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# ELECTROMAGNETIC MASS SPLITTINGS IN HEAVY MESONS

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The electromagnetic contribution to the isomultiplet mass splittings of heavy mesons is reanalyzed within the framework of the heavy mass expansion. It is shown that the leading term in the expansion is given to a good approximation by the elastic term. $1/m_Q$-corrections can only be estimated, the main source of uncertainty now being inelastic contributions. The $1/m_Q$-corrections to the elastic term turn out to be relatively small in both D and B mesons.

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## I. INTRODUCTION

The measurement of mass splittings in heavy pseudoscalar and vector mesons has attained a substantial degree of precision in the D-system [1, 2] and in the B-system is expected to improve over the presently available data [3]. In particular, the isospin splitting in D and D* mesons is now known within an error of ±0.2 MeV [2], in B mesons the error is larger and about ±1 MeV [3] and, in B* mesons its experimental determination is an important task still to be accomplished. The theoretical understanding of isospin splittings is less satisfactory. Neither the electromagnetic nor the quark mass contributions can be determined to a similar degree of precision. In both cases, the task boils down to a strong interaction problem where the extraction of a result cannot be at present achieved without assumptions and, to some extent, modeling [4-9]. Surprisingly good agreement with data was found [8] by an analysis in the limit of infinite heavy quark mass using a linear interpolation to determine the contribution $\Delta^m M$ to the splitting by the masses of the u and d quarks, and the elastic approximation to the electromagnetic contribution $\Delta^\gamma M$ with a VMD model for the elastic form factors associated to the light quark components of the electromagnetic current. There are, however, unanswered questions concerning each of the assumptions made. In the present work, we reconsider $\Delta^\gamma M$ in the light of the heavy mass expansion. We consider models beyond VMD and give an estimate of corrections of order $1/m_Q$. Some conclusions of general character are drawn from the analysis.

## II. ELECTROMAGNETIC MASS SPLITTINGS

The QED contribution to the mass of a hadron H is given to order $\alpha$ by:

$$\delta^\gamma M_H = \lim_{\Lambda\to\infty} \frac{i\,\alpha}{2\,M_H\,(2\pi)^3} \int d^4q\; \mathrm{D}_{\mu\nu}(q)\, \mathrm{T}_H^{\mu\nu}(q,P) \left[\frac{\Lambda^2}{\Lambda^2-q^2}\right] + \delta^\gamma M_H^{c.t.}(\Lambda) \tag{1}$$

where $P^2 = M_H^2$, $-i\,\mathrm{D}_{\mu\nu}(q)$ is the photon propagator in an arbitrary gauge, the tensor $\mathrm{T}_H^{\mu\nu}$, the covariant forward amplitude for virtual Compton scattering off H, is given by

$$\mathrm{T}_H^{\mu\nu}(q,P) \equiv i \int d^4x\, e^{i\,qx}\, \langle H,P,\sigma\,|\,T\, j^\mu(x) j^\nu(0)\,|\,H,P,\sigma\rangle \tag{2}$$

where $\sigma$ denotes the polarization of the hadron, and $\delta^\gamma M_H^{c.t.}(\Lambda)$ is the counterterm needed to render $\delta^\gamma M_H$ U.V. finite. The customary covariant normalization of states is used. In this work, we focus on heavy mesons containing a light quark (u, d) and a heavy antiquark (c, b). $H$ and $H^*$ will denote respectively the pseudoscalar and vector mesons. The electromagnetic current pertaining to our analysis is conveniently decomposed as follows:

$$\begin{aligned} j^\mu &= J_3^\mu + \frac{1}{2}\,J_Y^\mu + e_Q\,J_Q^\mu \\ J_3^\mu &= \frac{1}{2}\,(\bar u\gamma^\mu u - \bar d\gamma^\mu d) \\ J_Y^\mu &= \frac{1}{3}\,(\bar u\gamma^\mu u + \bar d\gamma^\mu d) \\ J_Q^\mu &= \bar Q\gamma^\mu Q \end{aligned} \tag{3}$$

where $Q$ denotes the heavy quark with charge $e_Q$.

Since QCD plus QED is a renormalizable and parity conserving theory, the counterterm $\delta^\gamma M_H^{c.t.}(\Lambda)$ can only involve the matrix elements of even parity gauge-invariant operators of dimension four or less [10]. At order $\alpha$, the only such operators are $G^{\mu\nu}G_{\mu\nu}$ ($G^{\mu\nu}$ is the QCD field strength), $\bar q_i q_i$ and $\bar Q Q$. The quark bilinear operators must appear multiplied by a coefficient proportional to the respective quark mass as demanded by a consistent chiral limit. In this limit, the difference of mass shifts relevant to this work

$$\Delta^\gamma M_H \equiv \delta^\gamma M_{H_d} - \delta^\gamma M_{H_u} \tag{4}$$

becomes U.V. finite. This results from the cancellation of the gluon- and heavy quark- operator contributions in the difference. Throughout this work, we take the chiral limit; this amounts to an error in $\Delta^\gamma M_H$ of order $\alpha m_q/\Lambda_{QCD}$. The $\bar s\gamma^\mu s$ component of the electromagnetic current drops off when considering (4). For this reason it was not included in (3).

