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The Heavy Mass Expansion In $B \to D$ Decays

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We examine the $1/m_c$ and $1/m_b$ expansions in $B^{(*)} \to D^{(*)}$ processes, and point out that there are relationships among the form factors that do not receive corrections in any order of $1/m_c$. A different set of relationships are unchanged by $(1/m_b)^n$ corrections, while the $(1/m_c)^n$ preserved relationships are also unaffected by terms proportional to powers of $\alpha_s(m_c)$. All relationships are modified when terms of the form $1/(m_b m_c)$ or radiative corrections proportional to powers of $\alpha_s(m_b)$ are included.

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I. INTRODUCTION

In a recent article [1], we considered the semileptonic decay $\Lambda_b \to \Lambda_c \ell \nu$ in the framework of the heavy quark effective theory (HQET) [2], and showed that there were predictions that received no corrections beyond leading order in the $1/m_c$ expansion. In addition, these predictions received no corrections from radiative terms proportional to powers of $\alpha_s(m_c)$. We further went on to show that a subset of these predictions were also preserved to all orders in the $1/m_b$ expansion, but that all predictions were modified when corrections of the type $1/(m_b m_c)$ were taken into account. We also estimated the sizes of the radiative corrections to one of the predictions mentioned above.

One obtains such predictions because the heavy quark symmetries restrict the number of independent form factors for baryonic heavy to light decays $\Lambda_H \to B$, where B is any light spin 1/2 baryon (in fact, the restriction occurs no matter what spin the light baryon has, but spin-1/2 baryons are somewhat more interesting pheomenologically). In this class of decays there are only two independent form factors. At scales between m_b and m_c the charm quark is still light and the decay $\Lambda_b \to \Lambda_c \ell \nu$ is a heavy to light decay at these scales. Matching at the scale m_c then yields predictions that are preserved to all orders in $1/m_c$.

In this note we extend the discussion to mesons by considering $B^{(*)} \to D^{(*)}$ transitions. Due largely to the more complex spin structure of mesons compared with Λ baryons, the mesonic heavy to light decays are in general given in terms of six independent form factors. Applying this fact to $B^{(*)} \to D^{(*)}$ decays and matching at the scale m_c to an effective theory with a heavy c quark, one may also obtain predictions which do not receive any $1/m_c$ correction.

However, the left handed currents for $B \to D^{(*)}$ decays are given in general by six independent form factors and hence the number of phenomenologically interesting predictions that can be made is somewhat limited. A further limitation on the scope of the relations we obtain is the fact that only the $B \to D^{(*)}$ decays are directly accessible experimentally. We emphasize, however, that the limitation on the scope of the predictions we obtain arises only when we consider the left-handed current in particular. As we shall see in what follows, many of our statements are valid independent of the current operator being considered. This means that the power of the predictions is greater if the Lorentz structure of the current operator being considered is more complicated. This will be illustrated explicitly below.

In addition to these motivations, the relationships we obtain are useful in another way. Since the relationships will hold to all orders in the inverse of the appropriate quark's mass ($1/m_c$ or $1/m_b$), and must also hold independent of the operator being considered, they can be used as a check on any calculation

of higher order power corrections. In addition, due to the unique nature of the $1/m_c$ expansion, radiative corrections proportional to powers of $\alpha_s(m_c)$ can also be checked using these relationships.

Another possible application of the relations given here may be the decays $B \to D^{(*)}\pi\ell\nu$, calculated in the framework of heavy quark and chiral symmetries. Recent analyses of the inelastic process $B \to D^{(*)}\pi\ell\nu$ by various authors [3] indicate that the dominant non-resonant contribution to this decay arises from the diagram shown in figure 1, where a virtual B^* decays into $D^{(*)}$ semileptonically.

