CEBAF

The Continuous Electron Beam Accelerator Facility

Theory Group Preprint Series

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150

CEBAF-TH-93-12

QUALITATIVE AND QUANTITATIVE ASPECTS OF THE QCD THEORY OF ELASTIC FORM FACTORS [*]

A.V.RADYUSHKIN[†]

Physics Department, Old Dominion University, Norfolk, VA 23529, USA

and

Continuous Electron Beam Accelerator Facility

Newport News, VA 23606, USA

A long-standing controversy about the early applicability of perturbative QCD to elastic form factors is reviewed with a particular emphasis on the qualitative and quantitative aspects of the modern QCD picture of hadronic structure.

I. INTRODUCTION: SOME HISTORY COMMENTS

The foundations of the perturbative QCD (pQCD) theory of hard elastic processes were developed in 1977-1978 in Irkutsk [1] and Dubna [2–4], however, both groups failed then to publish their results in the West. The summary of the Dubna 77-78 results (which include a complete derivation of the QCD factorization in Feynman gauge [3, 4], introduction of the relevant wave function and its relation to matrix elements of local operators [2], evolution equation for the pion wave function [3, 4], Gegenbauer expansion and its relation to conformal operators [4], derivation of the asymptotic wave function [3, 4], etc.) appeared in Physics Letters only in 1980 [5].

By 1979, when the pQCD approach was rediscovered in a more heuristic form by Brodsky and Lepage [6], the two Russian groups already knew that the asymptotic QCD formulas with the asymptotic wave functions cannot describe the form factor data at accessible momentum transfers. To find the way out, Chernyak and A. Zhitnitsky proposed a double-humped wave function for the pion and incorporated the QCD sum rules to justify it [7]. In essence, their claim was that the pQCD predictions based on the lowest order, lowest twist diagrams calculated with CZ wave function are phenomenologically sound and that no further improvements are necessary.

On the other hand, my concern was just how stable are the pQCD "successes" with respect to possible improvements like higher order and/or higher twist corrections. We started our analysis of the "improvement stability" by calculating the next-to-leading order α_s corrections to the pion form factor [8]. Our conclusion was that [8]

> "for accessible momentum transfers the average virtualness of the exchanged gluon is very small compared to the typical hadronic scale $M \sim 0.5\,GeV$. In other words, for $Q^2 \lesssim 100\,GeV^2$ the pion form factor is not a truly short-distance problem and to understand the behaviour of $F_\pi(Q)$ for moderately large Q^2 (in particular, to clarify the true nature of the quark counting rules ...) one should develop methods of taking into account the effects usually referred to as power (or higher twist) corrections."

In ref. [9], I performed a qualitative analysis of the higher twist effects, taking into account both the higher twist contributions due to the non-leading two-body operators $\bar{q}\gamma_5 q$ and $\bar{q}\gamma_5\sigma_{\mu\nu}q$ and those due to the intrinsic transverse momentum of the quarks. In the latter case, I proceeded by

1. modifying the hard propagators $xyQ^2 \to xyQ^2 + 2M^2$,
2. using a "frozen" coupling constant $\alpha_s(2M^2)$ for small virtualities and
3. incorporating the Sudakov suppression of the small virtuality regions.

This paper was also rejected and never published. Much later, in 1987, a model incorporating steps 1) and 2) was used by C.R. Ji et al. [10]. A recent work by Li and Sterman [11] incorporates (in a more sophisticated framework) all three steps.

I have learned a few lessons from my 1980 study. First, I realized that, after the transverse momentum modification, the perturbative QCD formula for the pion form factor might be self-consistent if Λ_{QCD} is small enough, $\Lambda \sim 100$ or $200\,MeV$ (such a low value was advocated then by the ITEP group, but most of the other QCD experts were still living in the world where Λ was around $0.5\,GeV$). I observed also that the numerical value of $Q^2F_\pi(Q^2)$ given by the

modified formula is too low to describe the data. Still, I managed to describe the data by adding the contribution of the higher-twist two-body operators $\bar{q}\gamma_5 q$ and $\bar{q}\gamma_5\sigma_{\mu\nu}q$ calculated using the same assumptions. However, an important observation was that these contributions were dominated by the region $x \sim 1/Q^2$. Hence, the essential gluon virtualities were $O(M^2)$ and constant with increasing Q^2, therefore, the correct way was to treat them as a part of soft contribution. The latter is usually suppressed by powers of $1/Q^2$ in the asymptotic $Q^2 \to \infty$ limit, but it cannot be classified as a higher-twist contribution, since no short-distance dynamics is involved. In other words, the soft contribution is completely nonperturbative, and one should use essentially nonperturbative methods to calculate it.

A year and a half later, the soft contribution to the pion pion form factor was calculated within the QCD sum rule approach [12, 13]. As expected, this contribution was large enough to describe the data. In 1983, the soft term for the nucleon form factors was estimated using the local duality version of the QCD sum rules [14]. A parameter-free formula, with a good accuracy, described the proton magnetic form factor data from $Q^2 \sim 3\,GeV^2$ till $Q^2 \sim 20\,GeV^2$. The last sentence of our paper [14] was:

> "the experimentally observed power-law fall-off of the nucleon form factors reflects only the finite size of the nucleons rather than the approximate short-distance scale invariance of the underlying theory."

In 1984, similar statements were made by Isgur and Llewellyn Smith [15].

Meanwhile, we extended the QCD sum rule analysis of the pion form factor into the region of small Q^2 [16]. Combining this calculation with the previous results [13], we obtained a continuous description of the pion form factor from the normalization point $Q^2 = 0$, where $F_\pi = 1$ till moderately large values $Q^2 \sim 3GeV^2$. For the nucleon form factors, similar QCD sum rule analyses of the low-Q^2 behaviour were developed at ITEP [17] and Gatchina [18].