Following Cottingham [11], one can perform in (1) an Euclidean rotation in the $\nu = \frac{P\cdot q}{M_H}$ complex plane and express:

$$\Delta^\gamma M_H = \frac{\alpha}{2\,M_H\,(2\pi)^3} \int \frac{dQ^2}{Q^2} \int_{-Q}^{Q} d\nu\, \sqrt{Q^2-\nu^2}\; T^\mu_\mu(Q^2,-i\,\nu) \tag{5}$$

where $Q^2 = -q^2$, and:

$$T^{\mu\nu} \equiv T^{\mu\nu}_{H_d} - T^{\mu\nu}_{H_u} \tag{6}$$

It is convenient to separate $T^{\mu\nu}$ into two pieces, one referring only to the light quark clouds and the other to the light quark cloud and the heavy quark:

$$T^{\mu\nu} = T^{\mu\nu}_A + T^{\mu\nu}_B \tag{7}$$
$$T^{\mu\nu}_i = i \int d^4x \, e^{iqx} \left( \langle H_d \mid T(J^\mu_i(0)\, J^\nu_3(x) + J^\nu_3(0)\, J^\mu_i(x)) \mid H_d \rangle - d \to u \right)$$

where $i = A, B$, and $J^\mu_A \equiv \frac{1}{2} J^\mu_V$ and $J^\mu_B \equiv e_Q J^\mu_Q$.

The lack of experimental access to the absorptive part of $T^{\mu\nu}_H$, which would permit the reconstruction of $T^{\mu\nu}$, compels us to resort to some approximations. The main approximation will consist in taking only the elastic terms in the absorptive part of $T^{\mu\nu}$. In the framework of the heavy quark expansion, we will show that inelastic terms are of leading order $(m_Q^0)$ for $i = A$ and of order $1/m_Q$ for i=B.

The elastic contributions are conveniently studied in the large $m_Q$ limit, where the pseudoscalar and vector mesons belong to a multiplet under the heavy quark symmetry [12]. It is therefore appropriate to keep as elastic terms in the absorptive part of $T^\mu_\mu$ not only the strictly elastic terms, but also those inelastic terms which involve a M1 transition to $H^*$ in the intermediate state. We can then write the elastic absorptive part of $T^{\mu\nu}_H$:

$$W^{el\,\mu\nu}_{H;i}(q,P) \equiv \frac{1}{2} \sum_{n=H,H^*} \int \frac{d^3p_n}{2p^0_n} (2\pi)\, \delta^4(p_n - P - q) \tag{8}$$
$$\cdot \left[ \langle H,P \mid J^\mu_i(0) \mid n,p_n \rangle \langle n,p_n \mid J^\nu_3(0) \mid H,P \rangle + \begin{matrix} i \leftrightarrow 3 \\ \mu \leftrightarrow \nu \end{matrix} \right]$$
$$W^{el\,\mu\nu}_i \equiv W^{el\,\mu\nu}_{H_d;i} - W^{el\,\mu\nu}_{H_u;i}$$

with the matrix elements

$$\begin{aligned}
\langle H_\beta, P' \mid J^\mu_i(0) \mid H_\alpha, P \rangle &= \delta_{\alpha\beta}\, (P+P')^\mu\, F_i(Q^2) \\
\langle H_\beta, P' \mid J^\mu_3(0) \mid H_\alpha, P \rangle &= \tau^3_{\alpha\beta}\, (P+P')^\mu\, F_3(Q^2) \\
\langle H^*_\beta, \epsilon, P' \mid J^\mu_i(0) \mid H_\alpha, P \rangle &= i\delta_{\alpha\beta}\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon_\nu P_\rho P'_\sigma\, g_i(Q^2) \\
\langle H^*_\beta, \epsilon, P' \mid J^\mu_3(0) \mid H_\alpha, P \rangle &= i\tau^3_{\alpha\beta}\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon_\nu P_\rho P'_\sigma\, g_3(Q^2)
\end{aligned} \tag{9}$$
$$F_V(0) = \frac{1}{3} F_3(0) = \tfrac{1}{2} \qquad \alpha,\, \beta = u,\, d$$

where $\epsilon_\nu$ is the polarization four-vector of $H^*$. In the large $m_Q$ limit one has [12]:

$$F_Q(Q^2) = -e_Q\, \xi(v.v') \tag{10}$$
$$g_Q(Q^2) = e_Q\, \frac{\xi(v.v')}{M_H} \qquad \xi(1) = 1$$



$\xi$ is the universal I-W form factor, and $v$ denotes the four-velocity of the meson. The sign convention for $g_Q$ given by this equation implies that $\mathrm{sgn} F_V = \mathrm{sgn} g_V$.

