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The Heavy Mass Expansion In $\Lambda_b \to \Lambda_c$ Decays

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We point out that in the decays of the Λ_b to Λ_c one can find predictions which – in the framework of the $1/m_c$ expansion – do not receive corrections in any order of $1/m_c$. We discuss QCD corrections to these predictions and examine some of the consequences for nonleptonic decays.

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I. INTRODUCTION

Recently, heavy hadrons have come under rather heavy scrutiny [1–6]. This has been largely due to the realization that, in the limit when the mass of one of the quarks in a hadron becomes infinite, symmetries above and beyond those usually associated with quantum chromodynamics (QCD) result. These new symmetries have been used, for example, to obtain relations among form factors for electroweak decays. When applied to the specific example of $B \to D$ transitions, the framework of the heavy quark effective theory (HQET) that has been formulated [1–6] allows an extraction of $|V_{cb}|$ from data in a model independent way [7].

In practice, neither the charm nor bottom quark are infinitely massive. This means that one should perform expansions, with the inverse of the quark mass serving as expansion parameter. Such an expansion introduces new operators into the effective theory, and this effective theory must be matched onto the full theory (QCD) at every step in the expansion [8–11]. Matrix elements of the new operators will then be described by form factors, which need not be related to the form factors that describe the leading order matrix element.

One would thus expect that any leading-order predictions should be modified as one proceeds to higher order in the $1/m_Q$ expansion. If the effective theory makes sense, these higher order contributions should become progressively smaller, so that the expansion converges. The $1/m_b$ corrections for a $b \to c$ transition fall into two classes, characterized by the two parameters $\Lambda_{QCD}/m_b \leq 10\%$ and $(\alpha_s(m_b)/\pi)(m_c/m_b) \sim 7\%$ which are small enough to justify an expansion. The corrections proportional to $1/m_c$ are given in terms of the parameter Λ_{QCD}/m_c which lies in the range 20 - 30 %. This is still small enough for an expansion, but it may be necessary to include higher orders in this parameter.

The aim of this article is to point out that there is a class of predictions that are protected from any $1/m_c$ corrections (in $b \to c$ transitions), to any order in the expansion. This means that for such predictions, corrections are of order $1/m_b$, which are considered safer than the corresponding $1/m_c$ corrections. The 'protected' predictions we discuss occur for transitions between baryons, so that they will not be immediately testable.

These predictions are modified by QCD radiative corrections of the order $\alpha_s(m_b)$, and by recoil corrections for the b quark. The latter fall into two classes. One class contains corrections of order $(\Lambda_{QCD}/m_b)^n$ and $(\Lambda_{QCD}/m_b)^i(\Lambda_{QCD}/m_c)^j$, which are small compared to radiative corrections and thus may be neglected safely. The second class are corrections of order $(\alpha_s/\pi)(m_c/m_b)$ which may be of the order of 10%. However, like the QCD radiative corrections, these may be systematically calculated.

In the next section we discuss heavy-to-light transitions, and show what some of their implications are for the corresponding heavy-to-heavy transitions. In section III we examine further consequences for the semileptonic decays of the Λ_b, by working to order $1/m_c$. In section IV, we present a proof of the results of section III, valid to all orders in the $1/m_c$ expansion. Section V deals with the corrections to the protected predictions. In section VI, we examine some consequences for two-body nonleptonic decays, and in section VII we present our conclusions.

II. HEAVY TO HEAVY VERSUS HEAVY TO LIGHT TRANSITIONS

Let us focus on the transition of a Λ_b into a Λ_c mediated by a current of the form $\bar{c}\Gamma b$, where Γ is an arbitrary combination of Dirac matrices. At mass scales $\mu > m_b$ both quarks are treated as light and the number of independent form factors is given mainly by Lorentz covariance considerations. At the scale $\mu = m_b$ the b quark is replaced – to leading order in $1/m_b$ – by a static heavy quark $h_v^{(b)}$ and the matrix element we have to consider at scales $m_c < \mu < m_b$ is of the heavy to light type, for which heavy quark symmetries already reduce the number of form factors. This is illustrated in figure 1. It has been shown in [12] that this heavy to light transition, to leading order in the $1/m_b$ expansion, is given in terms of two form factors,

$$\left\langle \Lambda_c(p') \left| \bar{c}\Gamma h_v^{(b)} \right| \Lambda_b(v) \right\rangle = \bar{u}(p') \left[F_1(v \cdot p') + \not{v} F_2(v \cdot p') \right] \Gamma u(v). \tag{1}$$

Eq. (1) holds up to corrections of the order Λ_{QCD}/m_b and $(\alpha_s(m_b)/\pi)(m_c/m_b)$.