Thus, the soft contributions alone were sufficient to describe the data: apparently, there was no need for sizable hard contributions, and, hence, there was no place for broad Chernyak-Zhitnitsky type wave functions: otherwise the total "soft plus hard" contribution would be too large. However, the CZ wave functions were also supported by a QCD sum rule calculation, and it was not clear why we should not believe in them. The answer to this question was found in 1986: the CZ analysis implied a specific model of the QCD vacuum, in which the correlation length of the nonperturbative vacuum fluctuations is essentially larger than the hadronic size [19]. In such a case, it is legitimate to take into account only the simplest parameters $\langle 0|\bar{q}q|0\rangle$, $\langle 0|GG|0\rangle$ (local condensates - all fields are taken at the same point) describing the structure of the QCD vacuum.

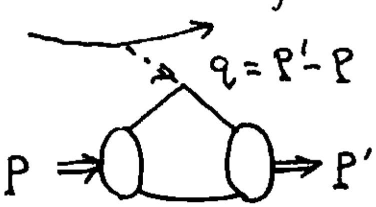
However, if the correlation length of the vacuum fluctuations is comparable with the observed hadronic size (this is just the situation realized in the real world), one should know the whole function $\langle 0|\bar{q}(0)q(x)|0\rangle$, *i.e.*, the nonlocal condensate. The wave function we obtained using a standard value for the correlation length parameter is rather narrow, and the relevant hard contribution is sufficiently small. For self-consistency, we also applied the formalism of nonlocal condensates to the calculation of the soft contribution to the pion form factor [20] and enjoyed a reasonably good agreement with the data.

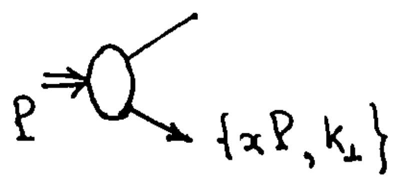
In what follows, I will concentrate on emphasizing that applicability of perturbative QCD to elastic processes is a quantitative rather than a qualitative problem. There is no question of whether pQCD is applicable to elastic form factors at very large Q^2, the question is whether it is applicable now, at accessible Q^2. And the question is not whether a particular approximation producing a curve describing the data is in general compatible with QCD, the question is whether such an approximation is compatible with all information about QCD extracted from all we know about the real dynamics of the strong interactions.

II. WAVE FUNCTIONS AND FORM FACTORS

In nonrelativistic quantum mechanics, the form factor is given by a Fourier transform of the relevant wave function squared. In (light-cone) quantum field theory, the form factor of a two-body bound state is given by a convolution

$$F(q^2) \sim \int \psi_P(x, k_\perp)\, \psi_P(x, k_\perp + xq)\, d^2k_\perp dx \tag{2.1}$$





involving initial and final state wave functions $\psi_P(x, k_\perp)$ depending on the longitudinal xP and transverse $k_\perp$ momenta carried by the quarks. At $Q^2 = 0$, one has the normalization condition

$$1 = F(0) \sim \int |\psi_P(x, k_\perp)|^2\, d^2k_\perp dx. \tag{2.2}$$

The form of the wave function is determined by the interactions between the constituents. As far as we know, in real QCD these interactions can be represented by a potential which is essentially Coulombic ($\sim \alpha_s(1/r)/r$) at short distances r and increases linearly at large r. The confining part of the potential is responsible for the major nonperturbative effect of chiral symmetry breaking, converting almost massless light quarks into the constituent quarks with masses around $300\,MeV$. It is this effect that establishes the basic mass scale for the hadronic spectrum. The role of the Coulombic part is secondary, and one can treat it as a small correction. Consequently,one should expect that the $k_\perp$-dependence of the wave function can be visualized as a sum of two components: the soft component due to the dominant confinement part of the potential and the hard part due to the short-distance Coulomb interactions:

$$\psi = \psi^{hard} + \psi^{soft}. \tag{2.3}$$

The soft component should be a fast-decreasing function concentrated in the region of small $k_\perp$. On the other hand, the hard component dominates at large $k_\perp$, and it is proportional to the Fourier transform of the Coulomb potential: $\psi^{hard} \sim \alpha_s(k_\perp^2)/k_\perp^2$.

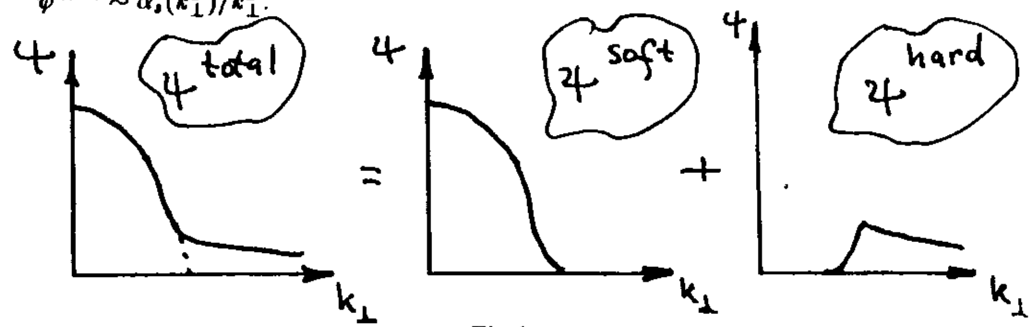
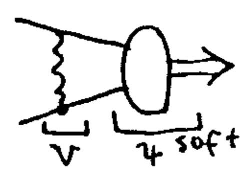