Part of the motivation for the consideration of the inelastic process described above, and hence for the study of the form factors for the semileptonic decays of heavy vector mesons, arises from the sum rule of Bjorken, Dunietz and Taron (BDT) [4], and its implication for the slope of the Isgur-Wise function at the non-recoil point. The slope of this function at this kinematic point can be expressed in terms of a 'radius', ρ, and the sum rule implies that

$$\rho \geq \frac{1}{2}, \tag{1}$$

with the equality occuring if the elastic semileptonic decays of the B meson are sufficient to saturate the sum rule. Experimental evidence suggests that ρ is in excess of unity, which implies that inelastic channels and corrections to the sum rule are important. Among the many possible inelastic channels, $B \to D^{(*)}\pi\ell\nu$ is likely to be the channel that most influences the value of ρ.

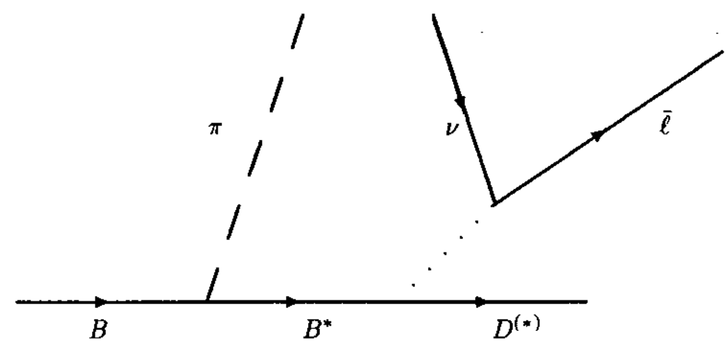

Figure 1: One of the continuum contributions to the decay $B \to D^{(*)}\pi\bar{\ell}\nu$.

II. GENERAL RELATIONS FOR HEAVY TO LIGHT DECAYS

In order to extract the relations among form factors that are unchanged by $1/m_c$ corrections, we proceed in a manner similar to that used in [1].

We begin at the scale $\mu = m_b$, where we integrate out the b quark, treating it as heavy, but leave the charm quark as a light quark. At leading order in the $1/m_b$ expansion, the $B^{(*)}$ mesons may be represented by Dirac matrices as

$$\begin{aligned} B(v) &\to M_B(v) = \left(\frac{1+\not{v}}{2}\right)(-\gamma_5)\,, \\ B^*(v) &\to M_{B^*}(v) = \left(\frac{1+\not{v}}{2}\right)\not{\varepsilon}. \end{aligned} \tag{2}$$

To leading order in $1/m_b$, and ignoring radiative corrections, we can write the matrix elements of a general current operator $\bar{c}\Gamma h_v^{(b)}$, where Γ is some collection of Dirac matrices, in terms of six unknown functions, a_i.

$$\begin{aligned} &\left\langle D(v')\left|\bar{c}\Gamma h_v^{(b)}\right| B(v)\right\rangle = \frac{\sqrt{m_B m_D}}{2}\mathrm{Tr}\left[\gamma_5\left(a_1 + a_2\not{v}'\right)\Gamma M_B(v)\right], \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\Gamma h_v^{(b)}\right| B(v)\right\rangle = \frac{\sqrt{m_B m_{D^*}}}{2}\mathrm{Tr}\left\{\left[\not{\varepsilon}'^*\left(a_3 + a_4\not{v}'\right)\right.\right. \\ &\left.\left. + \,\varepsilon'^*\cdot v\left(a_5 + a_6\not{v}'\right)\right]\Gamma M_B(v)\right\}, \\ &\left\langle D(v')\left|\bar{c}\Gamma h_v^{(b)}\right| B^*(v,\varepsilon)\right\rangle = \frac{\sqrt{m_{B^*} m_D}}{2}\mathrm{Tr}\left[\gamma_5\left(a_1 + a_2\not{v}'\right)\Gamma M_{B^*}(v)\right], \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\Gamma h_v^{(b)}\right| B^*(v,\varepsilon)\right\rangle = \frac{\sqrt{m_{B^*} m_{D^*}}}{2}\mathrm{Tr}\left\{\left[\not{\varepsilon}'^*\left(a_3 + a_4\not{v}'\right)\right.\right. \\ &\left.\left. + \,\varepsilon'^*\cdot v\left(a_5 + a_6\not{v}'\right)\right]\Gamma M_{B^*}(v)\right\}. \end{aligned} \tag{3}$$