In the customary tensorial decomposition of the hadronic tensor, (8) and (9) give the following expressions for the elastic structure functions:

$$W^{el\,\mu\nu}_{1,i}(q,P) = -4\pi\, M_H^2\, \delta(2\,M_H\,\nu - Q^2)\, \theta((P+q)^0)\, g_i(Q^2)\, g_3(Q^2)\, (\nu^2 + Q^2)$$
$$W^{el\,\mu\nu}_{2,i}(q,P) = -4\pi\, M_H^2\, \delta(2\,M_H\,\nu - Q^2)\, \theta((P+q)^0) \tag{11}$$
$$\times \left[ 4\,F_i(Q^2)\, F_3(Q^2) + g_i(Q^2)\, g_3(Q^2)\, \frac{\nu^2 + Q^2}{1 + \frac{Q^2}{4M_H^2}} \right]$$

At fixed $Q^2 \geq 0$ the dispersion integral over the variable $\nu$ finally gives:

$$T^{el}_{1,i}(Q^2,\nu) = -2\, g_i(Q^2)\, g_3(Q^2) \frac{Q^4(1 + \frac{Q^2}{4M_H^2})}{\frac{Q^4}{4M_H^2} - \nu^2 - i\epsilon\, Q^2} \tag{12}$$
$$T^{el}_{2,i}(Q^2,\nu) = -2\left(4F_i(Q^2)\, F_3(Q^2) + Q^2\, g_i(Q^2)\, g_3(Q^2)\right) \frac{Q^2}{\frac{Q^4}{4M_H^2} - \nu^2 - i\epsilon\, Q^2}$$

Using that:

$$T^\mu_\mu(q,\nu) = -3\,T_1(Q^2,\nu) + T_2(Q^2,\nu)(1 + \frac{\nu^2}{Q^2}) \tag{13}$$

replacing in (5), and performing the integration over $d\nu$ we finally obtain:

$$\Delta^{\gamma\, el} M_H = \Delta^{\gamma\, el}_A M_H + \Delta^{\gamma\, el}_B M_H$$
$$\Delta^{\gamma\, el}_i M_H = \frac{\alpha}{\pi M_H} \int dQ^2 \left( F_i(Q^2)\, F_3(Q^2) \left[ (1 + \frac{Q^2}{4\,M_H^2})\,(1 - \sqrt{1 + 4\frac{M_H^2}{Q^2}}) + \frac{1}{2} \right] \right.$$
$$\left. - g_i(Q^2)\, g_3(Q^2)\, \frac{Q^2}{2} \left[ (1 + \frac{Q^2}{4\,M_H^2})\,(1 - \sqrt{1 + 4\frac{M_H^2}{Q^2}}) - \frac{1}{4} \right] \right) \tag{14}$$

To leading order in the heavy quark expansion the result becomes physically very simple:

$$\Delta^{\gamma\, el} M_H \big|_0 = -\frac{4\alpha}{\pi} \int_0^\infty dQ \left( \frac{1}{2} F_V(Q^2)\, F_3(Q^2) \right.$$
$$\left. - e_Q\, F_3(Q^2) - \frac{Q^2}{4}\, g_V(Q^2)\, g_3(Q^2) \right) \tag{15}$$

where the form factors are taken in the $m_Q \to \infty$ limit. The first term is the difference between the Coulomb self-energies of the light quark clouds with the u and d flavor quantum numbers, the second term is the difference between the Coulomb interaction of these clouds with the heavy quark, and the third term is the result of M1 transitions $H \leftrightarrow H^*\gamma$ mediated by the light quark component of the e.m. current. Since the relevant contributions to the integral in (15) stem from values of $Q^2$ of the order of light hadronic masses, one can neglect the $Q^2$-dependence in $F_Q(Q^2)$ which amounts to an error of order $m_Q^{-2}$. Since $g_Q$ is already of order $1/m_Q$, it does not appear in (15).