Fig.1

The most natural decomposition of ψ into its soft and hard parts is shown in fig.(1). Under such a convention, ψ^{hard} is zero (or very small) below some value of $k_\perp \sim 1\,GeV$. Since one can apply perturbative QCD in the large-$k_\perp$ region, one can represent the hard tail of the wave function as a convolution of the Coulomb (one-gluon exchange) kernel $V^{hard} \sim \alpha_s(k_\perp^2)/k_\perp^2$ with the soft part of the wave function:

$$\psi^{hard} = V^{hard} \otimes \psi^{soft}. \tag{2.4}$$

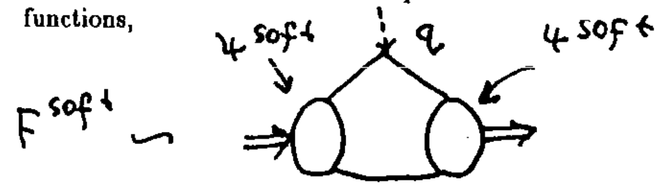


Now, one can rewrite the form factor formula as

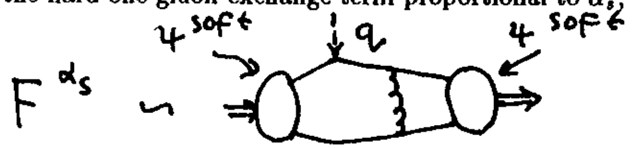
$$F = \int (\psi^{soft} + \psi^{hard}) \times (\psi^{soft} + \psi^{hard}) = F^{soft} + F^{\alpha_s} + \ldots . \qquad (2.5)$$

This is just the QCD factorization expansion which states that the pion form factor in QCD is given by

1. the soft term which can be interpreted as a convolution of two soft wave functions,



2. the hard one-gluon-exchange term proportional to α_s,



3. various radiative and higher-twist corrections to the hard term.

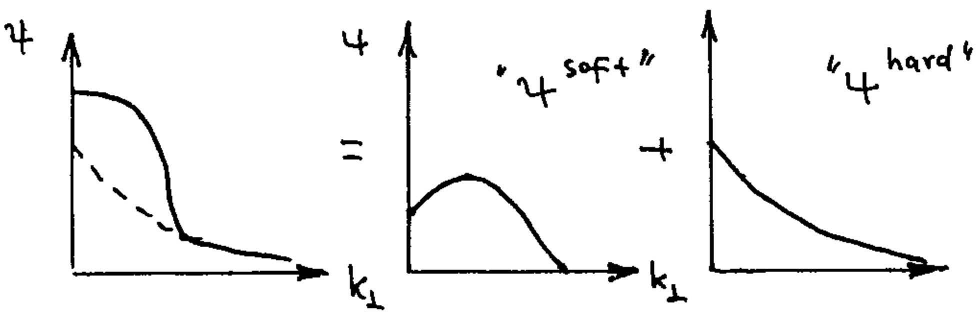


Fig.2

From the above discussion, it is clear that the separation of the wave function into soft and hard components is prescription-dependent. An extreme version (see Fig.2) is to define the hard component for all $k_\perp$ as a smooth extrapolation from high-$k_\perp$ values using, *e.g.*, the prescription $\psi^{hard} \sim \alpha_s(k_\perp^2+M^2)/(k_\perp^2+M^2)$. This simple trick of increasing the "perturbatively calculable "hard"-gluon-exchange contribution" is very popular among the advocates of the early applicability of perturbative QCD.

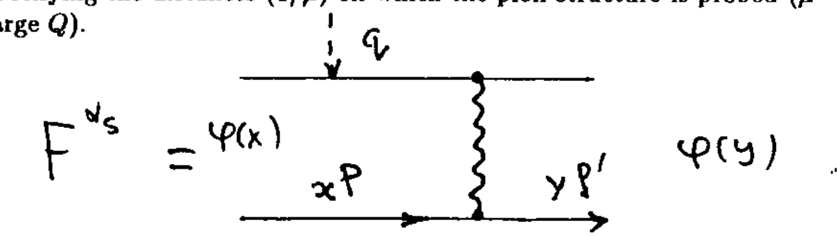
There are different ways to introduce and/or interpret the scale M: it can be treated either as an effective gluon mass or as an averaged transverse momentum (with variations: the transverse momentum can be either primordial, *i.e.*, induced by the finite size of the system or generated by radiative effects). Of course, it is the numerical value of M rather than its interpretation which is crucial: if M is $500\,MeV$ or larger, the one-gluon-exchange term is too small to describe the data. That is why the pQCD proponents favor small values $M \lesssim 300 MeV$ and sometimes even $M \sim 100\,MeV$. However, any introduction of M motivated by physics of the real pion, *e.g.*, interpretation of M as an averaged transverse momentum $M^2 = \langle k_\perp^2 \rangle + \langle k_\perp'^2 \rangle$ or as an effective gluonic mass, excludes low values of M.

III. INTERPLAY BETWEEN SOFT AND HARD CONTRIBUTIONS

The asymptotic expression for the hard contribution $F^{\alpha_s}(Q^2)$ in the pion case reads [2]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\ \varphi(x,\mu)\ \varphi(y,\mu) \frac{8\pi}{9} \frac{\alpha_s(\mu)}{xyQ^2}, \qquad (3.1)$$

where $\varphi(x,\mu)$ is the pion wave function [2] giving the probability amplitude to find the pion in a state where quarks carry fractions xP and $(1-x)P$ of its longitudinal momentum P and μ is the evolution parameter of the wave function specifying the distances $(1/\mu)$ on which the pion structure is probed ($\mu \sim Q$ for large Q).



