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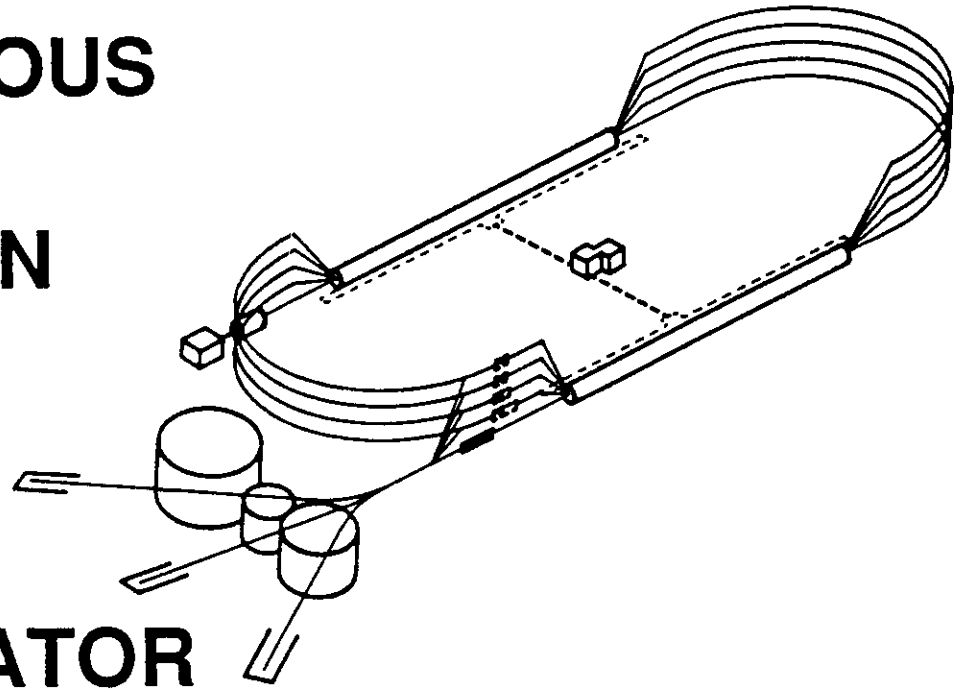

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Abstract

The value of power corrections to the integrals of the polarization structure functions of proton and neutron $\int g_{1_{p,n}}(x)dx$ measured by the EMC, SMC and E142 groups, is determined based on a model which accounts for higher twist terms, has the correct asymptotic behavior at large Q^2 and satisfies the Gerasimov-Drell-Hearn sum rule at $Q^2 = 0$. The contribution of resonances up to $W = 1.8GeV$ at $Q^2 = 0$ is taken into account based on the analysis of electroproduction data. It is shown that when taking into account these higher twist terms, the experimental data agree with the Bjorken sum rule and the part of the proton spin projection carried by quarks, is consistent with the natural estimate of $\sim 50\%$.

Recent measurements of the deep inelastic polarized structure function by the SMC goup at CERN[1] using polarized muon scattering from polarized deuterium, and by the SLAC E142[2] experiment using polarized electrons and polarized ^3He, respectively, allowed determination of the polarized struxture function $g_{1n}(x)$. The analogous proton structure function $g_{1p}(x)$ was measured previously by the EMC[3] and SLAC [4] groups. Thereby, it became possible to test the important Bjorken sum rule [5].

$$\Gamma_p(Q^2) - \Gamma_p(Q^2) \equiv \int_0^1 dx[g_{1p}(x,Q^2) - g_{1n}(x,Q^2)] = \frac{g_A}{6}(1 - \frac{\alpha_s(Q^2)}{\pi}) \quad (1)$$

where Q^2 is the four momentum transfer from the scattered lepton to the target, g_A is the axial constant of β-decay. The relation (1) is written taking into account the first order perturbative correction in QCD [6]. The higher order corrections are also known [7].