At the scale $\mu = m_c$ the c quark is also integrated out and replaced by a static quark $h_{v'}^{(c)}$. Matching the two effective theories at the scale $\mu = m_c$ one finds, to leading order in $1/m_b$ and $1/m_c$ [12–14],

$$\left\langle \Lambda_c(v') \left| \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} \right| \Lambda_b(v) \right\rangle = \eta(v \cdot v') \bar{u}(v') \Gamma u(v), \tag{2}$$

However, as is clear from the form of (1) the matching at the scale $\mu = m_c$, *to any arbitrary order in the $1/m_c$ expansion* will at most introduce two form factors. Note that this statement is independent of the particular form of Γ. Furthermore, it is not affected by QCD radiative corrections of the order $\alpha_s(m_c)$ which arise from the matching at the scale m_c.

In the following we exploit this statement to obtain relations which are 'protected' against $1/m_c$ corrections, i.e., to find relations which are valid to leading order in Λ_{QCD}/m_b and $(\alpha_s(m_b)/\pi)(m_c/m_b)$ and to all orders in Λ_{QCD}/m_c.

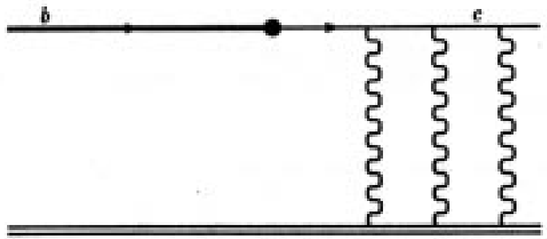

Figure 1: $\Lambda_b \to \Lambda_c$ at the scale $m_c \le \mu \le m_b$. The thick line represents the effective heavy quark field $h_v^{(b)}$.

III. DEMONSTRATION TO ORDER $1/M_C$

To be explicit, let us examine the matrix element of the left handed current $\Gamma = \gamma_\mu(1-\gamma_5)$. The leading order (in the $1/m_b$ expansion) hadronic matrix element for the $\Lambda_b \to \Lambda_c \ell \nu$ decay, treating the charm quark as light, is given in eqn. (1). This form is to be compared with the most general form of the matrix element that can be written, based on Lorentz invariance, which is

$$\begin{aligned}\langle \Lambda_c(p')\,|\bar{c}\gamma_\mu\,(1-\gamma_5)\,b|\,\Lambda_b(p)\rangle = \bar{u}(p')\,[(f_1\gamma_\mu + i f_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\\ - (g_1\gamma_\mu + i g_2\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\,\gamma_5]\,,\end{aligned} \tag{3}$$

with $q = p - p'$.

For later convenience, we recast this into a slightly different form as

$$\begin{aligned}&\langle \Lambda_c(p')\,|\bar{c}\gamma_\mu\,(1-\gamma_5)\,b|\,\Lambda_b(p)\rangle\\ &= \bar{u}(p')\left[\gamma_\mu\left[f_1 - f_2\left(m_{\Lambda_b} + m_{\Lambda_c}\right)\right] + p_\mu\left(f_2+f_3\right) - p'_\mu\left(f_3 - f_2\right)\right.\\ &\left. - \left\{\gamma_\mu\left[g_1 + g_2\left(m_{\Lambda_b} - m_{\Lambda_c}\right)\right] + p_\mu\left(g_2+g_3\right) - p'_\mu\left(g_3-g_2\right)\right\}\gamma_5\right] u(p)\\ &\equiv \bar{u}(p')\left[\mathcal{F}_1\gamma_\mu + \mathcal{F}_2 v_\mu + \mathcal{F}_3 v'_\mu - \left(\mathcal{G}_1\gamma_\mu + \mathcal{G}_2 v_\mu + \mathcal{G}_3 v'_\mu\right)\gamma_5\right] u(p).\end{aligned} \tag{4}$$