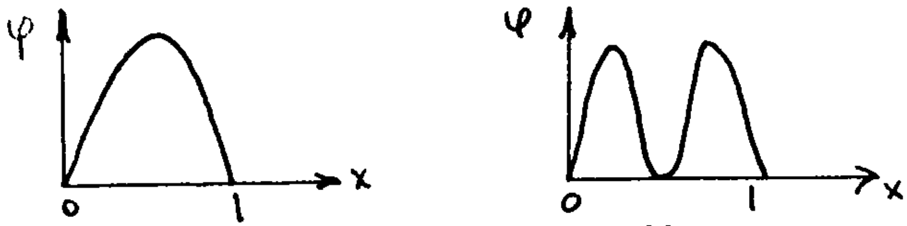
The logarithmic dependence of $\varphi(x,\mu)$ on the factorization scale μ is given by the renormalization group [3,4]. In particular, as $\mu \to \infty$, the pion wave function $\varphi(x,\mu)$ evolves to a very simple and natural form [3–5]

$$\varphi_\pi(x, \mu \to \infty) \to \varphi_\pi^{as}(x) = 6 f_\pi x(1-x), \qquad (3.2)$$

where $f_\pi = 133 MeV$ is the pion decay constant setting the wave function normalization [2]. In practice, however, the logarithmic evolution of the wave function is very mild and one can usually neglect it. In this approximation , the large-Q^2 value of the combination $Q^2 F_\pi(Q^2)$ is determined by the magnitude of I [2]:

$$I = \left| \int_0^1 \frac{\varphi(x)}{x} \right|^2 . \qquad (3.3)$$

For the asymptotic wave function [3, 4], $I = \frac{1}{4}$, and the resulting value for $Q^2F_\pi(Q^2)$ is [3, 4] $8\pi^2 f_\pi^2 \left(\frac{\alpha_s}{\pi}\right) \approx \frac{\alpha_s}{\pi} 1.4\,GeV^2 \lesssim 0.14\,GeV^2$, while the experimental data suggest that $Q^2F_\pi(Q^2) \approx 0.4\,GeV^2$ for $Q^2 \sim 3\,GeV^2$.



The use of the Chernyak-Zhitnitsky wave function [7]

$$\varphi_\pi^{CZ}(x) = 30 f_\pi x(1-x)(1-2x)^2, \qquad (3.4)$$

increases I by factor $\frac{25}{9}$. The increase is due to the enhancement of the small-x region provided by the CZ wave function. The large-Q^2 value of $Q^2F_\pi(Q^2)$ in this case coincides with that corresponding to data at $Q^2 \sim 3\,GeV^2$. Of course, whether the hard term reaches its large-Q^2 value at $Q^2 \sim 3GeV^2$, is an open question.

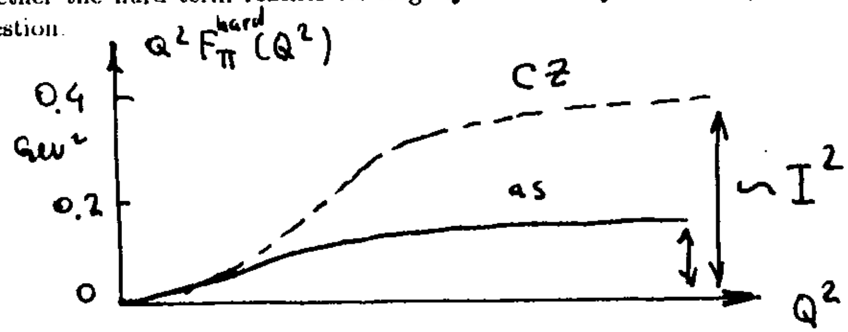


Fig.3

Let us turn now to the soft term. By construction, the soft part of the wave function vanishes faster than $1/k_\perp^2$ for large $k_\perp$. This means the soft contribution is suppressed by at least one power of $1/Q^2$ compared to the hard term. Furthermore, the large-Q behaviour of the form factor integral (2.1) in this case is is dominated by the $x \sim 1/Q$ region, and the result is determined by the small-x behaviour of the hadronic wave function. Physically, this means that the large-q behaviour of $F^{soft}(Q^2)$ is determined by the configuration when the active (quark) parton carries the bulk of the hadron momentum while the spectator(s) take a wee $\sim 1/Q$ fraction of it. This is the essence of the mechanism formulated by Feynman in his 1972 book [21]. Depending on a particular model for the soft wave function, one can obtain different curves for $Q^2F_\pi(Q^2)$ (see Fig.4)

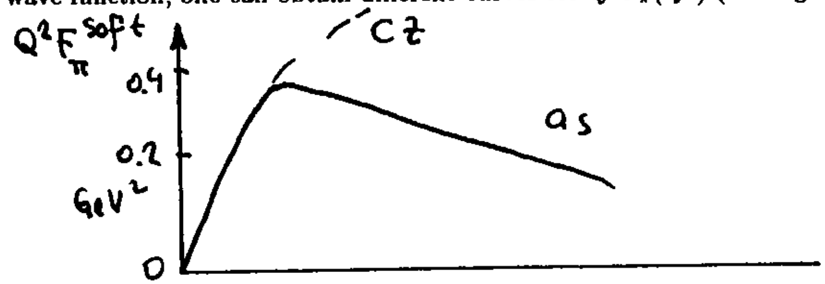
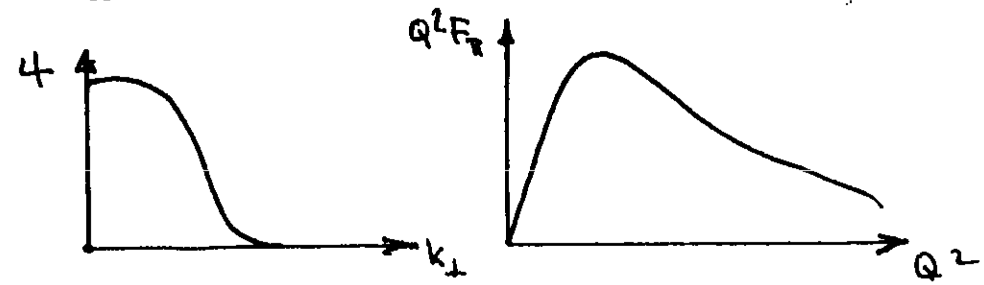


