, the pion's weak decay constant and its two photon decay width are calculated for several choices of parameters. In this work, a different choice from the IBG work is employed, namely, $f_\pi = 98.3\ MeV$, $m_{u,d} = 250\ MeV$ and $\Lambda_\pi = 600\ MeV$. The strange quark mass is chosen so that the neutral kaon form factor is zero at $Q^2 = 0$. The result, as stated earlier, is $m_s = 430\ MeV$ and $\Lambda_K = 690\ MeV$. Having fixed these parameters, predictions for the pion and kaon elastic form factors can be made in addition to the pion's transition form factor. The results for the pion charge radius of $r_\pi^2 = 0.449\ fm^2$ (experimental value $= 0.450 \pm 0.099\ fm^2$), a K^0 charge radius of $r_{K^0}^2 = -0.086\ fm^2$ (experimental value = -0.054 $\pm$ 0.101 fm^2). Our results for the K^+ charge radius, absolute value, is 0.358 fm^2 (experimental value $= 0.340 \pm 0.310\ fm^2$). As for the decay constants, the pion decay constant is calculated as 98.3 MeV (experimental value = 93 MeV), the kaon decay constant is calculated as 119.9 MeV (experimental value = 113.4 MeV) and the ratio of the kaon decay constant to that of the pion's is calculated to be 1.22 (experimental value = 1.22). These results are summarized in Table I. The data employed in this work is taken from refs [5, 6].

Once the parameters are fixed, the model presented here provides simultaneous predictions for the pion elastic charge and transition form factors [1], and the kaon elastic form factors (both charged and neutral) that are in very good agreement with the available data. The CEBAF experiments to measure, independently, the kaon (E-93-018) and pion (E-93-021) charge form factors in the

range of $Q^2 < 3\ GeV^2$ will provide further guidance to our understanding of these phenomena. In particular, the experiments may be able to distinguish between the work presented here and that of other theoretical work [7]. Figure 1 illustrates our Q^2-dependent predictions for the pion and kaon form factors. As a final note, a more complete analysis should incorporate an explicit calculation of the weak $K \to \pi$ transition form factors. This has not been performed here; however, we expect good agreement with the data to be achieved if one employs the pion and kaon wave functions generated with our formalism.

Table I. Charge radii and Weak decay constants of the pion and kaon predicted by our model with $m_{u,d} = 250\ MeV$, $m_s = 430\ MeV$, $\Lambda_\pi = 600\ MeV$ and $\Lambda_K = 690\ MeV$.

Observable	Calculation	Experiment
$r_{\pi^+}^2\ [fm^2]$	0.449	0.450 ± 0.099
$r_{K^+}^2\ [fm^2]$	0.358	0.340 ± 0.310
$r_{K^0}^2\ [fm^2]$	-0.086	-0.054 ± 0.101
$f_\pi\ [MeV]$	98.3	92.4
$f_K\ [MeV]$	119.9	113.4
f_K/f_π	1.22	1.22

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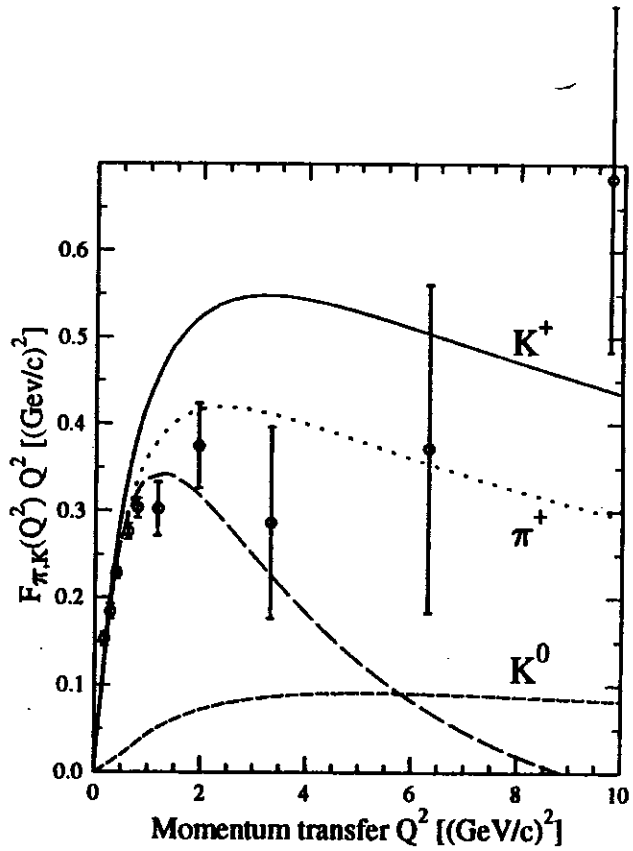

Figure 1: Elastic charge form factor predictions of the $K^{\pm}$, $\pi^{\pm}$, and K^0. The solid, dotted and short dashed lines represent the theoretical predictions for the $K^{\pm}$, $\pi^{\pm}$, and K^0, respectively, whereas the long dashed line is a prediction for the K^+ of ref. 7. The data is that of the π^+ from refs. 5 and 6 shown for comparison.

REFERENCES

[1] H. Ito, W. W. Buck, F. Gross, *Phys. Lett.* **B** 287, 23-28 (1992); H. Ito and W. W. Buck, Blois Workshop on Elastic and Diffractive Scattering, Brown University, 8-12 June 1993.

[2] H. Ito, W. W. Buck, F. Gross, *Phys. Rev.* **C** 43, 2483 (1991); 45, 1918 (1992).

[3] J. Bardeen, L. N. Cooper and J. R. Schrieffer, *Phys. Rev* 108, 1175 (1957); A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill, NY, 1971).

[4] W. W. Buck, H. Ito and C. Rankin, in progress.

[5] C. J. Bebek et al., *Phys. Rev.* **D** 17, 1693 (1978).

[6] T. Azemoon et al., *Nucl. Phys.* **B** 95, 77 (1975); S. R. Amendoria et al., *Phys. Lett.* **178B** 435 (1986).

[7] See, for example : C.-R. Ji and S. R. Cotanch, *Phys. Rev.* **D** 41, 2319 (1990); A. Radyushkin, private communication.