The form factors a_i depend only on the degrees of freedom with masses lighter than m_b and hence contain all information on the dependence on m_c. Matching at the scale m_c to a theory with a heavy c quark will then yield relations which hold to all orders in the $1/m_c$ and $\alpha_s(m_c)$ expansions.

However, the phenomenological relevance of this result is limited, since for the case of the left handed current the decays $B \to D^{(*)}$ are in general given by six form factors. However, if the the Lorentz structure of the current is more complicated one may find useful relations. Let us imagine that we are interested in matrix elements of the currents $\bar{c}\gamma_\mu\gamma_\nu b$ and $\bar{c}\gamma_\mu\gamma_\nu\gamma_5 b$. We would then write, for example

$$\begin{aligned} &\left\langle D(v')\left|\bar{c}\gamma_\mu\gamma_\nu b\right| B(v)\right\rangle = \sqrt{m_B m_D}\left[h_1 g_{\mu\nu} + h_2 v_\mu v_\nu + h_3 v'_\mu v'_\nu\right. \\ &\left. + h_4\left(v_\mu v'_\nu + v_\nu v'_\nu\right) + h_5\left(v_\mu v'_\nu - v_\nu v'_\nu\right)\right], \\ &\left\langle D(v')\left|\bar{c}\gamma_\mu\gamma_\nu\gamma_5 b\right| B(v)\right\rangle = \sqrt{m_B m_D}\, h_6 \varepsilon_{\mu\nu\alpha\beta} v^\alpha v'^\beta, \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\gamma_\mu\gamma_\nu b\right| B(v)\right\rangle = i\sqrt{m_B m_{D^*}}\left[\varepsilon_{\mu\nu\alpha\beta}\left(h_7 \varepsilon'^*\cdot v v^\alpha v'^\beta + h_8 v^\alpha \varepsilon'^{*\beta}\right.\right. \\ &\left.\left. + h_9 v'^\alpha \varepsilon'^{*\beta}\right) + \varepsilon_{\mu\alpha\beta\gamma} v^\alpha v'^\beta \varepsilon'^{*\lambda}\left(h_{10} v^\nu + h_{11} v'^\nu\right)\right], \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\gamma_\mu\gamma_\nu\gamma_5 b\right| B(v)\right\rangle = \sqrt{m_B m_{D^*}}\left\{\varepsilon'^*\cdot v\left[h_{12} g_{\mu\nu} + h_{13} v_\mu v_\nu + h_{14} v'_\mu v'_\nu\right.\right. \\ &\left. + h_{15}\left(v_\mu v'_\nu + v_\nu v'_\nu\right) + h_{16}\left(v_\mu v'_\nu - v_\nu v'_\nu\right)\right] + h_{17}\left(\varepsilon'^*_\mu v_\nu + \varepsilon'^*\nu v_\nu\right) \\ &\left. + h_{18}\left(\varepsilon'^*_\mu v'_\nu + \varepsilon'^*\nu v'_\nu\right) + h_{19}\left(\varepsilon'^*_\mu v_\nu - \varepsilon'^*\nu v_\nu\right) + h_{20}\left(\varepsilon'^*_\mu v'_\nu - \varepsilon'^*\nu v'_\nu\right)\right\} \end{aligned} \tag{4}$$

A total of twenty form factors is needed to describe the transitions of a B meson into a $D^{(*)}$ meson via the currents $\bar{c}\gamma_\mu\gamma_\nu b$ and $\bar{c}\gamma_\mu\gamma_\nu\gamma_5 b$, another ten are required to describe the process $B^* \to D$, and an additional number in excess of twenty is required for $B^* \to D^*$; these numerous form factors may be related to the six form factors a_i and these relations will hold to any order in the $1/m_c$ expansion.