Fig.4

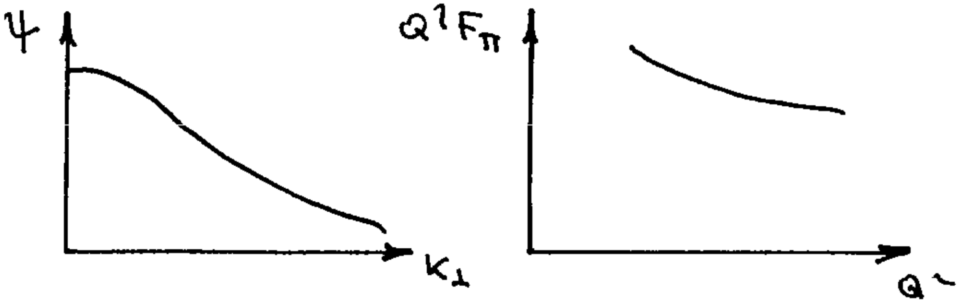
A particular prediction for the pion form factor is a sum of a soft term and of a hard term. Of course, a specific choice of the soft and hard terms should not be arbitrary: it must be correlated with the assumed structure of the total wave function. There are essentially three scenarios on the market.

1. "TAILLESS" W.F. In this approach, the high-$k_\perp$ tail of the wave function is completely neglected, and the total contribution is given by the soft term only. Almost all form factor calculations within the constituent quark models fall into this category. It should be mentioned that some authors reached a good agreement with existing data. A general trend is that in order to describe the data at highest available Q^2, one should use wave functions with a power-law behaviour at large $k_\perp$ rather than the exponential ones suggested by oscillator models.

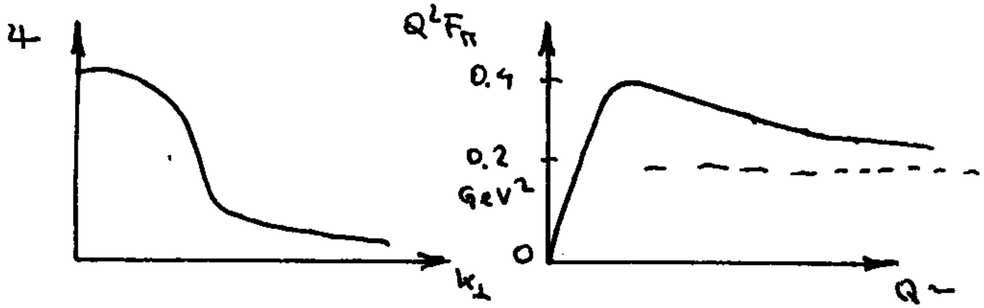


2. "HEADLESS" W.F. This is an opposite approach: the soft contribution is essentially ignored or claimed to be negligibly small - the point of view shared by the advocates of the early applicability of perturbative QCD.

Such a scenario can be realized when the total wave function is just a regularized extrapolation $\psi_\pi(x,k_\perp) \sim \alpha_s/(k_\perp^2+M^2)$ of its high-$k_\perp$ asymptotics. In this picture, the role of the nonperturbative effects reduces to the change $1/k^2 \to 1/(k^2+M^2)$ in the propagators.



3. W.F. WITH LARGE "HEAD" AND SMALL "TAIL". As argued above, just this form of the wave function is favored by the present understanding of how the hadrons are formed from the quarks in QCD.



To specify the quantitative aspects of the 3rd scenario one should use a particular QCD-based picture of the hadronic structure. Our picture is based on QCD sum rules [22] and quark-hadron duality considerations. In application to the pion form factor, our version of the 3rd scenario can be described in the following way:

- The effective coupling constant is always small inside the pion (and all other hadrons): $\alpha_s/\pi \lesssim 0.1$. This is the standard feature of any QCD sum rule based approach.
- The pion wave function $\varphi(x)$ (or $\psi(x,k_\perp)$ integrated over $k_\perp$) is rather close to its asymptotic form $\varphi^{as}(x) = 6f_\pi x(1-x)$. This statement means that the pion wave function is approximately dual to the wave function of free $\bar{q}q$ states.

- For small Q^2, when $Q^2 \lesssim 0.6\,GeV^2$, the pion form factor behaves like $1/(1+Q^2/(0.6\,GeV^2))$ [16]. This result was obtained using a specific version of the QCD sum rules adjusted to handle the small-Q^2 region and it is in agreement with the ρ-dominance. The whole contribution is soft: no α_s-corrections were included.
- At intermediate momentum transfers, $0.6\,GeV^2 \lesssim Q^2 \lesssim 3\,GeV^2$, the soft contribution can be approximated by the dipole formula [13]

$$F_\pi^{soft} = \frac{1}{\sqrt{2}} \frac{1}{\left(1+\frac{Q^2}{\sqrt{2}s_0}\right)^2} \tag{3.5}$$

where $s_0 \approx 4\pi^2 f_\pi^2 \approx 0.7\,GeV^2$ is the pion duality interval in the axial current channel (the effective threshold for higher states production).

