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NON-STANDARD PHYSICS AND NUCLEON STRANGENESS IN LOW-ENERGY PV ELECTRON SCATTERING[1]

M. J. Musolf

Department of Physics
Old Dominion University
Norfolk, VA 23529 U.S.A.
and
CEBAF Theory Group, MS-12H
12000 Jefferson Avenue
Newport News, VA 23606 U.S.A.

and

T. W. Donnelly

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

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ABSTRACT

Contributions from physics beyond the Standard Model, strange quarks in the nucleon, and nuclear structure effects to the left-right asymmetry measured in parity-violating (PV) electron scattering from ^{12}C and the proton are discussed. It is shown how lack of knowledge of the distribution of strange quarks in the nucleon, as well as theoretical uncertainties associated with higher-order dispersion amplitudes and nuclear isospin-mixing, enter the extraction of new limits on the electroweak parameters S and T from these PV observables. It is found that a series of elastic PV electron scattering measurements using ^{4}He could significantly constrain the s-quark electric form factor if other theoretical issues are resolved. Such constraints would reduce the associated form factor uncertainty in the carbon and proton asymmetries below a level needed to permit extraction of interesting low-energy constraints on S and T from these observables. For comparison, the much smaller scale of s-quark contributions to the weak charge measured in atomic PV is quantified. It is likely that only in the case of heavy muonic atoms could nucleon strangeness enter the weak charge at an observable level.

1. Introduction

It has recently been suggested that measurements of the "left-right" helicity difference asymmetry (A_{LR}) in parity-violating (PV) elastic electron scattering from ^{12}C nuclei and of the weak charge (Q_w) in atomic PV experiments using ^{133}Cs are potentially sensitive to certain extensions of the Standard Model at a significant level.[1] In particular, these observables carry a non-negligible dependence on the so-called S-parameter characterizing extensions of the Standard Model which involve degenerate multiplets of heavy fermions.[2] It is argued that a 1% measurement of $A_{LR}(^{12}\mathrm{C})$ or a 0.7% determination of $Q_w(^{133}\mathrm{Cs})$ (equivalent to a 1% determination of the weak mixing angle) would constrain S to $|\delta S| \leq 0.6$, a significant improvement over the present limit of $\delta S = \pm 2.0$ (exp't) ± 1.1 (th'y) obtained from $Q_w(^{133}\mathrm{Cs})$.[1] The level of systematic precision achieved in the recently completed MIT-Bates measurement of $A_{LR}(^{12}\mathrm{C})$,[3] along with prospects for improving statistical precision with longer run times at CEBAF or MIT-Bates, suggest that a 1% $A_{LR}(^{12}\mathrm{C})$ measurement could be feasible in the foreseeable future. Similarly, improvements in atomic structure calculations[4] have reduced the theoretical error in $Q_w(^{133}\mathrm{Cs})$ to roughly 1%, and the prospects for pushing the experimental uncertainty below this level also appear promising.[5] If such high-precision, low-energy measurements were achieved, the resultant constraints on non-standard physics would complement those obtainable from measurements in the high-energy sector. The latter are generally equally sensitive to both S and the T-parameter, where the latter characterizes standard model extensions involving non-degenerate heavy multiplets.[1, 2]

In this work, we point out the presence of terms in $A_{LR}(^{12}\mathrm{C})$ not considered in Ref. [1] involving nucleon and nuclear structure physics which must be experimentally and/or theoretically constrained in order to achieve the limits on S suggested above. Specifically, we consider contributions involving the distribution of strange quarks in the nucleon, multi-boson "dispersion corrections" to tree-level electromagnetic (EM) and weak neutral current (NC) amplitudes, and isospin impurities in the nuclear ground state. We show that lack of knowledge of ρ_s, the dimensionless mean square "strangeness radius", introduces uncertainties into $A_{LR}(^{12}\mathrm{C})$ at a potentially problematic level. We further show how a series of two measurements of A_{LR} for elastic scattering from ^{4}He could constrain ρ_s sufficiently to reduce the associated uncertainty in $A_{LR}(^{12}\mathrm{C})$ to below 1% . In addition, we observe that an improved theoretical understanding of dispersion corrections and isospin impurities for scattering from $(J^\pi, I) = (0^+, 0)$ nuclei is needed in order both to determine ρ_s at an interesting level and to constrain S to the level suggested in Ref. [1]. For comparison, we also discuss briefly the

interplay of nucleon strangeness and non-standard physics in PV elastic $\vec{e}p$ scattering and atomic PV. In the former instance, a 10% determination of $A_{LR}(\vec{e}p)$ at forward-angles could yield low-energy constraints on S and T complementary to those obtained from either atomic PV or $A_{LR}(^{12}\mathrm{C})$, if the strangeness radius were constrained to the same level as appears possible with the aforementioned series of ^{4}He measurements. A determination of ρ_s with PV $\vec{e}p$ scattering alone would not be sufficient for this purpose. In contrast, the impact of strangeness on the interpretation of $Q_w(^{133}\mathrm{Cs})$ is significantly smaller, down by at least an order of magnitude from the dominant atomic theory uncertainties. Only in the case of PV experiments with heavy muonic atoms might ρ_s enter at a potentially observable level. Other prospective PV electron scattering experiments – such as elastic scattering from the deuteron or quasielastic scattering – are discussed elsewhere.[6-9]

2. Hadronic neutral current, new physics and strangeness

The low-energy PV observables of interest here are dominated by the charge ($\mu = 0$) component of the hadronic vector NC. In terms of quark fields, the nuclear vector NC operator may be written in terms of the isoscalar and isovector EM currents and a strange quark current:[6]

$$J_\mu^{NC} = \xi_V^{I=1} J_\mu^{EM}(I=1) + \sqrt{3}\xi_V^{I=0} J_\mu^{EM}(I=0) + \xi_V^{(0)} V_\mu^{(s)} \quad , \tag{1}$$

where $V_\mu^{(s)} = \bar{s}\gamma_\mu s$ and the ξ_V's are couplings determined by the underlying electroweak gauge theory. In writing Eq. (1), we have eliminated terms involving (c, b, t) quarks, since their contributions to nuclear matrix elements of J_μ^{NC} are suppressed (see below). In the minimal Standard Model, one has

$$\begin{aligned} \xi_V^{(0)} &= -[1 + R_V^{(0)}] \\ \sqrt{3}\xi_V^{I=0} &= -4\sin^2\theta_w[1 + R_V^{I=0}] \\ \xi_V^{I=1} &= 2(1 - 2\sin^2\theta_w)[1 + R_V^{I=1}] \quad , \end{aligned} \tag{2}$$

where $\sin^2\theta_w$ is the weak mixing angle and the $R_V^{(a)}$ are higher-order corrections to tree-level electron-nucleus NC amplitudes. In addition, one may define couplings which govern the low-$|Q^2|$ NC charge scattering from the neutron and proton:

$$\begin{aligned} \xi_V^p &\equiv \tfrac{1}{2}[\sqrt{3}\xi_V^{I=0} + \xi_V^{I=1}] = (1 - 4\sin^2\theta_w)[1 + R_V^p] \\ \xi_V^n &\equiv \tfrac{1}{2}[\sqrt{3}\xi_V^{I=0} - \xi_V^{I=1}] = -[1 + R_V^n] \quad . \end{aligned} \tag{3}$$

At the operator level, the ξ_V's are determined entirely in terms of couplings of the Z^0 to the (u, d, s) quarks, including contributions from radiative corrections within or beyond the framework of the Standard Model, both of which may be included in the $R_V^{(a)}$.[10] Upon taking nuclear matrix elements of J_μ^{NC}, one must include in the $R_V^{(a)}$ additional contributions arising from strong interactions between quarks in intermediate states.[10, 11] Further contributions arising from isospin impurities in the nuclear ground state are discussed below. Corrections owing to neglect of the (c, b, t) quarks in writing Eq. (1) have been estimated in Ref. [12] and may be included in the $R_V^{(a)}$ for $a = 0$ and $I = 0$ as $R_V^{(a)} \to R_V^{(a)}(\mathrm{ewk}) - \Delta_V$, where $\Delta_V \sim 10^{-4}$. No such corrections enter $R_V^{I=1}$.