In what follows, we shall focus on the left handed current transitions between $B^{(*)} \to D^{(*)}$ for the reasons mentioned in the introduction. Such transitions are described in terms of twenty independent form factors, in general. In this case one may also find relations which are protected from $1/m_c$ correction.

III. THE LEFT HANDED CURRENT

We begin by listing the twenty form factors $g_1 \ldots g_{20}$ needed to describe the processes $B^{(*)} \to D^{(*)}$ via the left-handed current in general. We define these as

$$\begin{aligned} &\left\langle D(v')\left|\bar{c}\gamma_\mu b\right| B(v)\right\rangle = \sqrt{m_B m_D}\left[g_1\left(v+v'\right)_\mu + g_2\left(v-v'\right)_\mu\right], \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\gamma_\mu b\right| B(v)\right\rangle = i\sqrt{m_B m_{D^*}}\, g_3 \varepsilon_{\mu\nu\alpha\beta}\varepsilon'^{*\nu} v'^\alpha v^\beta \\ &\left\langle D^*(v',\varepsilon')\left|\bar{c}\gamma_\mu\gamma_5 b\right| B(v)\right\rangle = \sqrt{m_B m_{D^*}}\left\{g_4 \varepsilon'^*_\mu\right. \\ &\left. + \varepsilon'^*\cdot v\left[g_5\left(v+v'\right)_\mu + g_6\left(v-v'\right)_\mu\right]\right\}, \\ &\left\langle D(v')\left|\bar{c}\gamma_\mu b\right| B^*(v,\varepsilon)\right\rangle = i\sqrt{m_B m_{D^*}}\, g_7 \varepsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v'^\alpha v^\beta \\ &\left\langle D(v')\left|\bar{c}\gamma_\mu\gamma_5 b\right| B^*(v\varepsilon)\right\rangle = \sqrt{m_{B^*} m_D}\left\{g_8 \varepsilon_\mu\right. \end{aligned}$$

$$+\varepsilon\cdot v'\left[g_9(v+v')_\mu+g_{10}(v-v')_\mu\right]\Big\},$$

$$\left\langle D^*(v',\varepsilon')\left|\bar c\gamma_\mu b\right|B^*(v,\varepsilon)\right\rangle=\sqrt{m_{B^*}m_{D^*}}\Big\{\varepsilon\cdot\varepsilon'^*\left[g_{11}(v+v')_\mu+g_{12}(v-v')_\mu\right]$$

$$+g_{13}\varepsilon'^*\cdot v\varepsilon_\mu+g_{14}\varepsilon\cdot v'\varepsilon'^*_\mu+\varepsilon\cdot v'\varepsilon'^*\cdot v\left[g_{15}(v+v')_\mu+g_{16}(v-v')_\mu\right]\Big\},$$

$$\left\langle D^*(v',\varepsilon')\left|\bar c\gamma_\mu\gamma_5 b\right|B^*(v,\varepsilon)\right\rangle=\sqrt{m_{B^*}m_{D^*}}\Big\{\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\alpha\varepsilon'^{*\beta}\left[g_{17}(v+v')^\nu\right.$$

$$\left.+g_{18}(v-v')^\nu\right]+g_{19}\varepsilon\cdot v'\varepsilon_{\mu\nu\alpha\beta}\varepsilon'^{*\nu}v'^\alpha v^\beta+g_{20}\varepsilon'^*\cdot v\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v'^\alpha v^\beta\Big\}. \qquad (5)$$