In terms of the $\mathcal{F}_i$ and $\mathcal{G}_i$, the f_i and g_i are

$$f_1 = \mathcal{F}_1 + \frac{1}{2}\left(m_{\Lambda_b} + m_{\Lambda_c}\right)\left(\frac{\mathcal{F}_2}{m_{\Lambda_b}} + \frac{\mathcal{F}_3}{m_{\Lambda_c}}\right),$$
$$f_2 = \frac{1}{2}\left(\frac{\mathcal{F}_2}{m_{\Lambda_b}} + \frac{\mathcal{F}_3}{m_{\Lambda_c}}\right),\quad f_3 = \frac{1}{2}\left(\frac{\mathcal{F}_2}{m_{\Lambda_b}} - \frac{\mathcal{F}_3}{m_{\Lambda_c}}\right),$$

$$g_1 = \mathcal{G}_1 - \frac{1}{2}\left(m_{\Lambda_b} - m_{\Lambda_c}\right)\left(\frac{\mathcal{G}_2}{m_{\Lambda_b}} + \frac{\mathcal{G}_3}{m_{\Lambda_c}}\right),$$
$$g_2 = \frac{1}{2}\left(\frac{\mathcal{G}_2}{m_{\Lambda_b}} + \frac{\mathcal{G}_3}{m_{\Lambda_c}}\right),\quad g_3 = \frac{1}{2}\left(\frac{\mathcal{G}_2}{m_{\Lambda_b}} - \frac{\mathcal{G}_3}{m_{\Lambda_c}}\right). \tag{5}$$

In eqn. (3), one quantity of experimental interest is G_A/G_V, defined as

$$\frac{G_A}{G_V} \equiv \frac{g_1(q^2=0)}{f_1(q^2=0)}. \tag{6}$$

The six form factors of eqn. (3) may be written in terms of the two form factors of eqn. (1) as

$$\begin{aligned}f_1 = g_1 &= F_1 + \frac{m_{\Lambda_c}}{m_{\Lambda_b}}F_2,\\ f_2 = f_3 = g_2 = g_3 &= \frac{1}{m_{\Lambda_b}}F_2.\end{aligned} \tag{7}$$

Note that the factors $m_{\Lambda_c}/m_{\Lambda_b}$ and $1/m_{\Lambda_b}$ are of purely kinematic origin and are not related to corrections of order $1/m_b$. Thus, to leading order in the $1/m_b$ expansion, not only is $G_A = G_V$, but in fact $g_1 = f_1$ over the entire allowed kinematic region. This prediction of HQET is obtained at scales $m_c < \mu < m_b$ where the charm quark is treated as a light quark. At $\mu = m_c$ this quark is now integrated out and a $1/m_c$ expansion is performed. Matching the two effective theories at $\mu = m_c$ must leave these predictions inviolate, order by order, to all orders in the $1/m_c$ expansion. Furthermore, eqn. (7) tells us that the relations $f_2 = g_2 = f_3 = g_3$ must also be protected from corrections arising in the $1/m_c$ expansion. At leading order, the form factors of eqn. (2) lead to the relations $f_1 = g_1 = \eta$, $f_2 = g_2 = f_3 = g_3 = 0$, so the relationships are satisfied.

In the work of Georgi, Grinstein and Wise [9], it has been shown that to order $1/m_c$, no additional form factors arise in the description of this matrix element, and that the corrections may be expressed solely in terms of a dimensionful quantity $\bar{\Lambda} = m_{\Lambda_c} - m_c$. The form factors f_i and g_i are given as [9]

$$\begin{aligned}f_1 = g_1 &= \eta\left[1 + x v\cdot v' - x\frac{m_{\Lambda_c}}{m_{\Lambda_b}}\right],\\ f_2 = f_3 = g_2 = g_3 &= \frac{x}{m_{\Lambda_b}}\eta,\end{aligned} \tag{8}$$

where we have defined

$$x = \frac{\bar{\Lambda}}{2m_c(1+v\cdot v')}. \tag{9}$$

and indeed the relationships of eqn. (7) are preserved to this order. Let us now prove that these relationships hold to all orders in the $1/m_c$ expansion.