The motivation for considering PV electron scattering as a probe of new physics may be seen, for example, by noting the S- and T-dependencies of the $R_V^{(a)}$. Following Ref. [1], in which $\overline{\mathrm{MS}}$ renormalization was used in computing one-loop electroweak corrections, one has

$$\begin{aligned} R_V^{I=0}(\mathrm{new}) &= 0.016S - 0.003T \\ R_V^{I=1}(\mathrm{new}) &= -0.014S + 0.017T \\ R_V^p(\mathrm{new}) &= -0.206S + 0.152T \\ R_V^n(\mathrm{new}) &= 0.0078T \quad . \end{aligned} \tag{4}$$

Within the framework of Ref. [1], a value of the top-quark mass differing from 140 GeV would also generate a non-zero contribution to T. The different linear combinations of S and T appearing in Eqs. (4) suggest that a combination of PV electron scattering experiments could provide interesting low-energy constraints on these two parameters. One such scenario is illustrated in Fig. 1, where the constraints attainable from a 1% measurement of $A_{LR}(^{12}\mathrm{C})$ and a 10% determination of ξ_V^p from a forward-angle $A_{LR}(\vec{e}p)$ measurement are shown. For comparison, the present constraints from $Q_w(^{133}\mathrm{Cs})$ are also shown. One expects these constraints to be tightened by a factor of two to three with future measurements.[13] While $Q_w(^{133}\mathrm{Cs})$ is effectively independent of T, both $A_{LR}(^{12}\mathrm{C})$ and the forward-angle $\vec{e}p$ asymmetry carry a non-negligible dependence on T. Hence, one or both of the latter could complement the former as a low-energy probe of new physics.In addition, one might also consider PV electro-excitation of the $\Delta(1232)$ resonance as a means of extracting $R_V^{I=1}$. This quantity is relatively more sensitive to T than are $R_V^{I=0}$, $R_V^{(0)}$, and $Q_w(^{133}\mathrm{Cs})$, so that a determination of the former would further complement any low-energy constraints attained from the latter.[14] It is unlikely, however, that the experimental and theoretical uncertainties associated with $A_{LR}(N \to \Delta)$ will be reduced to the level necessary to make such a measurement relevant as an electroweak test in the near term.[8, 14] Consequently, a

combination of PV scattering experiments on ^{12}C and/or the proton, together with atomic PV, appear to hold the most promise for placing low-energy, semileptonic constraints on new physics. Before such a scenario is realized, however, other hadronic physics dependent terms entering the PV asymmetries must be analyzed. We now consider these additional contributions, focusing first on the simplest case of ^{12}C.

3. PV elastic scattering from carbon

In the limit that the ^{12}C ground state is an eigenstate of strong isospin, matrix elements of the isovector component of the current in Eq. (1) vanish. Moreover, since this nucleus has zero spin, only monopole matrix elements of the charge operator contribute. In the absence of the strange-quark term in Eq. (1), one has $\langle \text{g.s.} \| \rho^{NC} \| \text{g.s.} \rangle = \sqrt{3}\xi_V^{I=0} \langle \text{g.s.} \| \rho^{EM} \| \text{g.s.} \rangle$, so that $A_{LR}(^{12}\text{C}) \propto \langle \text{g.s.} \| \rho^{NC} \| \text{g.s.} \rangle / \langle \text{g.s.} \| \rho^{EM} \| \text{g.s.} \rangle = \sqrt{3}\xi_V^{I=0}$. In short, the asymmetry becomes independent of the nuclear physics contained in the EM and NC matrix elements[15, 16] and carries a dependence only on the underlying gauge theory coupling, $\xi_V^{I=0}$. Upon including the strange-quark term one has[6, 17]

$$A_{LR}(^{12}\text{C}) = A_0 Q^2 \left[4\sin^2\theta_w (1 + R_V^{I=0}) + \frac{G_E^{(s)}(Q^2)}{G_E^{I=0}} (1 + R_V^{(0)}) \right] \quad , \qquad (5)$$

where $A_0 = G_\mu/(4\sqrt{2}\pi\alpha) = 8.99 \times 10^{-5}\text{GeV}^{-2}$, G_μ is the Fermi constant measured in muon decay, $Q^2 = \omega^2 - |\vec{q}|^2 \leq 0$ is the four-momentum transfer squared, and $G_E^{(s)}(Q^2)$ and $G_E^{I=0}(Q^2)$ are the Sachs electric form factors[18] appearing in single-nucleon matrix elements of $V_\mu^{(s)}$ and $J_\mu^{EM}(I=0)$. Note that at the one-body level, the strangeness and EM charge density operators, $\hat{\rho}^{(s)}$ and $\hat{\rho}^{EM}(I=0)$, respectively, are identical, apart from the single nucleon form factors which enter multiplicatively. Consequently, any dependence on the nuclear wavefunction cancels from the asymmetry, leaving only the ratio of form factors in the second term of Eq. (5). For $R_V^{I=0}$ one has

$$R_V^{I=0} = R_V^{I=0}(\text{st'd}) + R_V^{I=0}(\text{new}) - R_V^{I=0}(\text{QED}) + R_V^{I=0}(\text{had}) + \Gamma - \Delta_V \quad , (6)$$

where $R_V^{I=0}$(st'd) are Standard Model electroweak radiative corrections to tree-level electron quark PV NC amplitudes, $R_V^{I=0}$(new) denote contributions from extensions of the Standard Model as in Eqs. (4), $R_V^{I=0}$(QED) are QED radiative corrections to the EM amplitude entering the denominator of $A_{LR}(^{12}\text{C})$ (hence, the minus sign in Eq. (6)), $R_V^{I=0}$(had) are strong-interaction hadronic contributions to higher-order electroweak amplitudes, Γ is a correction due to isospin

impurities in the ^{12}C ground state,[19] and Δ_V is the heavy-quark correction discussed previously. The correction $R_V^{(0)}$ appearing in the second term of Eq. (5) may be written in a similar form. For fixed top-quark and Higgs masses, the $R_V^{I=0}$(st'd) and $R_V^{I=0}$(QED) can be determined unambiguously, up to hadronic uncertainties associated with quark loops in the $Z^0 - \gamma$ mixing tensor and two boson-exchange "box" diagrams (see, *e.g.*, Ref. [11]).