These form factors may be related to the six unknown functions a_i in the way indicated in the last section

$$\left\langle D(v')\left|\bar c\gamma_\mu h_v^{(b)}\right|B(v)\right\rangle=\frac{\sqrt{m_Bm_D}}{2}\mathrm{Tr}\left[\gamma_5\left(a_1+a_2\not v'\right)\gamma_\mu M_B(v)\right]$$

$$=-\sqrt{m_Bm_D}\left(a_1v_\mu+a_2v'_\mu\right),$$

$$\left\langle D^*(v',\varepsilon')\left|\bar c\gamma_\mu\left(1-\gamma_5\right)h_v^{(b)}\right|B(v)\right\rangle$$

$$=\frac{\sqrt{m_Bm_{D^*}}}{2}\mathrm{Tr}\left\{\left[\not\varepsilon'^*\left(a_3+a_4\not v'\right)+\varepsilon'^*\cdot v\left(a_5+a_6\not v'\right)\right]\gamma_\mu\left(1-\gamma_5\right)M_B(v)\right\}$$

$$=\sqrt{m_Bm_{D^*}}\left[\varepsilon'^*_\mu\left(a_3+wa_4\right)-a_4\varepsilon_{\mu\nu\alpha\beta}\varepsilon'^{*\nu}v'^\alpha v^\beta-a_5\varepsilon'^*\cdot vv_\mu\right.$$

$$\left.+\varepsilon'^*\cdot vv'_\mu\left(a_6-a_4\right)\right],$$

$$\left\langle D(v')\left|\bar c\gamma_\mu\left(1-\gamma_5\right)h_v^{(b)}\right|B^*(v,\varepsilon)\right\rangle$$

$$=\frac{\sqrt{m_{B^*}m_D}}{2}\mathrm{Tr}\left[\gamma_5\left(a_1+a_2\not v'\right)\gamma_\mu\left(1-\gamma_5\right)M_{B^*}(v)\right]$$

$$=\sqrt{m_{B*}m_D}\left[-ia_2\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v'^\alpha v^\beta+\varepsilon_\mu\left(a_1+wa_2\right)-a_2\varepsilon\cdot v'v_\mu\right],$$

$$\left\langle D^*(v',\varepsilon')\left|\bar c\gamma_\mu\left(1-\gamma_5\right)h_v^{(b)}\right|B^*(v,\varepsilon)\right\rangle$$

$$=\frac{\sqrt{m_{B^*}m_{D^*}}}{2}\mathrm{Tr}\left\{\left[\not\varepsilon'^*\left(a_3+a_4\not v'\right)+\varepsilon'^*\cdot v\left(a_5+a_6\not v'\right)\right]\gamma_\mu(1-\gamma_5)M_{B^*}(v)\right\}$$

$$=\sqrt{m_{B^*}m_{D^*}}\Big\{a_6\varepsilon'^*\cdot v\varepsilon\cdot v'v_\mu+\varepsilon'^*\cdot\varepsilon\left(a_3v_\mu+a_4v'^*_\mu\right)$$

$$+\varepsilon'^*\cdot v\left(a_5-a_3-wa_6\right)-a_4\varepsilon\cdot v'\varepsilon'^*_\mu$$

$$+i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\alpha\varepsilon'^{*\beta}\left(a_3v^\nu+a_4v'^\nu\right)+ia_6\varepsilon'^*\cdot v\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\nu v^\alpha v'^\beta\Big\}, \qquad (6)$$

where $w=v\cdot v'$ In the above, we have used the spin symmetry that exists for the b quark to relate the vector-current matrix-elements to those of the axial-vector-current, and the form factors for the decays of the B meson to those of the B^* meson.

Note that the directly accessible decays $B\to D^{(*)}\ell\nu$, that proceed via the left-handed current, require six form factors, so that the information obtained here may appear to be of somewhat limited scope. However, if, for example, we let $\Gamma=\gamma_\mu\gamma_\nu$ and $\Gamma=\gamma_\mu\gamma_\nu\gamma_5$ in eqn. (3) and compare with eqn. (4), we see that the numerous form factors of (4) may be written in terms of the same six functions.