The SMC [1] and E142 [2] results are the following. The SMC measurements correspond to the averaged value $\bar{Q}^2 = 4.6GeV^2$. For scattering on polarized deuteron it was obtained that

$$\Gamma_d = 0.023 \pm 0.020 \pm 0.015 \ , \quad (2)$$

where Γ_d is related to Γ_p, Γ_n by

$$\Gamma_p + \Gamma_n = 2\Gamma_d(1 - 1.5\omega_D)^{-1} \ , \quad (3)$$

and $\omega = 0.058$ takes into account the D-wave admixture in the deuteron. From (2) and (3) follows that

$$\Gamma_p + \Gamma_n = 0.050 \pm 0.044 \pm 0.033 \qquad \bar{Q}^2 = 4.6GeV^2 \quad (4)$$

The EMC [3] experiment at $\bar{Q}^2 = 10.7GeV^2$ found

$$\Gamma_p = 0.126 \pm 0.010 \pm 0.015 \qquad \bar{Q}^2 = 10.7GeV^2 \quad (5)$$

and neglecting the Q^2 dependence, we get

$$\Gamma_n = -0.076 \pm 0.04 \pm 0.04 \quad (6)$$

and

$$SMC : \Gamma_p - \Gamma_n = 0.202 \pm 0.045 \pm 0.045 \tag{7}$$

The value (7) agrees with the Bjorken sum rule (1)

$$\Gamma_p - \Gamma_n = 0.186 \pm 0.003 \tag{8}$$

(at $\alpha_s \approx 0.25, \Lambda_{QCD} = 150 MeV$), though the agreement is not completely convincing because of large errors. (In eq.(8) we also took into account the corrections $\sim \alpha_s^2$ and α_s^3 [7]. The error represents an estimate of the uncertainty in these corrections.) The E142 group made their measurements at $Q^2 = \bar{2} GeV^2$. Since the spins of two protons in ^3He are compensated, the polarized ^3He scattering up to small corrections (taken into account in experiment) correspond to the scattering on a polarized neutron. In the E142 experiment it was obtained that:

$$\int_{0.03}^{0.6} dx g_{1n}(x) \quad = -0.019 \pm 0.006 \pm 0.006 \tag{9}$$

Performing extrapolation into small and large x region, the E142 group determined

$$\Gamma_n = -0.022 \pm 0.011 \qquad \bar{Q}^2 = 2\ GeV^2 \tag{10}$$

(To determine Γ_n from the E142 data Ellis and Karliner in their work [8] have used a different parametrization of $F_2(x, Q^2)$ and $R(x, Q^2)$ than was used in [2], as well as a different extrapolation at small x. $F_2(x, Q^2)$ and $R(x, Q^2)$ enter when finding $g_1(x)$ from the experimentally measured asymmetry. In doing so, instead of (10) they have obtained $\Gamma_n = -0.028 \pm 0.006 \pm 0.009$). Making use of (5) and (10), and, again, neglecting the Q^2 dependence, we find

$$E142 : \qquad \Gamma_p - \Gamma_n = 0.148 \pm 0.021 \tag{11}$$

which differs from the Bjorken sum rule (8) by two standard deviations. One can easily see that the account of perturbative effects, i.e. of Q^2 dependence of α_s does not eliminate this disagreement.

The main problem in comparing the EMC, SMC and E142 results and in checking the Bjorken sum rule is the account of non-perturbative Q^2 dependence, i.e. of higher twist term contributions. In their work Balitsky, Braun

and Kolesnichenko [9] have attempted to calculate the twist-4 contribution. It seems to us, however, that the results of [9] are not reliable (for the criticism see below).

To take into account the nonperturbative Q^2 dependence we use in this paper the idea, conjectured in ref. [10,11], on a connection of Γ_p, Γ_n at large Q^2 with the Gerasimov-Drell-Hearn (GDH) sum rule which holds at $Q^2 = 0$.

Following [10,11], we introduce the functions

$$I_{p,n}(Q^2) = \int\limits_{Q^2/2m_{p,n}}^{\infty} \frac{d\nu}{\nu} G_{1p,n}(\nu, Q^2) \equiv \frac{2m_{p,n}^2}{Q^2}\Gamma_{p,n}(Q^2) \tag{12}$$

At large ν and Q^2, the quantity νG_1 is related to g_1 by

$$\frac{\nu}{m_p} G_1(x, Q^2) \approx g_1(x, Q^2), \quad \nu \to \infty, \quad Q^2 \to \infty, \quad x = Q^2/2m_p\nu = const$$

Therefore, in the region of large ν and Q^2 , $\Gamma_{p,n}(Q^2)$ defined in (12) coincide with the ones introduced in eq.(1). At $Q^2 = 0$ $I_{p,n}(0)$ satisfy the GDH sum rules

$$I_{p,n}(0) = -\frac{1}{4}\kappa_{p,n}^2 \tag{13}$$

where κ_p, κ_n are anomalous magnetic moments of proton and neutron.