Before discussing the remaining terms in Eq. (6), we note here an additional feature of spin-0 nuclei which simplifies the interpretation of the PV asymmetry. In general, when working to one-loop order, one must also include bremsstrahlung contributions to the helicity-dependent (-independent) cross sections entering the numerator (denominator) of A_{LR}. These contributions, although not loop corrections, enter the cross section at the same order in α as one-loop amplitudes and should be formally included in the $R_V^{I=0}$(st'd) and $R_V^{I=0}$(QED). At low momentum transfer, one need only consider bremsstrahlung from the scattering electron (Fig. 2), since the target experiences very small recoil and is unlikely to radiate. The contributions to the EM and EM-NC interference cross sections from the amplitudes of Fig. 2 are

$$d\sigma_{EM}^{\text{brem}} \propto |M_a + M_b|^2 = M_a M_a^* + M_a M_b^* + M_b M_a^* + M_b M_b^* \qquad (7)$$

$$d\sigma_{INT}^{\text{brem}} \propto M_a M_c^* + M_a M_d^* + M_b M_c^* + M_b M_d^* + \text{c.c.} \qquad (8)$$

where the M_i are the amplitudes associated with the diagrams in Fig. 2. For simplicity, we consider only the first terms on the right side of Eqs. (7-8). The arguments for the remaining terms are similar. For these terms one has

$$M_a M_a^* = \frac{(4\pi\alpha)^3}{Q^4} \bar{L}_{\mu\nu}^{EM} W_{EM}^{\mu\nu} \qquad (9)$$

$$M_a M_c^* = -\frac{(4\pi\alpha)^2}{Q^2} \frac{G_\mu}{2\sqrt{2}} \bar{L}_{\mu\nu}^{INT} W_{INT}^{\mu\nu} \quad , \qquad (10)$$

where the $W^{\mu\nu}$ are hadronic tensors formed from products of the hadronic electromagnetic and weak neutral currents, and where the $\bar{L}_{\mu\nu}$ are the corresponding tensors formed from the leptonic side of the diagrams in Fig. 2. The $W^{\mu\nu}$ are identical to the tree-level hadronic tensors, since the only differences between the diagrams of Fig. 2 and the tree-level graphs involve the lepton line. For the leptonic tensors, one has after averaging over initial and summing over final states[20]

$$\bar{L}_{\mu\nu}^{EM} = \tfrac{1}{2}[(K'+q)^2 - m_e^2]^{-2} \text{Tr}\Big\{ \gamma_\lambda (K' + \not{q} + m_e)\gamma_\mu (1 + \gamma_5 \not{s}) \qquad (11)$$
$$\times (K + m_e)\gamma_\nu (K' + \not{q} + m_e)\gamma_\sigma (K + m_e) \Big\} \epsilon^\lambda \epsilon^\sigma$$

$$\bar{L}_{\mu\nu}^{INT} = \tfrac{1}{2}[(K'+q)^2 - m_e^2]^{-2} \text{Tr}\Big\{ \gamma_\lambda (K' + \not{q} + m_e)\gamma_\mu$$

$$\times(g_V^e + g_A^e\gamma_5)(1+\gamma_5\not{s})$$
$$(K+m_e)\gamma_\nu(K'+\not{q}+m_e)\gamma_\sigma(K+m_e)\Big\}\epsilon^\lambda\epsilon^\sigma \quad , \tag{12}$$

where K_μ (K'_μ) are the initial (final) electron momenta, q_μ is the momentum of the outgoing photon having polarization ϵ_μ, s_μ is the initial electron spin, and g_V^e (g_A^e) are the vector (axial vector) NC couplings of the electron.

Taking the electron and radiated photon on-shell ($K^2 = K'^2 = m_e^2$, $q^2 = 0$) and working in the extreme relativistic limit ($E_e/m_e >> 1$) for which $s_\mu \to (h/m_e)K_\mu$, with h being the electron helicity, one has

$$M_a M_a^* = \frac{(4\pi\alpha)^3}{Q^4}\frac{1}{2}\left(\frac{1}{2K'\cdot q}\right)^2$$
$$\times\mathrm{Tr}\Big\{\gamma_\nu(K'+\not{q})\gamma_\sigma K'\gamma_\lambda(K'+\not{q})\gamma_\mu K\Big\}\epsilon^\lambda\epsilon^\sigma W^{\mu\nu}_{RM} \tag{13}$$

$$M_a M_c^* = -\frac{(4\pi\alpha)^2}{Q^2}\frac{G_\mu}{2\sqrt{2}}\frac{h}{2}\left(\frac{1}{2K'\cdot q}\right)^2$$
$$\times\Big[-g_V^e\mathrm{Tr}\Big\{\gamma_\nu(K'+\not{q})\gamma_\sigma K'\gamma_\lambda(K'+\not{q})\gamma_\mu K\gamma_5\Big\} \tag{14}$$
$$+g_A^e\mathrm{Tr}\Big\{\gamma_\nu(K'+\not{q})\gamma_\sigma K'\gamma_\lambda(K'+\not{q})\gamma_\mu K\Big\}\Big]\epsilon^\lambda\epsilon^\sigma W^{\mu\nu}_{INT} \quad .$$

For elastic scattering from spin-0 nuclei, only the $\mu = \nu = 0$ components of the $W^{\mu\nu}$ are non-vanishing. Since the trace multiplying g_V^e in Eq. (14) is anti-symmetric in μ and ν, this term does not contribute. Adding Eqs. (13-14) to the absolute squares of the corresponding tree-level amplitudes leads to

$$d\sigma^{\mathrm{tree}}_{RM} + d\sigma^{\mathrm{brem}}_{RM} \sim \frac{1}{2}\frac{(4\pi\alpha)^2}{Q^4}\Big[\mathrm{Tr}\Big\{\gamma_0 K'\gamma_0 K\Big\} +$$
$$(4\pi\alpha)\mathrm{Tr}\Big\{\gamma_0(K'+\not{q})\gamma_\sigma K'\gamma_\lambda(K'+\not{q})\gamma_0 K\Big\}\epsilon^\lambda\epsilon^\sigma\Big]W^{00}_{RM} \tag{15}$$

$$d\sigma^{\mathrm{tree}}_{INT} + d\sigma^{\mathrm{brem}}_{INT} \sim -\frac{h}{2}\frac{(4\pi\alpha)}{Q^2}\frac{G_\mu}{2\sqrt{2}}g_A^e\Big[\mathrm{Tr}\Big\{\gamma_0 K'\gamma_0 K\Big\} +$$
$$(4\pi\alpha)\mathrm{Tr}\Big\{\gamma_0(K'+\not{q})\gamma_\sigma K'\gamma_\lambda(K'+\not{q})\gamma_0 K\Big\}\epsilon^\lambda\epsilon^\sigma\Big]W^{00}_{INT} \tag{16}$$

Since $A_{LR} = (d\sigma^+_{INT} - d\sigma^-_{INT})/d\sigma_{RM}$, and since the quantities inside the square brackets in Eqs. (15) and (16) are identical, they cancel from the asymmetry. It is straightforward to show that this cancellation occurs even when the remaining terms in Eqs. (7-8) are included. In short, the bremsstrahlung contributions drop out entirely from A_{LR}, leaving the expression of Eq. (5) unchanged. One could, of course, attempt to be more rigorous and integrate bremsstrahlung cross sections over the detector acceptances, *etc.* In doing so, however, one would only modify the form of the expressions inside the square brackets in Eqs. (15-16) and not change the fact that they are identical in the two equations. The cancellation of bremsstrahlung contributions to the asymmetry would still obtain in this case. We note that this result does not carry over to nuclei having spin > 0. In the latter case, A_{LR} receives contributions from the leptonic vector NC (first term on the right side of Eqs. (13-14)). There exists no term in $d\sigma^{\mathrm{brem}}_{RM}$ to cancel the corresponding contribution from $d\sigma^{\mathrm{brem}}_{INT}$.