There are two types of $1/m_Q$-corrections, kinematic ones which are obtained by expanding the terms in the square brackets in (14), and dynamical ones which reside in the heavy mass dependence of the form factors associated with the light quark currents. The latter are hard to determine; charge conservation assures us that the corrections vanish at $Q^2 = 0$ and can only affect the charge radius. Kinematic corrections due to the mass difference $M_{H^*} - M_H \propto 1/m_Q$ are of higher order. One thus obtains the following $1/m_Q$-corrections:

$$\Delta_A^{\gamma\, el} M_H \Big|_1 = \frac{3}{2}\frac{\alpha}{\pi M_H} \int_0^\Lambda dQ \left( Q\, F_Y(Q^2)\, F_3(Q^2) - \frac{1}{4} Q^3\, g_Y(Q^2)\, g_3(Q^2) \right.$$
$$\left. - \frac{8}{3} F_Y(Q^2) \frac{\partial}{\partial \frac{1}{M_H}} F_3(Q^2) + \frac{4}{3}\, Q^2\, g_Y(Q^2) \frac{\partial}{\partial \frac{1}{M_H}} g_3(Q^2) \right) \tag{16}$$
$$\Delta_B^{\gamma\, el} M_H \Big|_1 = -3\, e_Q \frac{\alpha}{\pi M_H} \int_0^\Lambda dQ \left( Q\, F_3(Q^2) - \frac{2}{3} Q^2\, g_3(Q^2) \right.$$
$$\left. - \frac{4}{3} \frac{\partial}{\partial \frac{1}{M_H}} F_3(Q^2) \right)$$

We have assumed here that the $m_Q$-dependences of $F_Y(Q^2)$ and $F_3(Q^2)$ are the same. A similar assumption applies to the M1 form factors. In the limit where the light quark is taken as a constituent quark, the term proportional to $g_3(Q^2)$ in $\Delta_B^{\gamma\, el} M_H$ is identified with the difference between hyperfine splitting. This clarifies our inclusion of the M1 transition terms and also the sign of the contribution.

It is important to notice that the expansion in powers of $1/m_Q$ we have implemented corresponds to that of the HQET (heavy quark effective theory). The integration over $Q$ must then be cut off at a scale $\Lambda << m_Q$, and the contributions which arise from the domain $Q > \Lambda$ must be included as local counter-terms of order $1/m_Q$. These terms are, moreover, responsible for lifting the cut off dependence of the results. A list of the possible counter-terms one



can add has been given in ref [9]. The size of the coefficients in front of these terms is not known a priori. In the case of the elastic contributions they could be determined once all the form factors are known above the cut off. Our ignorance about the counter-terms is manifested, therefore, in the cut off dependence of our results, which depends on the asymptotic behavior of the intergrands in (16), and which turns out to be more pronounced for $i = B$.

There are other terms of order $1/m_Q$ which have been disregarded; they are the tadpole terms resulting from seagull type contributions to $T_H^{\mu\nu}$, and which must be present by gauge invariance. In particular, their strength is fixed $Q^2 = 0$ in terms of the electric charge of the meson. This fact has the particular consequence that at order $1/m_Q$ the tadpole contributions to $\Delta^\gamma M_H$ and $\Delta^\gamma M_{H^*}$ are equal.

In conclusion, we are able to determine the $1/m_Q$-corrections of kinematical origin, up to the mentioned cut off dependence, and those originating in hyperfine type interactions. Those residing in the form factors will in any event be buried in our lack of knowledge of the form factors themselves, and those of tadpole origin will be disregarded.

For the sake of providing a rough quantitative estimate of the mass splitting we now resort to a model for the form factors. In the case of the proton and the pion, the asymptotic behaviour of the form factors seems to pervade the behavior at lower values of $Q^2$, giving rise to the respective dipole and monopole shapes. For heavy mesons the situation is more involved. Within the heavy quark expansion, the asymptotic behavior corresponds to $\Lambda_{QCD}^2 \ll Q^2 \ll m_Q^2$. To leading order in $\alpha_s$, the asymptotic behavior of the charge and M1 form factors coincide with those of the relativistic hydrogen atom with $\alpha = \frac{4}{3}\alpha_s$:

$$F(Q^2) \propto Q\, g(Q^2) \propto Q^{-\delta(\alpha_s)}$$
$$\alpha_s = \alpha_s(Q^2) \tag{17}$$

The exponent has the following values at some illustrative points: $\delta(0) = 3$, $\delta(0.3) = 2.8$, $\delta(0.5) = 2.5$, $\delta(0.6) = 2.2$, $\delta(0.64) = 2$. For those values of $\alpha_s$ where one can trust this estimate the form factors at large $Q^2$ fall off faster than for the pion. Since the asymptotic behavior is not definitely known, we choose the form factors to have the following form:

$$3\, F_Y(Q^2) = 2\, F_3(Q^2) = \left(1 + \frac{Q^2}{m_\rho^2}\right)^{-n} \tag{18}$$
$$3\, g_Y(Q^2) = 2\, g_3(Q^2) = \beta \left(1 + \frac{Q^2}{m_\rho^2}\right)^{-n-\frac{1}{2}}$$