The authors of ref.[10] proposed a vector dominance based model which described $I_p(Q^2)$ throughout the whole Q^2 region. At large Q^2 $I_{p,n}(Q^2)$ has the asymptotic form (12), and at $Q^2 = 0$ satisfies the GDH sum rules. In ref.[11] it was shown that in its original form the model is not satisfactory since at small Q^2 contribution of baryonic resonances to the integral (12) are important. These should be taken into account separately. We adopt here the model [11] with this refinement and write

$$I_{p,n}(Q^2) = I_{p,n}^{res}(Q^2) + I'_{p,n}(Q^2) \tag{14}$$

Here $I'_{p,n}$ is defined by [11]

$$I'_{p,n}(Q^2) = 2m_p^2\Gamma_{p,n}^{as}\left[\frac{1}{Q^2+\mu^2} - \frac{c_{p,n}\mu^2}{(Q^2+\mu^2)^2}\right] \tag{15}$$

$$c_{p,n} = 1 + \frac{1}{2}\frac{\mu^2}{m_p^2}\frac{1}{\Gamma^{as}_{p,n}}\left[\frac{1}{4}\kappa^2_{p,n} + I^{res}_{p,n}(0)\right], \tag{16}$$

where μ^2 is the vector (ρ,ω) meson mass, $\mu^2 = 0.6GeV^2$. In (15), (16) $\Gamma^{as}_{p,n}$ have the meaning of the integrals defined in eq.(1) or (12) at large Q^2, where higher order twist terms can be neglected. (We neglect the weak dependence $\alpha_s(Q^2)$ in the region $2 < Q^2 < 10GeV^2$, since its effect is below the accuracy limits of the estimates below).

The contribution of baryonic resonances $I^{res}_{p,n}$ with masses up to $W = 1.8GeV$ is known from the analysis of pion electroproduction experiments [12],[13] and is presented in Fig.1. Knowing $I^{res}_{p,n}$, we may find $c_{p,n}$ and thereby determine all the parameters of the model. ($\Gamma^{as}_{p,n}$ can be found from the EMC and E142 experimental data taking into account the $1/Q^2$ power corrections obtained in our model. We use the E142 data since their accuracy is better than of SMC). Using the resonance contributions from Fig.1, $I^{res}_p(0) = -1.028$ and $I^{res}_n(0) = -0.829$, we find:

$$c_p \simeq 0.43 \qquad c_n \simeq 0.0 \tag{17}$$

The value of c_n is defined with a significant error (+0.3,-1.2) which is due to the uncertainty in Γ_n in the E142 experiment as well as the uncertainty in $I^{res}(0)$. Substituting c_p into (15), we find that in the EMC experiment at $\bar{Q}^2 = 10.7GeV^2$ the power correction comprises 8%. Thus, the experimentally measured EMC value of Γ_p corresponds (after excluding the power corrections) to

$$\Gamma^{as}_p = 0.137 \pm 0.018 \tag{18}$$

Analogously, making use of the E142 data for neutron (10) and the value of c_n (17), we find

$$\Gamma^{as}_n = -0.028 \pm 0.015 \tag{19}$$

The error in eq.(19) includes the uncertainty due to the error in c_n.

The power corrections 8% in the EMC experiment and 23% in the E142 experiment are off the accuracy limit of these experiments and cannot be excluded by existing data. The power corrections for proton and neutron are consistent and yield 30% and 23%. respectively, at $Q^2 = 2$ GeV^2, while

at $Q^2 = 4.6$ GeV^2 the power corrections for the proton amount to 16% and 12%.

Thus, after excluding the power corrections the Bjorken sum rule takes the form

$$EMC, E142 \quad \text{, without } 1/Q^2 \text{ terms} \quad \Gamma_p - \Gamma_n = 0.165 \pm 0.024 \tag{20}$$

and differs from the theoretical value by less than one standard deviation. Note that the error in (20) is obtained by adding different statistical and systematic errors in quadrature. This, of course, may not be correct, and the real error could be larger. If, instead of using directly the E142 data we use the Ellis and Karliner result [8], the disagreement between experimental and theoretical value of the Bjorken's sum rule will be even smaller.