Returning to the remaining terms in Eq. (6), we emphasize that in contrast to the first three terms, the remaining terms are theoretically uncertain, due to the present lack of tractable methods for calculating low-energy strong interaction dynamics from first principles in QCD. Of particular concern are multi-boson-exchange dispersion contributions to $R_V^{I=0}$(had), such as those generated by the diagrams of Fig. 3. We note that neither the $G_E^{(s)}$-term of Eq. (5) nor the nuclear, many-body contributions to the dispersion corrections were included in the discussion of Ref. [1].

We first consider the impact of strangeness on the extraction of S from $A_{LR}(^{12}\mathrm{C})$. To that end, we employ an "extended" Galster parameterization[21] for the single-nucleon form factors appearing in Eq. (5): $G_E^{I=0} = \frac{1}{2}[G_E^p + G_E^n]$, $G_E^p = G_D^V$, $G_E^n = -\mu_n\tau G_D^V\xi_n$, $G_E^{(s)} = \rho_s\tau G_D^V\xi_s$, where μ_n is the neutron magnetic moment, $\tau = -Q^2/4m_N^2$, $G_D^V = (1+\lambda_D^V\tau)^{-2}$ is the standard dipole form factor appearing in nucleon form factors, and $\xi_{n,s} = (1+\lambda_E^{(n,s)}\tau)^{-1}$ allow for more rapid high-$|Q^2|$ fall-off than that given by the dipole form factor. From parity-conserving electron scattering, one has $\lambda_D^V \approx 4.97$ and $\lambda_n \approx 5.6$.[21] It is possible that $G_E^{(s)}$ falls off more rapidly at high-$|Q^2|$ than the $1/Q^4$ behavior exhibited by this parameterization, but for the momentum transfers of interest here,[22] this choice is sufficient. The parameters ρ_s and $\lambda_E^{(s)}$ characterize the low- and moderate-$|Q^2|$ behavior, respectively, of $G_E^{(s)}$ and are presently unconstrained. Because the nucleon has no net strangeness, $G_E^{(s)}$ must vanish at $Q^2 = 0 = \tau$. Hence, like G_E^n, which also must vanish at the photon point, the $G_E^{(s)}$ carries a linear dependence on $|Q^2|$ near the photon point. While no experimental information on $G_E^{(s)}$ exists, theoretical predictions for the mean-square strangeness radius (of which ρ_s is a dimensionless version) have been made using different models.[21-25, 14] Since these models generally predict qualitatively different behaviors of $G_E^{(s)}$ at moderate-$|Q^2|$, we choose the simple and convenient

Galster-like parameterization in which variations in this moderate-$|Q^2|$ behavior are characterized by a single parameter $\lambda_E^{(s)}$ to be constrained by experiment.

Under these choices, the strange-quark term in Eq. (5) induces a fractional shift in the $A_{LR}(^{12}\mathrm{C})$ asymmetry given by

$$\frac{\Delta A_{LR}}{A_{LR}} = \frac{\rho_s \tau \xi_s}{2\sin^2\theta_W[1-\mu_n\tau\xi_n]} \tag{17}$$

neglecting $R_V^{(0)}$. Taking the average value for ρ_s predicted in Ref. [22], choosing $\lambda_E^{(s)} = \lambda_n$, and working at the kinematics of the recent MIT-Bates $A_{LR}(^{12}\mathrm{C})$ measurement ($\tau \approx 0.007$), Eq. (17) indicates about a -3% shift in $A_{LR}(^{12}\mathrm{C})$. Any *uncertainty* in $G_E^{(s)}$ on this scale would weaken by a factor of three the limits on S predicted in Ref. [1].

From the standpoint of reducing the uncertainty in $A_{LR}(^{12}\mathrm{C})$ Standard Model tests, as well as that of learning about the distribution of strange quarks in the proton, it is clearly desirable to constrain $G_E^{(s)}$ as tightly as possible. To that end, a combination of two measurements of A_{LR} on a $(0^+,0)$ target could constrain $G_E^{(s)}$ sufficiently to reduce the $G_E^{(s)}$-induced error in a subsequent determination of S from $A_{LR}(^{12}\mathrm{C})$ to below $|\delta S| = 0.6$. For this purpose, we consider ^{4}He rather than ^{12}C. The statistical precision, $\delta A_{LR}/A_{LR}$, achievable for either nucleus goes as $\mathcal{F}^{-1/2}$, where the figure of merit $\mathcal{F} = \sigma A_{LR}{}^2$, with σ being the EM cross section.[6] For both nuclei, $\delta A_{LR}/A_{LR}$ displays a succession of local minima as a function of $|Q^2|$, corresponding to successive local maxima in the cross section. Since the relative sensitivity of $G_E^{(s)}$ to ρ_s and $\lambda_E^{(s)}$ changes with $|Q^2|$, a measurements of $A_{LR}(0^+,0)$ in the vicinity of different local minima in $\delta A_{LR}/A_{LR}$ would impose somewhat different joint constraints on ρ_s and $\lambda_E^{(s)}$. The EM cross section falls off more gently with $|Q^2|$ for ^{4}He than for ^{12}C, so that for the former, the first two $\delta A_{LR}/A_{LR}$ minima are more widely separated in $|Q^2|$ than for the latter. Consequently, the constraints on $G_E^{(s)}$ obtainable with two measurements carried out, respectively, at the first two $\delta A_{LR}/A_{LR}$ minima on ^{4}He could be more restrictive than with a similar series involving ^{12}C.

To complete this analysis, we consider a combination of two such $A_{LR}(^4\mathrm{He})$ experiments carried out roughly under conditions that are representative of what could be achievable with a moderate solid angle detector at CEBAF: luminosity $\mathcal{L} = 5\times10^{38}\mathrm{cm}^{-2}\mathrm{s}^{-1}$, scattering angle $\theta = 10°$, solid angle $\Delta\Omega = 0.01$ steradians, beam polarization $P_e = 100\%$, and run time $T = 1000$ hours.[26] The constraints resulting from these two prospective measurements are shown in Fig. 4. Since nothing at present is know experimentally about $G_E^{(s)}$, we assume two different models for illustrative purposes: (A) $(|\rho_s|, \lambda_E^{(s)}) = (0, \lambda_n)$ and (B) $(|\rho_s|, \lambda_E^{(s)}) = (2, \lambda_n)$. The value of $|\rho_s|$ in model (B) corresponds roughly to the average prediction of Ref. [22]. From these results, we find that for model (B), the uncertainty remaining in $G_E^{(s)}$ after the series of ^{4}He measurements would be sufficiently small to keep the associated error in a lower-$|Q^2|$ Standard Model test with either ^{12}C or ^{4}He below 1%. In the case of model (A), even though $\lambda_E^{(s)}$ is not constrained, the lower-$|Q^2|$ measurement appears to keep the $G_E^{(s)}$-induced error in a $(0^+,0)$ Standard Model test below 1% , independent of the value of $\lambda_E^{(s)}$.