The relationships between the form factors of eqn. (5) and the a_i are

$$g_1=-\frac{1}{2}(a_1+a_2),\quad g_2=\frac{1}{2}(a_2-a_1),\quad g_3=g_{14}=-a_4,\quad g_4=-(a_3+wa_4),$$

$$g_5=\frac{1}{2}(a_4+a_5-a_6);\quad g_6=\frac{1}{2}(a_5-a_4+a_6),\quad g_7=-2g_9=-2g_{10}=-a_2,$$

$$g_8=-(a_1+wa_2),\quad g_{11}=-g_{17}=\frac{1}{2}(a_3+a_4),\quad g_{12}=-g_{18}=\frac{1}{2}(a_3-a_4),$$

$$g_{13}=a_5-a_3-wa_6,\quad g_{15}=g_{16}=-\frac{1}{2}g_{20}=\frac{1}{2}a_6,\quad g_{19}=0. \qquad (7)$$

If we now proceed to the scale $\mu=m_c$, integrate out the c quark, and perform a $1/m_c$ expansion, all of the relationships among form factors shown above must remain intact. This must also be true if radiative corrections of order $\alpha_s(m_c)$ are considered. That these results are indeed preserved can be seen by noting that the effect of any new operator may be taken into account in a straightforward and intuitively obvious manner. At leading order, one uses the relations analogous to eqn. (2), and a typical matrix element is, for example

$$\left\langle D(v')\left|\bar h_{v'}^{(c)}\gamma_\mu h_v^{(b)}\right|B(v)\right\rangle=\frac{\sqrt{m_Bm_D}}{2}\mathrm{Tr}\left[\bar M_D(v')\gamma_\mu M_B(v)\right]. \qquad (8)$$

Beyond leading order in the $1/m_c$ expansion, Falk and Neubert [5] have shown that one can write

$$D^{(*)}(v')\to\left(\frac{1+\not v'}{2}\right)\left(\begin{matrix}-\gamma_5\\ \not\varepsilon'\end{matrix}+\frac{L_+^{D^{(*)}}}{2m_c}+\frac{\ell_+^{D^{(*)}}}{4m_c^2}+\ldots\right)$$

$$+\left(\frac{1-\not v'}{2}\right)\left(\frac{L_-^{D^{(*)}}}{2m_c}+\frac{\ell_-^{D^{(*)}}}{4m_c^2}+\ldots\right), \qquad (9)$$

where

$$L_+^D = L_1\left(-\gamma_5\right), \quad L_+^{D^*} = L_2 \not\varepsilon' + L_3 \varepsilon' \cdot v,$$
$$L_-^D = L_4\left(-\gamma_5\right), \quad L_-^{D^*} = L_5 \not\varepsilon' + L_6 \varepsilon' \cdot v, \tag{10}$$

with similar definitions for the ℓ's. The L's and ℓ's contain all the new form factors that arise. In an obvious way, we generalize and simplify this notation by writing

$$D^{(*)}(v') \to \left(\frac{1+\not v'}{2}\right)\mathcal{L}_+^{D^{(*)}} + \left(\frac{1-\not v'}{2}\right)\mathcal{L}_-^{D^{(*)}}, \tag{11}$$

with

$$\mathcal{L}_+^D = \mathcal{L}_1^c\left(-\gamma_5\right), \quad \mathcal{L}_+^{D^*} = \mathcal{L}_2^c \not\varepsilon' + \mathcal{L}_3^c \varepsilon' \cdot v,$$
$$\mathcal{L}_-^D = \mathcal{L}_4^c\left(-\gamma_5\right), \quad \mathcal{L}_-^{D^*} = \mathcal{L}_5^c \not\varepsilon' + \mathcal{L}_6^c \varepsilon' \cdot v. \tag{12}$$