- The one-gluon exchange contribution (without separation into soft and hard part) is approximately given by the monopole formula [23]

$$F_\pi^{\alpha_s}(Q^2) = \left(\frac{\alpha_s}{\pi}\right)\frac{1}{1+Q^2/2s_0} \tag{3.6}$$

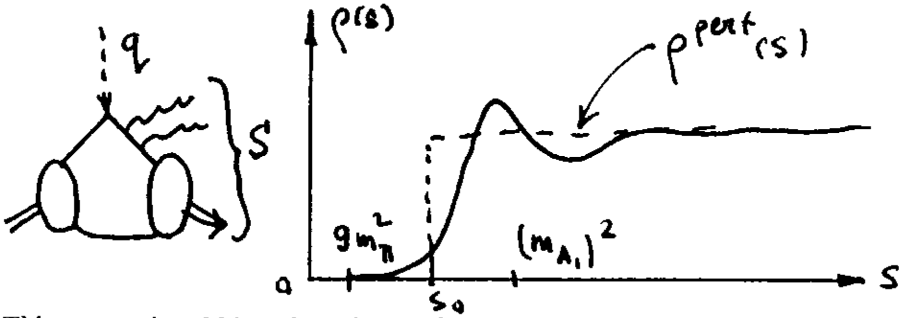
based on the interpolation of the local duality integral between the $Q^2=0$ value $F_\pi^{\alpha_s}(0) = \alpha_s/\pi$ (this value is known from the Ward identity between the 3-point and 2-point correlators) and the asymptotic behaviour $F_\pi^{\alpha_s}(Q^2) = 8\pi\alpha_s f_\pi^2/Q^2 = 2(s_0/Q^2)(\alpha_s/\pi)$ corresponding to the asymptotic wave function.

- The soft contribution is further suppressed by Sudakov form factor given by the formula [14]

$$S(Q^2) \approx \exp\left\{-\frac{\alpha_s}{2\pi}C_F \ln^2\left(\frac{Q^2}{s_0}\right)\right\}. \tag{3.7}$$

To stress our point, I ignored in this formula the next-to-leading order effects: they are not very significant and can be easily restored. The essential point is the value of the scale M^2 that appears in the double logarithm $\ln^2\left(\frac{Q^2}{M^2}\right)$. It is this value which determines when the suppression is numerically important. Our choice of the scale $M^2 = s_0 \approx 0.7\,GeV^2$ means that

Sudakov effects can be ignored in the accessible energy range $Q^2 \lesssim 10\, GeV^2$. There is a simple physical explanation of this fact. The Sudakov form factor characterizes the probability that the hard scattering process is not accompanied by the bremsstrahlung radiation. In perturbation theory, the gluons are easily radiated, and this strongly diminishes the probability of a purely elastic process. In the real world, however, nothing will be emitted till the invariant mass of the final state reaches the three-pion threshold.



This means that M is at least larger than $3m_\pi$. However, at the threshold, the pion emission probability is zero, and a more correct estimate is to put M^2 equal to the value at which the emission probability becomes sizable. This means one should take M^2 equal to the effective continuum threshold, *i.e.*, to s_0. This result is also supported by direct local duality estimates for the relevant two-loop diagrams.

The sum of soft and hard contributions displayed above is in good agreement with available experimental data.

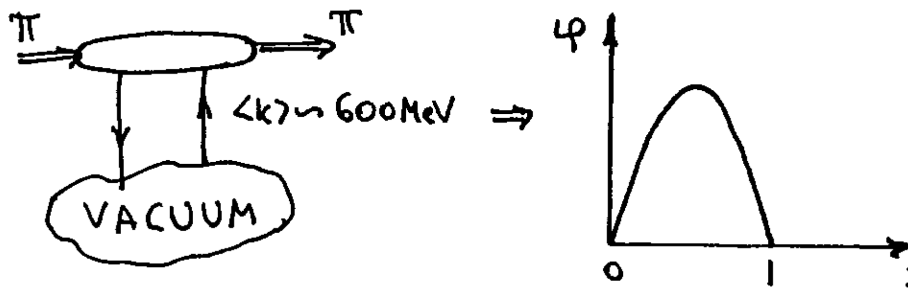
It should be emphasized that all the ingredients arise from one and the same physics:

- The hadron sizes are determined by nonperturbative effects.
- The Coulomb effects are treated as a small correction.
- Vacuum fluctuations responsible for the hadron formation have a small correlation length $r_c \lesssim 0.5\, fm$.

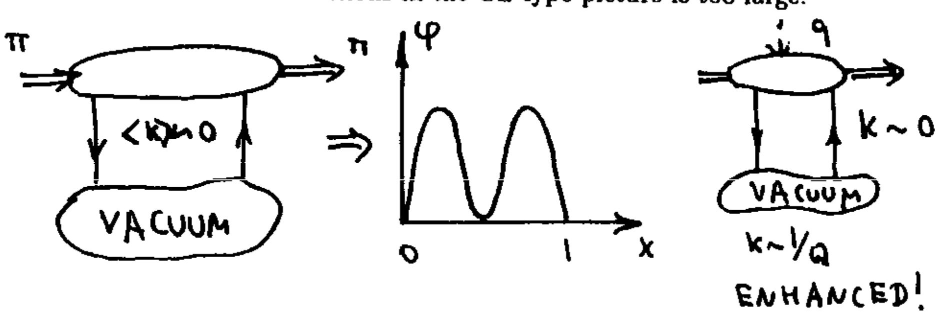
The last statement means that the QCD vacuum is populated by (virtual) quarks whose average momentum $\langle k \rangle \gtrsim 600\, MeV$ is not smaller than the momenta of the quarks inside the pion, and, hence, there is no enhancement of the end-point regions $x \sim 0$, $x \sim 1$ in the pion wave function caused by exchange of quarks between the pion and the QCD vacuum.