Before such ^{4}He constraints could be attained or a 1% Standard Model test performed, ambiguities associated with dispersion corrections in $R_V^{I=0}(\mathrm{had})$ and with the isospin-mixing parameter Γ must be resolved. Turning first to the former, we focus on *nuclear* many-body contributions to the amplitudes associated with Fig. 3. Since $A_{LR}(0^+,0) \sim M_{NC}^{PV}(I=0)/M_{EM}^{PC}(I=0)$, where $M_{NC}^{PV}(I=0)$ ($M_{EM}^{PC}(I=0)$) are the isoscalar parity-violating (-conserving) scattering amplitudes, and since the dispersion corrections enter as $\mathcal{O}(\alpha)$ corrections to the tree-level amplitudes, one has $R_V^{I=0}(\mathrm{disp}) \sim R_V^{VV'}(I=0) - R_V^{\gamma\gamma}(I=0)$, where $R_V^{VV'}$ is a dispersion correction to the tree-level Z^0-exchange amplitude involving one or more heavy vector bosons and $R_V^{\gamma\gamma}$ is the two-photon correction to the isoscalar electromagnetic amplitude. Although one might naïvely hope for some cancellation between these two corrections, the different Q^2-dependences carried by each makes such a possibility unlikely. Whereas $R_V^{\gamma\gamma} \to 0$ as $|Q^2| \to 0$, since the tree-level EM amplitude has a pole at $Q^2 = 0$, $R_V^{VV'}$ need not vanish in this limit since the tree-level NC amplitude has a pole at $Q^2 = M_Z^2$.

Generally speaking, one expects the scale of hadronic contributions to $R_V^{I=0}(\mathrm{disp})$ to be of $\mathcal{O}(\alpha/4\pi)$. Indeed, theoretical estimates of such contributions to the 2-γ, PC, ep scattering amplitude indicate that $R_V^{\gamma\gamma}(ep) \lesssim 1\%$ at intermediate energies.[27, 28] However, experimental information on $R_V^{\gamma\gamma}$ suggests that the dispersion corrections for scattering from *nuclei* can be significantly larger than the one-body (ep) scale. Results from the recent MIT-Bates measurement of $R_V^{\gamma\gamma}(I=0)$ for ^{12}C show that this correction could be as large as 20% in the first diffraction minimum and several percent in the regions outside the minimum where a $(0^+,0)$ Standard Model test or $G_E^{(s)}$-determination might be undertaken.[29] In the latter regions, the experimental error in $R_V^{\gamma\gamma}(I=0)$ is of the same order as the correction itself, and the overall level of agreement between these results and theoretical calculations[30] is rather poor. In short, experimentally and theoretically uncertain many-body effects appear to enhance the scale of $R_V^{\gamma\gamma}(I=0)$ to a level which is important for the interpretation of $A_{LR}(0^+,0)$.

In the case of PV amplitudes, no experimental information exists on $R_V^{VV'}(I=0)$. It is unlikely that this quantity will be measured directly, so that one must

rely on nuclear model-dependent theoretical estimates of its scale. Of particular concern is the $Z^0-\gamma$ dispersion amplitude which, for elementary $e-q$ scattering, contains logarithms involving the ratios $|M_Z^2/s|$ and $|M_Z^2/u|$, where the scale of the invariant variables s and u is set by the incoming electron momentum and the typical momentum of the quark bound in the target nucleus.[10] This logarithmic scale mismatch suggests that contributions from low-energy intermediate states involving hard-to-calculate hadronic collective excitations (*e.g.*, the nuclear giant resonance) could be important. Given the scale of the $R_V^{\Gamma\gamma}(I=0)$ results, the discrepancy with theory, and the need for a theoretical estimate of $R_V^{Z\gamma}(I=0)$, significant progress in theoretical understanding of many-body contributions to the dispersion corrections is needed in order to keep the corresponding uncertainty in $A_{LR}(0^+,0)$ below one percent.

The quantity $\Gamma(q)$ ($q \equiv |\vec{q}|$) in Eq. (6) has been introduced to take into account the fact that nuclei such as ^{4}He and ^{12}C are not exact eigenstates of strong isospin with $I=0$. Since, the EM interaction does not conserve isospin, one expects states having $I \neq 0$ to be present as small [O(α)] components in the nuclear ground states. For nuclei whose major configurations involve either the 1s shell (^{4}He) or the 1p shell (^{12}C) the isospin-mixing correction $\Gamma(q)$ is likely to be quite small at low momentum transfer ($|\Gamma| \lesssim 1\%$).[19] This special situation arises because of the difficulty of supporting isovector breathing modes in the relevant nuclear model spaces; since primarily a single type of radial wave function plays a role, radial excitations are suppressed.

We emphasize that this conclusion need not apply to spin-0 nuclei beyond the $1s-1p$ shell. For nuclei in the $2s-1d$ shell, for example, one has wave functions which display different radial distributions (*viz.*, 2s and 1d), making it possible to have important isovector breathing-mode admixtures introduced into the nuclear ground states. For nuclei beyond ^{40}Ca an additional issue arises. Since in this region the stable 0^+ nuclei have $N > Z$ and, thus, $I \neq 0$ from the outset, both isoscalar and isovector matrix elements of the monopole operators enter (even in the absence of isospin-mixing). In this case, isospin-mixing effects appear in two ways: (1) several eigenstates of isospin can mix to form the physical states (as above) and (2) the mean fields in which the protons and neutrons in the nucleus move may be slightly different. This latter effect was explored in Ref. [19], where it was found that A_{LR} for elastic scattering from 0^+ $N > Z$ nuclei is rather sensitive to the difference between R_p and R_n, the radii of the proton and neutron distributions in the nuclear ground state, respectively. The reason for this sensitivity is that $|\xi_V^n| >> |\xi_V^p|$, making the NC "charge" densities for the neutron and proton roughly comparable in magnitude.

These observations imply that the extraction of interesting constraints on S, T, and $G_E^{(s)}$ from measurements of A_{LR} for spin-0 nuclei in this region is likely to be more difficult than for spin-0 nuclei in the 1s-1p shell. On the other hand, such measurements could provide a new window on certain aspects of nuclear structure. Since the EM charge radius can be determined quite precisely using, *e.g.*, parity-conserving (PC) electron scattering, a measurement of A_{LR} would provide a way to determine R_n. A 1% determination of R_n appears to be achievable. For a nucleus such as ^{133}Cs, with its importance for atomic PV, it may prove useful to employ electron scattering to explore some of these issues. The charge and neutron distributions could be studied, thereby helping to reduce R_n uncertainties appearing in $Q_W(^{133}\text{Cs})$ (see Eqs. (21-24) below), and some indication concerning the degree of isospin-mixing [Eq. (24)] could be obtained.

4. PV elastic scattering from the proton

As illustrated in Fig. 1, $A_{LR}(\vec{e}p)$ carries a stronger dependence on T than either $Q_W(^{133}\text{Cs})$ or $A_{LR}(^{12}\text{C})$, so that a measurement of the former, in combination of one or both of the latter, could provide an interesting set of low-energy constraints on S and T. Naïvely, one might expect the interpretation of $A_{LR}(\vec{e}p)$ to be simpler than that of $A_{LR}(^{12}\text{C})$, since one has no many-body nuclear effects to take into account. However, the spin and isospin quantum numbers of the proton allow for the presence of several form factors in $A_{LR}(\vec{e}p)$ not appearing in the ^{12}C asymmetry, with the result that the interpretation of PV $\vec{e}p$ scattering is in some respects more involved than that of elastic scattering from $(0^+,0)$ nuclei. A detailed discussion of PV elastic $\vec{e}p$ scattering can be found in Refs. [6, 31, 32], and we focus here solely on scattering in the forward direction.