At leading order $\mathcal{L}_1$ and $\mathcal{L}_2$ are unity, while all the other $\mathcal{L}_i$ vanish. Each of the $\mathcal{L}_i$ will contain terms that arise at different orders in the $1/m_c$ expansion. Generally, as discussed in [5], $\mathcal{L}_1$, $\mathcal{L}_2$ and $\mathcal{L}_3$ arise from higher order Lagrangian terms, while $\mathcal{L}_4$, $\mathcal{L}_5$ and $\mathcal{L}_6$ arise from local corrections to the current. With these form factors, the general form factors are obtained from those for the heavy to light transitions by identifying

$$a_1 = \mathcal{L}_1^c + \mathcal{L}_4^c, \quad a_2 = \mathcal{L}_1^c - \mathcal{L}_4^c, \quad a_3 = \mathcal{L}_2^c + \mathcal{L}_5^c,$$
$$a_4 = \mathcal{L}_2^c - \mathcal{L}_5^c, \quad a_5 = \mathcal{L}_3^c + \mathcal{L}_6^c, \quad a_6 = \mathcal{L}_3^c - \mathcal{L}_6^c. \tag{13}$$

Detailed comparison of (7) with (12) reveals two facts that may be worth noting: g_1, g_{11} and g_{17} receive contributions only from Lagrangian corrections and leading order terms; g_2, g_{12} and g_{18} receive contributions only from terms involving local corrections to the current.

To illustrate the scope of the relationships among the form factors, let us choose the first six, $g_1 \ldots g_6$, to be independent, and express the remaining fourteen form factors in terms of these six. The resulting relationships that are untouched by $1/m_c$ corrections are

$$g_7 = -2g_9 = -2g_{10} = g_1 - g_2, \quad g_8 = (1+w)\,g_1 + (1-w)\,g_2,$$
$$g_{11} = -g_{17} = -\frac{1}{2}\left[(1-w)\,g_3 + g_4\right], \quad g_{12} = -g_{18} = \frac{1}{2}\left[(1+w)\,g_3 - g_4\right],$$
$$g_{13} = g_4 + (1+w)\,g_5 + (1-w)\,g_6, \quad g_{14} = g_3,$$
$$g_{15} = g_{16} = -\frac{1}{2}g_{20} = \frac{1}{2}\left(g_6 - g_3 - g_5\right), \quad g_{19} = 0. \tag{14}$$

Clearly, if we were interested in the matrix elements of $\gamma_\mu\gamma_\nu$ and $\gamma_\mu\gamma_\nu\gamma_5$, or some other more complicated current, the number of independent form factors would remain at six, so that the number of relationships that are preserved to all orders in the $1/m_c$ expansion would increase.

We have used recent results [5] to check our claims for the left-handed current. Identifying the terms that arise in the $1/m_c$ expansion in the results of [5] ($1/m_c$, $1/m_c^2$), translating between form factors, and substituting into (7) or (14), we find that all the form factor relationships that can be compared are indeed satisfied. Furthermore, we claim that the relationships of (7) and (14) are still satisfied even when radiative corrections proportional to powers of $\alpha_s(m_c)$ are taken into account.

The reason for this is as discussed in [1]: between $\mu = m_b$ and $\mu = m_c$ no radiative corrections proportional to powers of $\alpha_s(m_c)$ to these relationships can arise. At $\mu = m_c$, the effective theory with only a heavy b quark must be matched onto the effective theory with heavy b and c quarks. In the latter effective theory, there can be radiative terms proportional to powers of $\alpha_s(m_c)$, but matching on to the former effective theory requires that the corrections from these terms to the relationships above must vanish.