Let us now determine which values of the part of the proton spin projection carried by quarks and gluons correspond to the values of $\Gamma_{p,n}$ (18) and (19). Taking into account the first QCD correction [6] we have the equality

$$\Gamma^{as}_{p,n} = \frac{1}{12}\left\{(1-\frac{\alpha_s}{\pi})\left[\pm g_A + \frac{1}{3}a_8\right] + \frac{4}{3}\left[1 - C_f\frac{\alpha_s}{\pi}\right]\Sigma\right\} - \frac{N_f}{18\pi}\alpha_s\Delta g \tag{21}$$

where

$$a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D \qquad \Sigma = \Delta u + \Delta d + \Delta s \tag{22}$$

$\Delta u, \Delta d, \Delta s, \Delta g$ - are the parts of the proton spin projection carried by u, d, s-quarks and gluons, respectively, g_A - is the axial constant of β decay, $g_A = 1.257$ related to $\Delta u, \Delta d$ by the relation

$$g_A = \Delta u - \Delta d, \tag{23}$$

F and D - are the β-decay constants in the baryon octet, N_f is the flavor number, $C_f = (33 - 8N_f)/(33 - 2N_f) = 1/3$ at $N_f = 3$. We adopt [14]:

$$3F - D = 0.59 \pm 0.02 \tag{24}$$

and the portion of the nucleon spin carried by gluons to be $\Delta g \approx 0.5$. This value will be confirmed in what follows. From equalities (18), (20-23) one

can readily find $\Sigma, \Delta u, \Delta d, \Delta s$ which correspond to the EMC experiment ($\Lambda_{QCD} = 150 MeV$):

$$\Sigma = 0.28 \pm 0.17 \quad \Delta u = 0.82 \pm 0.06 \quad \Delta d = -0.44 \pm 0.06 \tag{25}$$
$$\Delta s = -0.10 \pm 0.06$$

From (19), (20-23) we find the values of the same quantities following from the E142 data:

$$\Sigma = 0.53 \pm 0.14 \quad \Delta u = 0.905 \pm 0.05 \quad \Delta d = -0.355 \pm 0.05 \tag{26}$$
$$\Delta s = -0.02 \pm 0.05$$

As is seen from a comparison of (25) and (26), after accounting for the power corrections, the EMC and E142 results agree with each other within overlapping errors of the two experiments. The value of the part of the nucleon spin carried by all quarks agrees with an intuitively expected value $\Sigma \approx 0.5$ (by analogy with the nucleon momentum fraction carried by quarks). If,now, in accordance with the quark model, we neglect orbital moments, then the gluon share goes to $\Delta g = 0.5$ - the value we have adopted above. The spin fraction carried by s- quarks, agrees with $\Delta s \approx -0.05$ which is consistent with naive expectations.

Let us finally compare the value $\Gamma_p + \Gamma_n$ at $\bar{Q}^2 = 4.6 GeV^2$ (4) measured by SMC with theoretical expectation in our model. The power corrections deviating Γ_p, Γ_n at $\bar{Q}^2 = 4.6 GeV^2$ from their asymptotic values $\Gamma_p^{as}, \Gamma_n^{as}$ are, correspondingly 16% and 12%. Taking them into account, using eq.(21), the values $3F - D$ given by (24) and $\Sigma = \Delta g = 0.5$ we obtain at $\bar{Q}^2 = 4.6 GeV^2$

$$(\Gamma_p + \Gamma_n)_{theor} = 0.10 \tag{27}$$

in comparison with the experimental value (4). Again, the difference is less than one standard deviation.

The value of the power correction which follows from our model is a factor of several larger than the twist-4 contribution calculated in ref. [9].

When comparing these two approaches one should take into consideration the following: In finding the vacuum expectation values induced by external axial field - a quantity, which essentially determines the final answer - the

authors of ref.[9] have really taken the octet field instead of the singlet one and used the dominance of massless goldstones (π or η) which is incorrect for the singlet field case. And finally, the result of [9] (unlike the other results obtained by the QCD sum rule) depends on the ultraviolet cut-off. Such a dependence implies that operators in the operator expansion which are accounted for in the calculation mix with operators whose contributions are neglected. This circumstance introduces a noncontrollable uncertainty into the calculation. We therefore do not believe that the twist-4 results obtained in [9] are justifiable. The crucial factor for checking our approach would be a precision study of the Q^2 dependence in the polarized deep-inelastic $e(\mu)$ nucleon scattering.

In a recent preprint Ji and Unrau [15] attempted to construct a model for the Q^2 dependence of $\Gamma(Q^2)$ which satisfies the GDH sum rule at $Q^2 = 0$. They found high twist corrections to the EMC, SMC and E142 experiments much less than in our model. We disagree with Ji and Unrau considerations at low Q^2. The main ingredient of their approach is the dominating role played by elastic contributions (nucleon pole) at low Q^2. However, as follows from original derivation [16] (see also [17]) the polarized Compton amplitude with account of crossing terms, which appears in the l.h.s. of GDH sum rule has no nucleon pole term at $Q^2 = 0$ and is a constant. For this reason at $Q^2 = 0$ the elastic term is absent in the r.h.s. At $Q^2 > 0$ the pole term in the l.h.s. is completely compensated by the pole term in the r.h.s. and as a consequence the elastic contribution can be omitted in constructing a model discribing the Q^2 dependence of $\Gamma(Q^2)$. It must also be mentioned that elastic contributions are not measured in any experiment on deep inelasic scattering.