At low momentum transfer and in the forward direction, the $\vec{e}p$ asymmetry has the form[6]

$$A_{LR}(\vec{e}p) \approx a_o\tau\left[\xi_V^p - \left\{G_E^n + G_E^{(s)} + \tau\mu_p(G_M^n + G_M^{(s)})\right\}\right] + O(\tau^2) \quad , \qquad (18)$$

where $a_o \approx 3\times10^{-4}$. The first term on the right side of Eq. (18) (containing ξ_V^p) is nominally independent of hadronic physics for essentially the same reasons as is the first term in the carbon asymmetry of Eq. (5). The terms contained inside the curly brackets all enter at O(τ), since both G_E^n and $G_E^{(s)}$ vanish at the photon point. From Eq. (18) one sees immediately the additional complexity of the proton asymmetry in comparison with that of carbon. The neutron EM form factors appear in $A_{LR}(\vec{e}p)$, since the isovector and isoscalar EM currents enter the hadronic neutral current (Eq. (1)) with different weightings than in the hadronic EM current. The presence of these form factors introduces one source of uncertainty not present at the same level in $A_{LR}(^{12}\text{C})$. In addition,

both the electric and magnetic strangeness form factors contribute at $O(\tau)$, and their presence also complicates the interpretation of the asymmetry.

As in the case of $A_{LR}(^{12}C)$, the τ-dependence of the terms in Eq. (18) suggests a two-fold strategy of measurements: (a) a very low-τ measurement to determine ξ_V^p, with an eye to obtaining the constraints indicated in Fig. 1, and (b) a moderate-τ measurement aimed at constraining the linear combination of form factors appearing in the second term of Eq. (18). The second of these measurements could be of interest for a number of reasons: to extract limits on the strangeness form factors, to constrain $G_E^{(s)}$ for purposes of interpreting $A_{LR}(^{12}C)$ as a Standard Model test, or to constrain this term for the same purpose but with a very low-τ $A_{LR}(\vec{e}p)$ measurement. Considering first scenario (a), we note that it is not possible to perform a Standard Model test at arbitrarily low-τ, since the statistical uncertainty *increases* for decreasing momentum transfer. For purposes of illustration, then, we analyze a prospective measurement at the limits of τ and forward scattering angle expected to be achievable at CEBAF Hall C. In order to achieve the 10% statistical uncertainty needed for the constraints in Fig. 1, a 1000 hour experiment would be needed, assuming 100% beam polarization. Under these conditions, the impact of form factor uncertainties on a determination of ξ_V^p is non-negligible. The dominant uncertainty is introduced by $G_E^{(s)}$. An uncertainty in the strangeness radius of $\delta\rho_s = \pm 2$ (corresponding to the magnitude of the prediction in Ref. [22]) would induce nearly a 30% uncertainty in the extracted value of ξ_V^p, a factor of three greater than the uncertainty assumed in Fig. 1. Similarly, an uncertainty in the value of μ_s of ± 0.3, also corresponding to the magnitude of the prediction in Ref. [22], would generate roughly a 20% error in ξ_V^p.

These statements point to the need for better constraints on the strangeness form factors if an interesting Standard Model test is to be performed with PV $\vec{e}p$ scattering. Turning, then, to strategy (b), we consider the constraints one might place on these form factors with a moderate-τ $A_{LR}(\vec{e}p)$ measurement. The difficulty here is that it is not possible to separate the form factors with $\vec{e}p$ scattering alone. As discussed in Ref. [6], a "perfect" backward-angle $A_{LR}(\vec{e}p)$ measurement (0% experimental error) might ultimately allow a determination of μ_s with an error of ± 0.12, thereby reducing the μ_s-induced uncertainty in a forward-angle Standard Model test below a problematic level. A subsequent determination of the second term in Eq. (18) might then allow a determination of $G_E^{(s)}$. We show in Fig. 4 the constraints in $(\rho_s, \lambda_E^{(s)})$ space such a measurement might achieve, assuming experimental conditions similar to those of recent CEBAF proposals.[33–35] We note that these constraints would not be sufficient to permit either a 10% determination of ξ_V^p from a low-τ $A_{LR}(\vec{e}p)$ measurement or a 1% Standard Model test with elastic scattering from ^{12}C. In the former case, the $G_E^{(s)}$-induced uncertainty in ξ_V^p would still be on the order of 20% . In fact, as Fig. 4 illustrates, it appears that a series of $A_{LR}(^4He)$ measurements could place far more stringent limits on $G_E^{(s)}$ than appears possible with PV $\vec{e}p$ scattering alone. Indeed, these limits would be sufficient to permit one to probe new physics with both $A_{LR}(\vec{e}p)$ and $A_{LR}(^{12}C)$ at the level assumed in Fig. 1.

5. Atomic PV

One should expect the impact of form factor uncertainties on the interpretation of Q_W to be considerably smaller than for electron scattering asymmetries, due to the very small effective momentum-transfer associated with the interaction of an atomic electron with the nucleus. Below, we quantify this statement with regard to the strangeness form factors, and note that only in the case of PV experiments with heavy muonic atoms might nucleon strangeness contribute at an observable level. To that end, consider the PV atomic hamiltonian which induces mixing of opposite-parity atomic states and leads to the presence of Q_W-dependent atomic PV observables:

$$\hat{H}_{PV}^{atom} = \frac{G_\mu}{2\sqrt{2}} \int d^3x \hat{\psi}_e^\dagger(\vec{x})\gamma_5\hat{\psi}_e(\vec{x})\rho^{NC}(\vec{x}) + \cdots \quad , \tag{19}$$

where $\hat{\psi}_e(\vec{x})$ is the electron field and $\rho^{NC}(\vec{x})$ is the Fourier Transform of $\rho^{NC}(\vec{q})$, the matrix element of the charge component of Eq. (1). For simplicity, we have omitted terms involving the spatial components of the nuclear vector NC as well as the nuclear axial vector NC. For a heavy atom, the leading term in Eq. (19) is significantly enhanced relative to the remaining terms by the coherent behavior of the nuclear charge operator. Consequently, one typically ignores the contribution from all magnetic form factors. Following Ref. [36], we write the matrix element of the leading term in $\hat{H}_{PV}^{atom}$ between atomic $S_{1/2}$ and $P_{1/2}$ states in the form $\langle P|\hat{\psi}_e^\dagger(\vec{x})\gamma_5\hat{\psi}_e(\vec{x})|S\rangle = \mathcal{N}C_{sp}(Z)f(x)$, where $\mathcal{N}$ is a known overall normalization, $C_{sp}(Z)$ is an atomic structure-dependent function, and $f(x) = 1 - \frac{1}{3}(x/x_o)^2 + \cdots$ gives the spatial-dependence of the electron axial charge density. In a simple model where a charge-Z nucleus is taken as a sphere of constant electric charge density out to radius R, one has $x_o = R/Z\alpha$ neglecting small corrections involving the electron mass. In this case, atomic matrix elements of Eq. (19) become