The choice of $m_\rho$ as the relevant scale is natural, and likely to be a reasonably good approximation. The constant $\beta$ determines the rate of the transition $H^* \to H\gamma$. We will choose here a value suggested by quark models [9]: $\beta \sim 3\ \mathrm{GeV}^{-1}$. $\kappa$ will be varied between 1, which corresponds to the vector meson dominance model [8], and 1.5, which asymptotically corresponds to a loosely bound light quark. Neglecting the $1/m_Q$ corrections to the form factors one obtains:

$$
\begin{aligned}
\Delta_A^{\gamma\,el} M_H &= -\frac{\alpha\, m_\rho}{2\pi} \left\{ \frac{\sqrt{\pi}}{3} \frac{\Gamma(2\kappa - \frac{1}{2})}{\Gamma(2\kappa)} \left(1 - \frac{\beta^2 m_\rho^2}{8\kappa}\right) \right. \\
&\qquad \left. + \frac{m_\rho}{32\, M_H} \frac{\beta^2 m_\rho^2 - 8\kappa}{\kappa(2\kappa - 1)} \right\} \qquad \kappa \geq \frac{1}{2} \\
\Delta_B^{\gamma\,el} M_H &= e_Q\, \frac{\alpha\, m_\rho}{\pi} \left\{ \sqrt{\pi} \frac{\Gamma(\kappa - \frac{1}{2})}{\Gamma(\kappa)} \right. \\
&\qquad + \frac{3 m_\rho}{4 M_H} \left( \frac{1}{1-\kappa} + \beta\, m_\rho \frac{\sqrt{\pi}}{3} \frac{\Gamma(\kappa - 1)}{\Gamma(\kappa + \frac{1}{2})} \right. \\
&\qquad \left. \left. + \left(\frac{m_\rho}{\Lambda}\right)^{2(\kappa-1)} \frac{1}{\kappa - 1} \left(1 - \frac{2}{3} \beta\, m_\rho\right) \right) \right\}
\end{aligned}
\tag{19}
$$

For the chosen range of values for $\kappa$, only $\Delta_B^{\gamma\,el} M_H$ shows non-analytic contributions when setting $\Lambda \sim M_H$.

| H | $\kappa$ | $\beta = 0$ | | | $\beta = 3\ \mathrm{GeV}^{-1}$ | | |
|---|---|---|---|---|---|---|---|
| | | $\Delta_A^{\gamma\,el} M_H$ MeV | $\Delta_B^{\gamma\,el} M_H$ MeV | $\Delta^{\gamma\,el} M_H$ MeV | $\Delta_A^{\gamma\,el} M_H$ MeV | $\Delta_B^{\gamma\,el} M_H$ MeV | $\Delta^{\gamma\,el} M_H$ MeV |
| D | 1 | **-0.39** | **3.0** | **2.6** | **-0.12** | **3.8** | **3.7± 0.2** |
| | | -0.47 | 3.7 | $\boxed{3.2}$ | -0.15 | 3.7 | 3.6 |
| | 1.25 | **-0.34** | **2.3** | **1.9** | **-0.15** | **2.9** | **2.7± 0.2** |
| | | -0.40 | 2.8 | 2.5 | -0.18 | 2.8 | 2.6 |
| | 1.5 | **-0.30** | **1.9** | **1.6** | **-0.17** | **2.4** | **2.2± 0.1** |
| | | -0.35 | 2.4 | 2.0 | -0.19 | 2.4 | 2.2 |
| B | 1 | **-0.44** | **-1.6** | **-2.1** | **-0.14** | **-1.9** | **-2.1± 0.1** |
| | | -0.47 | -1.9 | $\boxed{-2.4}$ | -0.15 | -1.9 | -2.1 |
| | 1.25 | **-0.38** | **-1.3** | **-1.7** | **-0.17** | **- 1.6** | **-1.6± 0.1** |
| | | -0.40 | -1.4 | -1.8 | -0.18 | -1.4 | -1.6 |
| | 1.5 | **-0.33** | **-1.1** | **-1.4** | **-0.19** | **-1.2** | **-1.4±0.05** |
| | | -0.35 | -1.2 | -1.5 | -0.19 | -1.2 | -1.4 |

**Table:** *Results for electromagnetic mass splittings in the elastic approximation as defined in the text. Bold type numbers refer to $1/m_Q$ corrected values. The results correspond to the choice $\Lambda \sim M_H$ in eq. (19), and the error shown in the last column results from a change in $\Lambda$ by a factor of 2. For comparison the results in the $m_Q \to \infty$ limit are also shown. The numbers designated by the square are the results of the VMD model* [8].