Another ansatz concerning the QCD vacuum structure is implied by the Chernyak-Zhitnitsky approach which is equivalent to assuming that the correlation length of vacuum fluctuations is much larger than a typical hadronic size. In this picture, the QCD vacuum is populated by quarks with very small momenta, and the exchange of quarks between the pion and the vacuum produces an enhancement of the low-momentum component of the pion wave function: that is why their wave function has the bumps in the end-point region. As we discussed above, using the CZ wave function increases the large-Q^2 magnitude of the one-gluon-exchange term by factor $\frac{25}{9}$:

$$\left[F^{\alpha_s}(Q^2)\right]^{CZ}\Big|_{Q^2\to\infty} = \frac{25}{9}\left[F^{\alpha_s}(Q^2)\right]^{as}. \tag{3.8}$$



However, the soft contribution is also governed by the small-x behaviour of the wave function, and the small-x enhancement produces an increase in the magnitude of $F^{soft}(Q^2)$ compared to that obtained in our picture. As a result, the sum of soft and hard contributions in the CZ-type picture is too large.



In other words, the early pQCD dominance requires *both*

1. a large hard term and
2. a small soft term.

Our observation is that within the QCD sum rule approach these requirements cannot be satisfied simultaneosly. The same conclusion was made within the framework of the constituent quark model [27].

IV. EFFECTIVE SIZE OF THE PION AND SUDAKOV EFFECTS

The explicit expressions for the soft and hard terms given above allow one to get an estimate of the effective size of the pion (or of its dominant $\bar{q}q$ component) in different situations.

1. At small Q^2, the pion form factor behaves like $1/(1+Q^2/(0.6\,GeV^2)$, which corresponds to $\langle r_\pi^2\rangle \approx (0.65\,fm)^2$.

2. At intermediate Q^2, a specific low-Q^2 contribution dies out, and the remaining part of the soft contribution behaves like $F_\pi^{soft}(Q^2) \sim 1/(1+Q^2/(2\,GeV^2))^2$. Hence, the effective radius of the $\bar{q}q$ component in the Feynman mechanism (estimated by the slope of this contribution at $Q^2=0$) is $\langle r_{\bar{q}q}^2\rangle \approx (0.5\,fm)^2$.

3. Finally, the one-gluon-exchange term is characterized by the factor $1/(1+Q^2/(1.4\,GeV^2))$, and the effective radius of the $\bar{q}q$ pair in the one-gluon-exchange subprocess at low and intermediate Q^2 can be estimated as $\langle r_{\bar{q}q}^2\rangle \approx (0.4\,fm)^2$. At large Q^2, of course, the effective size of such $\bar{q}q$ pair diminishes as $1/Q$.

Recently, Li and Sterman [11] demonstrated (under some assumptions subject to further inspection) that Sudakov effects suppress large-size $\bar{q}q$ configurations in the α_s-contributions to form factors. The suppression is described by a Sudakov factor like

$$\exp\left\{-\frac{\alpha_s(1/b)}{2\pi}\ln^2\left(\frac{xQ}{\Lambda}\right)\right\} \tag{4.1}$$

where b is the transverse size of the $\bar{q}q$ pair. Their expression is more complicated due to inclusion of the next-to-leading effects, but I again neglected them to make the formula more comprehensible. The suppression at large b is generated essentially by the growth of α_s according to the asymptotic freedom formula $\alpha_s(1/b) = 2\pi/9\ln(1/b\Lambda)$. The coupling constant α_s explodes when $b \to 1/\Lambda$ from below. Furthermore, because of the $\ln^2\left(\frac{xQ}{\Lambda}\right)$ factor, at higher Q^2 the same suppression occurs at smaller b, *i.e.*, due to the Sudakov effects, the effective size of the $\bar{q}q$ pair decreases with increasing Q. For accessible Q, the cut-off in b-space occurs at $b \approx 0.8/\Lambda$, which at $\Lambda = 100\,MeV$ used in ref.[11] corresponds

to distance $1.6\,fm$. This means that the Sudakov activity takes place at distances marginally larger than the actual size of the pion ($2/3\,fm$) and even more essentially larger than our estimate of the size of the relevant $\bar{q}q$ configuration ($1/2\,fm$). This result of the Li-Sterman analysis is in a complete agreement with our statement that the Sudakov effects play no significant role at accessible Q^2. Furthermore, if the Li-Sterman integrals over b are cut-off at $0.5\,fm$, the resulting curves for $F^{\alpha_s}(Q^2)$ are very close to our estimate based on the QCD sum rules. In particular, if one takes the asymptotic wave function, $F^{\alpha_s}(Q^2)$ is a factor 5 below the data for $Q^2 = 1-3\,GeV^2$. Use of the CZ wave function increases $F^{\alpha_s}(Q^2)$ in this region by a factor of two, but the result is still considerably below the data. In any case, the analysis is incomplete as the soft contribution was not included.