$$\langle P|\hat{H}_{PV}^{atom}|S\rangle = \frac{G_\mu}{2\sqrt{2}}\mathcal{N}C_{sp}(Z)\left[Q_W^{(0)} + \Delta Q_W^{(n,\,p)} + \Delta Q_W^{(s)} + \Delta Q_W^{(I)}\right] + \cdots \tag{20}$$

where

$$Q_w^{(0)} = \left(\frac{Z-N}{2}\right)\xi_v^{I=1} + \sqrt{3}\left(\frac{Z+N}{2}\right)\xi_v^{I=0} \tag{21}$$

$$\Delta Q_w^{(n,\,p)} = \tfrac{1}{2}\left[\sqrt{3}\xi_v^{I=0} + \xi_v^{I=1}\right]\langle I_0 \,\|\sum_{k=1}^{A} \tfrac{1}{2}[1+\tau_3(k)]h(x_k)\|\, I_0\rangle$$

$$+\tfrac{1}{2}\left[\sqrt{3}\xi_v^{I=0} - \xi_v^{I=1}\right]\langle I_0 \,\|\sum_{k=1}^{A} \tfrac{1}{2}[1-\tau_3(k)]h(x_k)\|\, I_0\rangle \tag{22}$$

$$\Delta Q_w^{(s)} = -\xi_v^{(0)}\left(\frac{\rho_s}{4m_\mu^2}\right)\langle I_0 \,\|\sum_{k=1}^{A} \nabla_k^2 h(x_k)\|\, I_0\rangle \tag{23}$$

$$\Delta Q_w^{(I)} = \lambda\xi_v^{I=1}\left[\langle I_0 \,\|\sum_{k=1}^{A} h(x_k)\tau_3(k)\|\, I_1\rangle + (I_1 \leftrightarrow I_0)\right] + \cdots \quad , \tag{24}$$

with $h(x) = f(x) - 1$, and with $\langle I_0 \,\|\hat{O}\|\, I_0\rangle$ denoting reduced matrix elements of a nuclear operator $\hat{O}$ in a nuclear ground state having nominal isospin I_0. The terms in Eq. (21) are those usually considered in analyses of Q_w. The term $\Delta Q_w^{(n,\,p)}$ carries a dependence on the ground-state neutron radius, R_n. The impact of uncertainties in R_n on the use of Q_w for high-precision electroweak tests has been discussed in Refs. [36, 37]. Eqs. (23) and (24) give, respectively, the leading contributions to Q_w from $G_E^{(s)}$ and from isospin impurities in the nuclear ground state. In arriving at Eqs. (21-24), we have kept terms in $f(x)$ only up through quadratic order and employed $R = r_o A^{1/3}$, $r_o \approx 1$ fm, for the nuclear radius. We have shown explicitly only the contribution to $\Delta Q_w^{(I)}$ arising from the mixing of a single state of isospin I_1 into the ground state of nominal isospin I_0 with strength λ. Additional contributions to Q_w arising from the single-nucleon EM charge radii are discussed elsewhere.[37]

According to Ref. [1], neglect of all but Eq. (21) leads to the prediction $Q_w(^{133}\mathrm{Cs}) = -73.20 - 0.8\,S - 0.005\,T$, so that a 0.7% determination of $Q_w(^{133}\mathrm{Cs})$ would constrain S to $|\delta S| \leq 0.6$. As noted in Ref. [36], a 10% uncertainty in R_n would generate a 0.7% error in $Q_w(^{133}\mathrm{Cs})$. While hadron-nucleus scattering typically permits a 5 - 10% determination of R_n for heavy nuclei,[19, 36] no experimental information on R_n for cesium isotopes presently exists. A series of PC and PV electron scattering experiments on ^{133}Cs could determine its neutron radius to roughly 1% accuracy.[6] In the meantime, one must rely on nuclear model calculations of R_n. The scale of the associated theoretical uncertainty in $Q_w(^{133}\mathrm{Cs})$ is presently the subject of debate.[37]

From Eq. (23), we find that an uncertainty in the strangeness radius induces an error in the weak charge of $\delta Q_w(^{133}\mathrm{Cs}) = -0.025\delta\rho_s$. For $\delta\rho_s$ on the order

of the average value of Ref. [22], the corresponding uncertainty in $Q_w(^{133}\mathrm{Cs})$ is slightly less than 0.1% , more than an order of magnitude below the dominant theoretical error associated with atomic structure[1, 4] and well below the level needed for an interesting $Q_w(^{133}\mathrm{Cs})$ Standard Model test. As expected, the situation differs sharply from that of PV electron scattering. Indeed, a measurement of $A_{LR}(^{12}\mathrm{C})$ would have to be carried out at $|\vec{q}| \approx 30$ MeV/c — roughly an order of magnitude smaller than in the experiment of Ref. [3] – to be equally insensitive to $G_E^{(s)}$.

We close with observations on the possibility of observing $G_E^{(s)}$ using PV experiments on muonic atoms. It has been noted recently that 1 - 10% measurements of PV observables for muonic boron may be feasible in the future at PSI.[13, 38] Since the ratio of Bohr radii $a_0^e/a_0^\mu = m_e/m_\mu \sim 207$, the muon in these atoms is more tightly bound for a given set of radial and angular momentum quantum numbers. One might expect, then, an enhanced sensitivity to short-range contributions to Q_w, such as those associated with R_n or ρ_s. To analyze the latter possibility, we solve the Dirac equation for a muon orbiting a spherically-symmetric nuclear charge distribution, keeping terms involving m_μ.[39] The result of this procedure is to make the replacement $x_o = R/Z\alpha \to [3R/4m_\mu Z\alpha]^{1/2}$ in the function $h(x)$ in Eqs. (21-24). The scale of $\Delta Q_w^{(s)}$ is correspondingly enhanced by $4m_\mu R/3Z\alpha \sim 4m_\mu r_o A^{1/3}/3Z\alpha$ over its magnitude for an electronic atom. In the case of ^{133}Cs, this enhancement factor is ≈ 8, making $Q_w(\mu\mathrm{Cs})$ roughly as sensitive to ρ_s as is $A_{LR}(\vec{e}p)$. The sensitivity of $\Delta Q_w^{(s)}$ for a muonic lead atom is roughly two times greater than $\Delta Q_w^{(s)}(\mu\mathrm{Cs})$. For light muonic atoms, on the other hand, the ρ_s contribution is still suppressed. In the case of muonic boron, for example, uncertainties associated with ρ_s would not enter the parameters ξ_v^p and ξ_v^n at an observable level. Consequently, one must go to heavy muonic atoms. While the sensitivity of the latter to R_n-uncertainties is also enhanced, these uncertainties could be reduced through a combination of PC and PV elastic electron scattering experiments.[6, 19] Given the simplicity of atomic structure calculations for muonic Cs or Pb (essentially a one-lepton problem), the theoretical atomic structure uncertainties entering Q_w-determinations should not enter at a level problematic for $G_E^{(s)}$ determinations. Thus, an experiment of this type could complement PV electron scattering as a probe of strange quarks in the nucleon. The remaining obstacle is the experimental one of achieving sufficient precision. To this end, it would be desirable to find a heavy muonium transition for which the PV signal is enhanced by accidental near degeneracies between opposite-parity atomic levels.