Numerical results for three different choices of $\kappa$ are shown in the table. In all cases $\Delta_A^{\gamma\,el} M_H$ is only a fraction of $\Delta_B^{\gamma\,el} M_H$. For $\beta = 0$ (i.e., in the strictly elastic approximation), the considered $1/m_Q$-corrections to $\Delta_A^{\gamma\,el} M_H$ are small: 0.1 MeV for D mesons and 0.03 MeV for B mesons, while the corrections to $\Delta_B^{\gamma\,el} M_H$ are larger: -0.7 to -0.5 MeV for D mesons and -0.3 to -0.1 MeV for B mesons, and tend to suppress the splitting. For $\beta = 3\ \mathrm{GeV}^{-1}$, the additional contributions to $\Delta_A^{\gamma\,el} M_H$ and $\Delta_B^{\gamma\,el} M_H$ due to the M1 transitions are of order $m_Q^0$ and $1/m_Q$ respectively. They lead to a suppression of $\Delta_A^{\gamma\,el} M_H$ by a factor 1/3 to 1/2; this, however, affects the overall mass splitting by less than 0.35 MeV. The corrections to $\Delta_B^{\gamma\,el} M_H$, identified before to be of hyperfine nature, have the sign opposite to that of the kinematical $1/m_Q$-corrections, and they are about 0.8 to 0.5 MeV for D mesons and about -0.3 to -0.2 MeV for B mesons. The cut off dependence is a function of $\beta$: $\left|\Delta^{\gamma\,el} M_H\right|$ increases (decreases) with increasing $\Lambda$ if $\beta = 0$ ($\beta = 3\ \mathrm{GeV}^{-1}$), and according with (19) the sensitivity diminishes as

$\kappa$ increases. We estimate that the cut off dependence affects the elastic term of D and B mesons by $\pm 0.2$ Mev and $\pm 0.1$ Mev respectively. The dependence on $\beta$ is important in D mesons, due to the fact that the dependences of $\Delta_A^{\gamma\, el} M_D$ and $\Delta_B^{\gamma\, el} M_D$ add up, while the contrary occurs in B mesons. $\Delta^{\gamma\, el} M_D$ increases by about 1 MeV as $\beta$ increases from 0 to 3 GeV$^{-1}$, and $\Delta^{\gamma\, el} M_B$ varies by at most $\pm 0.1$ MeV in this interval. Overall, for the chosen value of $\beta$, $1/m_Q$-corrections are small even for D mesons, and most of the uncertainty in $\Delta_B^{\gamma\, el} M_{D,B}$ resides in our lack of theoretical knowledge of the form factors.

At this point, it is appropriate to briefly address the problem of determining the total isospin splittings. $\Delta^m M_H$ is determined from the mass difference $M_{H_s} - M_{H_d}$, which after the inclusion of chiral corrections reads [13]:

$$M_{H_s} - M_{H_d} = C_H\,(m_s - m_d) - \frac{3\,g^2}{128\,F_0^2}\,(2\,M_K^2 - M_\eta^2) + \mathcal{O}(m_s^2) \qquad (20)$$

where $g$ determines the $H^* \to H\pi$ amplitude, $F_0$=93 MeV is the pion decay constant in the chiral limit, and $C_H$ is an unknown constant. Using the ratio $R \equiv (m_s - \hat{m})/(m_d - m_u) \sim 43$ [7], with $\hat{m}$ the average of the u and d quark masses, one obtains:

$$\Delta^m M_H = \frac{1}{R - 1/2}\,(M_{H_s} - M_{H_d} + \frac{3\,g^2}{128\,F_0^2}\,(2\,M_K^2 - M_\eta^2) + \mathcal{O}(m_s^2)) \qquad (21)$$

Thus, the leading chiral corrections, which are non-analytic in $m_s$, tend to increase the magnitude of $\Delta^m M_H$. Clearly, (21) makes sense only as far as $g$ is small enough; the corrections are large for $g > 0.3$ (for determinations of $g$ in the quark model and in QCD sum rules see refs. [16] and for constraints see refs. [17]). Otherwise, the unknown $\mathcal{O}(m_s^2)$ terms must also be included. We conclude that there is a large degree of theoretical uncertainty in $\Delta^m M_H$.

For $g = 0$ and the ratio $R \sim 43$ [7] one obtains $\Delta^m M_D \simeq 2.3$ MeV. The positivity of the non-analytic term and the experimental result $\Delta M_H = 4.8$ MeV [2] imply that $\Delta^\gamma M_D < 2.5$ MeV.