The pion, as a simple system, has been most thouroghly studied within the QCD sum rule approach. However, there is no doubt that the same physical arguments are applicable for the nucleon. In particular, the local duality formula

$$G_M^p(Q^2) = \frac{8}{3}\sqrt{T^2-1}\left\{(4T^2-1)(T^2-1)+(4T^2-3)T\sqrt{T^2-1}\right\}^{-1}, \tag{4.2}$$

(where $T = 1+Q^2/2S_0$ and $S_0 = 2.3\,GeV^2$) describes the proton magnetic form factor till $Q^2 = 15\,GeV^2$ and only slightly deviates from the data till $30\,GeV^2$.

Our claim that the soft term alone can be sufficiently large to describe the data in the whole accessible energy range was confirmed recently within a relativistic quark model by Schlumpf [24] who fitted the data on proton magnetic form factor till $Q^2 = 30\,GeV^2$ using for the proton a power-behaved soft wave function $\psi(x,k_\perp) \sim 1/(k_\perp^2)^{3.5}$.

On the other hand, Li [25] extended the analysis of ref.[11] to study the Sudakov effects in the hard contribution for the proton form factor. However, if one imposes a physically reasonable cut-off $b \lesssim 0.4/\Lambda$ in the b-integrals, his results for the hard term are by factors 3 - 5 lower than the data, even in the case of the CZ wave function, in full agreement with our qualitative analysis of the problem.

V. CONCLUSIONS

The criticism [8, 14, 26, 15, 27, 23, 28, 29] of the early applicability of perturbative QCD to hadronic form factors is now as healthy as it ever was. Moreover, our main statements received an extra support from a number of recent investigations [30, 31, 24], sometimes even from the studies [11, 25] intended to disprove our analysis. In any case, new experimental data unambigously resolving the controversy are most welcome.

VI. ACKNOWLEDGENMENT

I acknowledge numerous stimulating discussions with Nathan Isgur and thank him for careful reading of the manuscript and comments.

REFERENCES

* Based on talk "Reality Therapy" presented at CEBAF User Group Retreat, Snowshoe, West Virginia, May 21 - 25, 1993

† Also at the Laboratory of Theoretical Physics, JINR - Dubna, Russian Federation

[1] V.L.Chernyak and A.R.Zhitnitsky, JETP Letters, 25 (1977) 510; V.L.Chernyak, A.R.Zhitnitsky and V.G.Serbo, JETP Letters 26, (1977) 594

[2] A.V.Radyushkin, JINR preprint P2-10717, Dubna (June, 1977) (rejected from Physics Letters B)

[3] A.V.Efremov and A.V.Radyushkin, JINR preprint E2-11535, Dubna (April 1978), submitted to XIX Intern. Conf. on High Energy Physics (Tokyo)

[4] A.V.Efremov and A.V.Radyushkin,JINR preprint E2-11983, Dubna (October 1978), published in Theor. Math. Phys. 42 (1980) 97

[5] A.V.Efremov and A.V.Radyushkin, Phys.Lett., 94B (1980) 245

[6] S.J.Brodsky and G.P.Lepage, preprint SLAC-PUB-2294, Stanford (1979)

[7] V.L.Chernyak and A.R.Zhitnitsky, Nucl.Phys., B201 (1982) 492; B214 (1983) 547(E)

[8] F.M.Dittes and A.V.Radyushkin, Sov.J.Nucl.Phys., 34 (1981) 293

[9] A.V.Radyushkin, JINR preprint P2-80-687, Dubna (1980) (rejected from Physics Letters B)

[10] C.R. Ji, A.F. Sill and R.N. Lombard-Nelsen, Phys. Rev. D36 (1987) 165

[11] H.Li and G.Sterman, Nucl.Phys. B381 (1992) 129

[12] B.L.Ioffe and A.V. Smilga, Phys. Lett. 114B (1982) 353

[13] V.A.Nesterenko and A.V.Radyushkin, Physics Letters, 115B (1982) 410

[14] V.A.Nesterenko and A.V.Radyushkin, Physics Letters, 128B (1983) 439

[15] N.Isgur and C.H.Llewellyn Smith, Phys.Rev.Lett., 52 (1984) 1080

[16] V.A.Nesterenko and A.V.Radyushkin, JETP Letters, 39 (1984) 707

[17] V.M. Belyaev and Ya.I. Kogan, preprints ITEP-12-1984, Moscow (1984) and UBCTP-92-4, Vancouver (1992); Int.J.Mod.Phys.A8 (1993) 153

[18] I.I. Balitsky, A.V. Kolesnichenko and A.V. Yung, Phys. Letters, 157B (1985) 309

[19] S.V.Mikhailov and A.V.Radyushkin, JETP Letters, 43 (1986) 712

[20] A.P.Bakulev and A.V.Radyushkin, Physics Letters, B271 (1991) 223

[21] R.P.Feynman, Photon-Hadron Interaction; W.A. Benjamin, Reading, Mass. (1972)

[22] M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl. Phys. B147 (1979) 385, 447

[23] A.V.Radyushkin, Nucl.Phys. A527 (1991) 153c

[24] F. Schlumpf, preprint SLAC-PUB-5998, Stanford, (1992)

[25] H.Li, preprint ITP-SB-92-25, Stony Brook (1992)

[26] A.V.Radyushkin, Acta Physica Polonica, B15 (1984) 403

[27] N.Isgur and C.H.Llewellyn Smith, Phys.Lett., 217B (1989) 535; Nucl.Phys., B317 (1989) 526

[28] A.V.Radyushkin, Nucl.Phys. A532 (1991) 141c

[29] A.V.Radyushkin, In: "Baryons-92" (M.Gai, ed.), World Scientific, (1993) 366

[30] N.N. Nikolaev, preprint DFTT-10-92, Turin (1992)

[31] A. Szczepaniak and A. G. Williams, Phys.Lett. B302 (1993) 87