6. Conclusions

With any attempt at a precision electroweak test involving a low-energy hadronic system, one must ensure that all sources of theoretical hadronic physics uncertainties fall below the requisite level. The situation contrasts with purely leptonic or high-energy electroweak tests. In the former case, given a model of electroweak interactions, one can make precise and unambiguous predicitions for different observables, up to uncertainties associated with unknown parameters (*e.g.*, m_t and M_H) and with hadronic loops. In the latter instance, strong-interaction uncertainties are controllable through the use of a perturbative expansion and QCD. In the non-perturbative low-energy regime, however, one must rely on the use of symmetries as well as model estimates of, or independent experimental constraints on, hadronic effects. The scale of uncertainty in a low-energy semi-leptonic electroweak test, then, is set by experimental input and, where such is lacking, any reasonable model estimate. In the foregoing discussion, we have noted that completion of one or more PV electron scattering experiments has the potential to complement atomic PV as a low-energy probe of new physics. At present, however, experimental limits on nuclear dispersion corrections, as well as theoretical predictions for the nucleon's strangeness form factors, indicate that these two sources of hadronic physics uncertainty are too large to make interesting electroweak tests possible with low-energy polarized electrons. We have shown how a series of PV elastic scattering experiments with ^{4}He could reduce the uncertainty associated with the strangeness radius below a problematic level. Achieving a better understanding of nuclear dispersion corrections remains a challenge for both experiment and theory.

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39. We are indebted to Prof. L. Wilets for discussions on this procedure.

FIGURE CAPTIONS

Fig. 1. Present and prospective constraints on S, T parameterization of non-standard physics from low- and intermediate-energy PV observables. Short-dashed lines give present constraints from cesium atomic PV.[1, 4, 5] Solid lines give constraints from a 1% $A_{LR}(^{12}C)$ measurement. Long-dashed lines correspond to a 10% determination of ξ_V^p from a forward-angle measurement of $A_{LR}(\vec{e}p)$. For simplicity, it is assumed that all experiments agree on common central values for S and T, so that only the deviations from these values are plotted.

Fig. 2. Electron bremsstrahlung for electromagnetic (Fig. 2a,b) and weak neutral current (Fig. 2c,d) scattering from a hadronic target. Target bremsstrahlung is assumed to be negligible for low-energy (small recoil) processes.

Fig. 3. Dispersion corrections to tree-level EM and NC electron-nucleus scattering amplitudes. Here, V, V' are any one of the $Z^0, W^{\pm}, \gamma$ vector bosons and $|i\rangle$ ($|f\rangle$) are initial (final) nuclear states.

Fig. 4. Constraints imposed on $G_E^{(s)}$ from prospective PV elastic scattering experiments. Dashed-dot curves and solid curves give, respectively, constraints from possible low- and moderate-$|Q^2|$ measurements of $A_{LR}(^4\mathrm{He})$. Dashed lines give constraints from series of forward- and backward-angle $A_{LR}(\vec{e}p)$ measurements. Panels (a) and (b) correspond to two models for $G_E^{(s)}$ discussed in the text, where the canonical values of $(|\rho_s|, \lambda_E^{(s)})$ are indicated by the large dot.

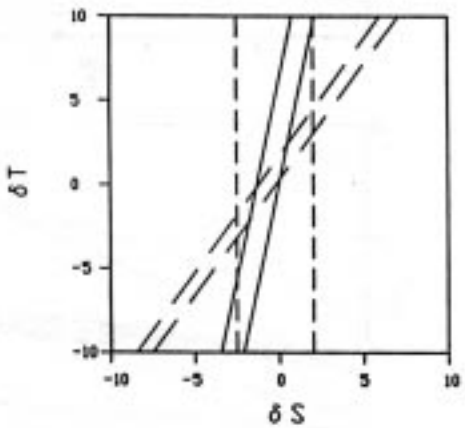

Fig. 1.

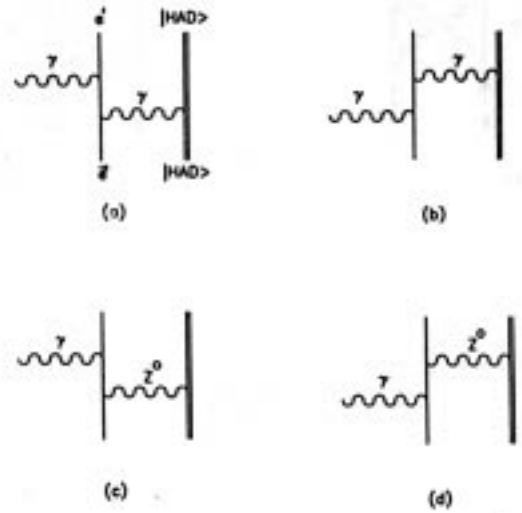

Fig. 2.

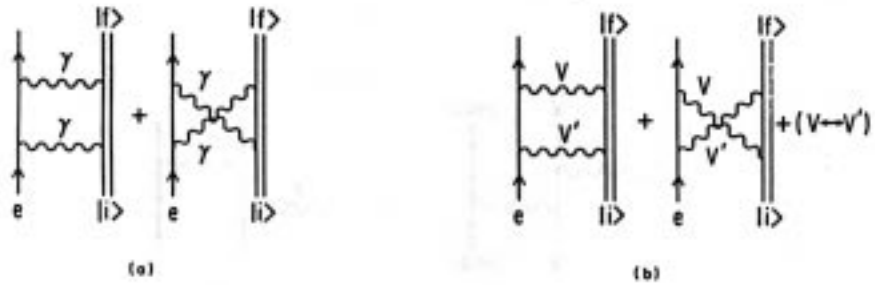

Fig. 3.

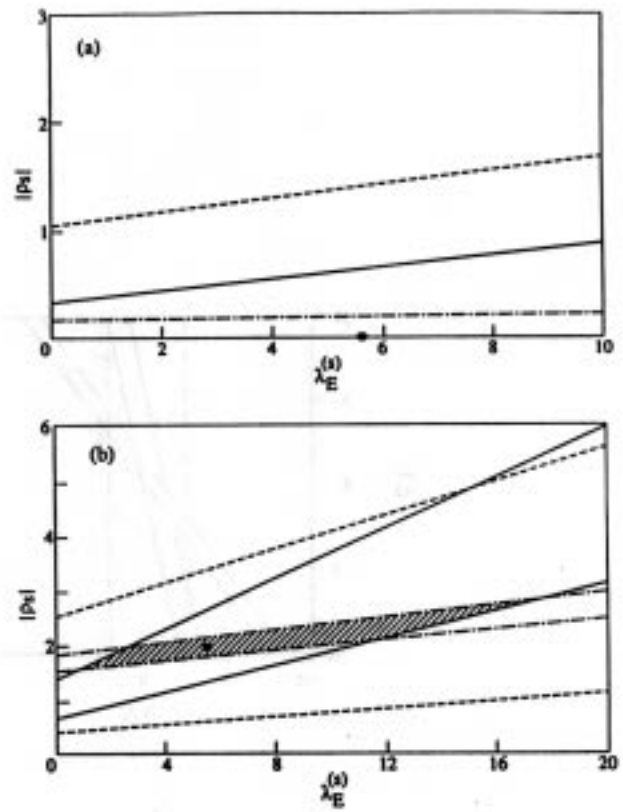

Fig. 4.