The main omission of the analysis presented here is that of inelastic contributions. We only kept those involving $H^*$ in the intermediate state. The inelastic contributions to $\Delta_A^\gamma M_H$ appear at leading order in the heavy quark expansion. Since $\Delta_A^{\gamma\, el} M_H$ is substantially smaller than $\Delta_B^{\gamma\, el} M_H$, one might hope that they will lead to numerically small contributions to the mass shift. As already mentioned, those considered amount to less than 0.35 MeV. As a curiosity, in the case of the p-n electromagnetic mass difference it has been noticed long ago [14] that



inelastic terms give only a modest contribution. On the other hand, for $\Delta_B^\gamma M_H$ we can prove the following statement: *the inelastic contributions to $\Delta_B^\gamma M_H$ are of order $1/m_Q$*. The proof is as follows: inelastic terms in the absorptive part of $T_\mu^\mu$ involve products of matrix elements of the form:

$$\langle H, P \mid J_S^\mu(0) \mid P'; X\rangle\langle P'; X \mid J_Q^\nu(0) \mid H, P\rangle \qquad (22)$$

where the intermediate state involves a heavy hadron with momentum $P'$ and light hadrons denoted by $X$ carrying momentum $P_X$. Let us analyze the matrix elements separately. For $\langle P'; X \mid J_Q^\nu(0) \mid H, P\rangle$ we use Bjorken's sum rule [15]:

$$1 = \frac{1}{2}(1 + v.v')\,|\xi(v.v')|^2 + \int_0^\infty d\epsilon\; \omega_{\rm inel}(\epsilon, v.v') \qquad (23)$$

where $\omega_{\rm inel}$ is given by:

$$\omega_{\rm inel}(\epsilon, v.v') = {\sum_{n;X}}' \int \frac{d^3 P'}{(2\pi)^3\, 2\, P'_0}\,(2\pi)^4 \delta^4(P - q - P' - P_X)\, \left|\langle n, P'; X \mid J_Q^\mu \mid H, P\rangle\right|^2$$
$$+\, Z - graph \qquad (24)$$

where $n$ denotes a heavy hadron and the prime on the sum implies that for $X = |\,0\rangle$ one must take $n \neq H$. Since each term on the RHS is positive, and the form factor satisfies the normalization condition $|\xi(1)| = 1$, near the point of zero recoil (23) implies the Cabibbo-Radicati type sum rule [15]:

$$\int_0^\infty d\epsilon\; \omega_{\rm inel}(\epsilon, v.v') = \left(\frac{1}{2} + 2\,|\xi|'(1)\right)(1 - v.v') \qquad (25)$$

which, supplemented with the assumption that $\xi'$ is non-singular at zero recoil and mild requirements of continuity for $\omega_{\rm inel}$, implies that

$$\left|\langle n, P'; X \mid J_Q^\nu(0) \mid H, P\rangle\right| \propto (v.v' - 1)^\eta \qquad \eta \geq \frac{1}{2} \qquad (26)$$

The behavior of $\langle H, P \mid J_S^\mu(0) \mid P'; X\rangle$ for $v \neq v'$ is determined as follows: in order to change the velocity of the heavy quark by a finite amount the large space-like momentum transfer $q$ must flow as shown in the figure. The magnitude of the momentum is $Q^2 \sim 4\,m_Q^2\,(v.v' - 1)$ in the cases where $P_X^2 \ll m_Q^2$, which are indeed the case in our problem. As illustrated in the figure, the flow of the momentum $q$ gives rise to a suppression factor of the order of $1/Q^3$ stemming from the light quark and gluon propagators. This clearly holds for all possible diagrams contributing to the matrix element. Thus:

$$|\langle H, P \mid J_5^{\mu}(0) \mid P'; X\rangle| \propto \frac{1}{m_Q^2 \, (v.v' - 1)^{3/2}} \tag{27}$$

which is valid as far as $m_Q \, (v.v' - 1)^{1/2} >> \Lambda_{QCD}$. Thus, according to (26) and (27), in this regime the product (22) becomes proportional to $m_Q^{-3} \, (v.v' - 1)^{-1}$. When $m_Q \, (v.v' - 1)^{1/2} \sim \Lambda_{QCD}$ the LHS of (27) is not suppressed, and (22) becomes proportional to $m_Q^{-1}$ since $(v.v' - 1)^{1/2} \propto m_Q^{-1}$ in (26). This latter case corresponds precisely to the inelastic term, which we included as elastic, with $H^*$ in the intermediate state (hyperfine term). We therefore conclude that $T^{inel}_{B;\mu\nu}(q, \nu)$ is suppressed with respect to the strictly elastic one by a factor $m_Q^{-3} \, (v.v' - 1)^{-1}$, which upon replacement in (5) leads to the stated $1/m_Q$ suppression for $\Delta_B^{\gamma\, inel} M_H$.