In a similar manner, we can examine the $1/m_b$ expansion. Applying the discussion between equations (9) and (13) to the B meson, we write

$$B(v) \to \mathcal{L}_1^b\left(\frac{1+\not v}{2}\right)\left(-\gamma_5\right) + \mathcal{L}_4^b\left(\frac{1-\not v}{2}\right)\left(-\gamma_5\right),$$
$$B^*(v,\varepsilon) \to \mathcal{L}_2^b\left(\frac{1+\not v}{2}\right)\not\varepsilon + \mathcal{L}_3^b\left(\frac{1+\not v}{2}\right)\varepsilon\cdot v'$$
$$+\,\mathcal{L}_5^b\left(\frac{1-\not v}{2}\right)\not\varepsilon + \mathcal{L}_6^b\left(\frac{1-\not v}{2}\right)\varepsilon\cdot v'. \tag{15}$$

With these forms, the contributions of terms in the $1/m_b$ expansion to the general form factors are

$$g_1 = -\mathcal{L}_1^b, \quad g_2 = \mathcal{L}_4^b, \quad g_3 = -2g_5 = -2g_6 = \mathcal{L}_4^b - \mathcal{L}_1^b,$$
$$g_4 = (w-1)\,\mathcal{L}_4^b - (w+1)\,\mathcal{L}_1^b, \quad g_7 = g_{13} = \mathcal{L}_5^b - \mathcal{L}_2^b,$$
$$g_8 = (w-1)\,\mathcal{L}_5^b - (w+1)\,\mathcal{L}_2^b, \quad g_9 = \frac{1}{2}\left(\mathcal{L}_2^b - \mathcal{L}_5^b\right) + \mathcal{L}_6^b,$$
$$g_{10} = \frac{1}{2}\left(\mathcal{L}_2^b - \mathcal{L}_5^b\right) - \mathcal{L}_3^b, \quad g_{11} = -g_{17} = \mathcal{L}_2^b, \quad g_{12} = -g_{18} = -\mathcal{L}_5^b,$$
$$g_{14} = (1-w)\,\mathcal{L}_3^b + (1+w)\,\mathcal{L}_6^b - \mathcal{L}_2^b - \mathcal{L}_5^b,$$
$$g_{15} = -g_{16} = -\frac{1}{2}g_{19} = \frac{1}{2}\left(\mathcal{L}_3^b - \mathcal{L}_6^b\right), \quad g_{20} = 0. \tag{16}$$

One could also write the $1/m_b$ analogues of (14) for these equations. Now, however, we must choose six different form factors as the independent ones, since $g_1 \dots g_6$ are no longer independent. Using the results of [5], we find that these relationships among the form factors are indeed satisfied up to order $1/m_b^2$. In the case of the $1/m_b$ expansion, all of the relationships above will, in general receive corrections from terms proportional to powers of $\alpha_s(m_b)$. We can say nothing about the effects of such radiative corrections without further study.

IV. CONCLUSION

We have used HQET to show that the set of matrix elements of the form $< D^{(*)}(v')|\bar{c}\Gamma b|B^{(*)}(v) >$ can be described by at most six form factors, to any order in the $1/m_c$ expansion, to leading order in $1/m_b$. This implies, in general, many relationships among the usual form factors that may be defined in terms of Lorentz covariance alone. Furthermore, we have argued that while the form factors themselves may receive radiative corrections from terms proportional to powers of $\alpha_s(m_c)$, the relationships among the form factors do not. We have also shown that a different set of relationships are preserved in the $1/m_b$ expansion. All of the relationships among form factors are modified by terms of the form $1/m_b m_c$, and by radiative terms proportional to powers of $\alpha_s(m_b)$.

Although we have focused mainly on the form factors appropriate to the left-handed current, our statements above are independent of the form of the particular current being considered. We illustrated this by briefly pointing out the increased power of the predictions if we considered the cases $\Gamma = \gamma_\mu\gamma_\nu$ and $\Gamma = \gamma_\mu\gamma_\nu\gamma_5$.

The results of this work, and of the work of [1], implies the existence of some underlying general symmetry principle, which must result from the group structure appropriate to HQET. In fact, by examining the group structure of HQET in detail, one should be able to see which relationships among form factors will receive no corrections in the $1/m_b$ or $1/m_c$ expansions. Such a study may be undertaken in the near future.

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