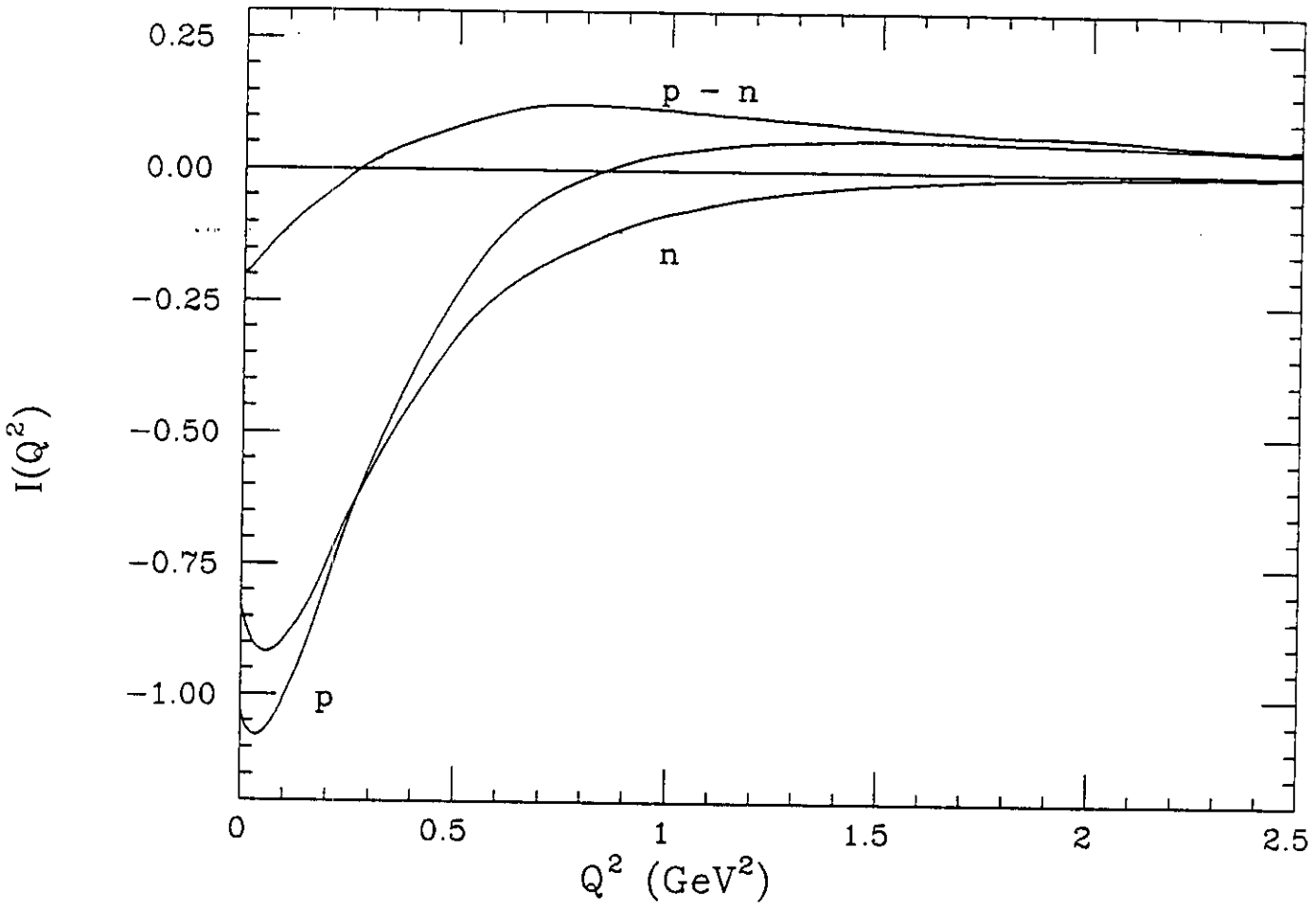

Fig.1. The contributions of resonances up to the masses $W = 1.8GeV$ in the r.h.s. of eq.(14), the indices $p, n, p-n$ refer to the cases of the deep inelastic scattering on proton, neutron and proton-neutron difference.

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