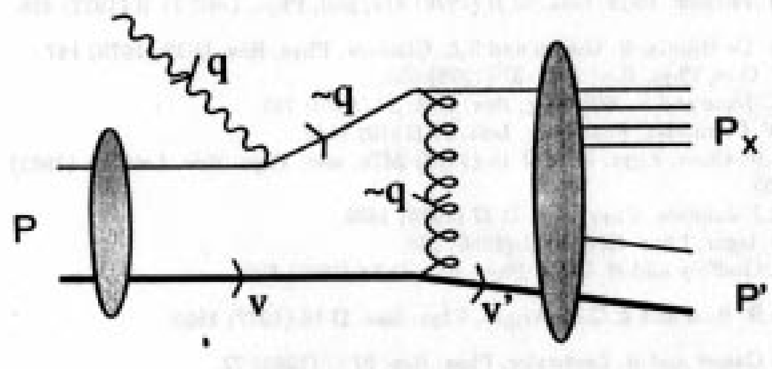


**Figure:** *The diagram shows the flow of the large space like momentum q in the process $H \to H' + X$ mediated by the light quark component of the electromagnetic current.*

Finally, a few comments on vector mesons. Their isospin mass splittings are identical to the ones of pseudoscalar mesons at leading order in $1/m_Q$. They differ at order $1/m_Q$, where a number of terms can be identified as possible contributions [9]. In particular, for the electromagnetic component there are the following terms: the hyperfine term, which breaks the heavy quark symmetry and contributes a factor (-1/3) times the hyperfine term for the pseudoscalars (relative factor as in quark models); charge form factors $F_{3,V}$ for pseudoscalar and vector mesons differ at this order (we expect this effect to be smaller than the hyperfine); $1/m_Q$ corrections to the elastic dipole form factor of the light quark components of the electromagnetic current between vector meson states which lead to corrections to $\Delta_A^{\gamma\, el} M_{H^*}$ (expected to be relatively small). The electric-quadrupole transitions give corrections of order $1/m_Q^2$, and, as mentioned before, the tadpole contributions are the same as for the pseudoscalars up to $1/m_Q^2$-corrections.



In D mesons the data is summarized by [1, 2]:

$$(a) \qquad \begin{aligned} M_{D_s^*} - M_{D_s} &= 141.5 \pm 1.9 \text{ MeV} \\ M_{D^{*0}} - M_{D^0} &= 142.12 \pm 0.07 \text{ MeV} \\ M_{D^{*+}} - M_{D^+} &= 140.64 \pm 0.1 \text{ MeV} \end{aligned} \tag{28}$$

$$(b) \qquad \begin{aligned} M_{D^{*+}} - M_{D^{*0}} &= 3.32 \pm 0.1 \text{ MeV} \\ M_{D^+} - M_{D^0} &= 4.8 \pm 0.1 \text{MeV} \end{aligned}$$

The results (a) show that the QCD hyperfine splitting is remarkably SU(3) symmetric, as emphasized in [8]: there is a change of less than 2% as the quark masses vary from $m_{u,d}$ to $m_s$. Unless this is an accident resulting from the fact that non-linear terms in $m_s$ are large, one concludes that [8] $\Delta^m M_D = \Delta^m M_{D^*}$ to a good degree of precision. Thus, $\Delta M_{D^*} - \Delta M_D \simeq \Delta^\gamma M_{D^*} - \Delta^\gamma M_D \simeq -1.48 \pm 0.15$ MeV, which suggests a hyperfine term in $\Delta^{\gamma\, el} M_D$ of about 1 MeV, a value which is not very different from the ones we obtain, which for $\beta = 3$ GeV$^{-1}$ vary between 0.5 to 0.8 MeV. Clearly, the possibility of isolating the hyperfine contribution provides a useful constraint for further theoretical understanding of $1/m_Q$-corections.

## III. CONCLUSIONS

Electromagnetic contributions to mass splittings in D and B mesons are conveniently analyzed in the framework of the large mass expansion. Leading order terms are determined, up to inelastic corrections to $\Delta_A^\gamma M_H$, by elastic contributions which are given in terms of the elastic form factors associated with the light quark components of the electromagnetic current. A precise theoretical determination of the latter from QCD is thus very important. $1/m_Q$ corrections suffer from uncertainties of different sorts, and a precise treatment is difficult. We have shown that the $1/m_Q$-corrections to elastic contributions can be estimated to a good extent, and, within the used approximations, they turn out to be relatively small dur to the fact that kinematic and hyperfine corrections conspire to cancel each other. For $\beta = 3$ GeV$^{-1}$ the results are remarkably close to those obtained at leading order. To fully pin down the corrections to the elastic term knowledge of the $m_Q$ dependence of $F_{Y,3}$ and $g_{Y,3}$ is required, and moreover, the difficult to estimate tadpole terms should be included. Much more work is needed in order to roughly estimate the importance of the disregarded inelastic terms.

Our main aim was to conceptually clarify the analysis of electromagnetic splittings, and therefore, we refrained from making definite numerical claims, which are sensitive to the model chosen for the elastic form factors and other assumptions. Our estimates for the relative sizes of the $1/m_Q$ corrections to $\Delta^{\gamma el} M_H$ should however be taken seriously.

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