IV. PROOF TO ALL ORDERS

To see how the relationships of eqn. (7) are preserved to all orders in $1/m_c$, let us first consider a local term that may arise in the $1/m_c$ expansion, due to some operator $\mathcal{O}_1$. Such a term can be written

$$\left\langle \Lambda_c(v') \left| \bar{h}_{v'}^{(c)} \mathcal{O}_1 \Gamma h_v^{(b)} \right| \Lambda_b(v) \right\rangle = \bar{u}(v')\mathcal{M}\Gamma u(v). \tag{10}$$

The matrix $\mathcal{M}$ contains all the new form factors that may arise due to the operator $\mathcal{O}_1$. The key point to note here is that, since $\mathcal{O}_1$ arises in the $1/m_c$ expansion, it *must* appear on the *left* of the matrix Γ. The 'form factor matrix' $\mathcal{M}$ will contain terms that reflect the Lorentz structure of the operator $\mathcal{O}_1$. In particular, it may contain terms proportional to $\not{v}'$. However, since $\bar{u}(v')\not{v}' = \bar{u}(v')$, such terms may be simplified without commutation through the Γ. Thus, there will be no v' terms that reflect the Lorentz structure of the Γ: there will be no v' terms that have the Lorentz indices associated with Γ.

For the non-local operators, a generic term may be written

$$\left\langle \Lambda_c(v') \left| T \int d^4x d^4y \ldots \left(\bar{h}_{v'}^{(c)} \mathcal{O}_2 h_{v'}^{(c)}\right)(x) \left(\bar{h}_{v'}^{(c)} \mathcal{O}_3 h_{v'}^{(c)}\right)(y) \ldots \right.\right.$$
$$\left.\left. \times \left(\bar{h}_{v'}^{(c)} \Gamma h_v^{(b)}\right)(0) \right| \Lambda_b(v) \right\rangle = \bar{u}(v')\mathcal{M}'\Gamma u(v), \tag{11}$$

where

$$\mathcal{M}' = \theta_2 \left(\frac{1+\not{v}'}{2}\right) \theta_3 \left(\frac{1+\not{v}'}{2}\right) \ldots, \tag{12}$$

and θ_2 has the same Dirac structure as $\mathcal{O}_2$, etc. As before, the key point is that all of the new operator insertions only have meaning when they appear on the left of Γ. Thus, as before, no v' terms carrying any of the indices of Γ will appear. In the case of mixed local/non-local terms, such as the one shown in figure 2, one can see that the argument proceeds in exactly the same manner.

These results mean that in the case of $\Gamma = \gamma_\mu(1-\gamma_5)$, the matrix element will contain no terms proportional to v'_μ, so that $\mathcal{F}_3$ and $\mathcal{G}_3$ both vanish. In terms of the form factors of eqn. (4), this means that $f_2 = f_3$ and $g_2 = g_3$, to all orders in the $1/m_c$ expansion. To see how the other relationships arise, it is sufficient to consider a generic matrix element that may arise at order $1/m_c^n$, which may be written as

$$\mathcal{M} = \bar{u}(v') \left(\alpha^{(n)} + \beta^{(n)}\not{v}\right) \gamma_\mu (1-\gamma_5) u(v). \tag{13}$$

In writing this form, we are making use of the spin symmetry which still exists for the heavy b quark to relate the vector and axial vector form factors. In addition, the form written in eqn. (13) is valid for the matrix element of *any* operator that may arise in the expansion.

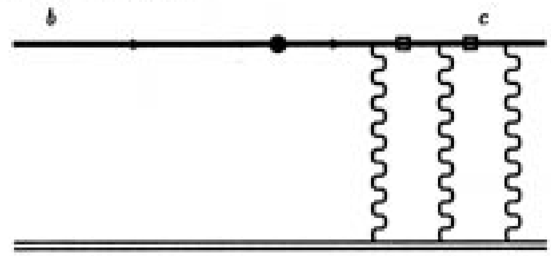

Figure 2: Diagrammatic representation of a typical term that may arise in the $1/m_c$ expansion.

The contribution of such a matrix element to the form factors of eqn. (3) is

$$f_1^{(n)} = g_1^{(n)} = \alpha^{(n)} + \frac{m_{\Lambda_c}}{m_{\Lambda_b}}\beta^{(n)}, \quad f_2^{(n)} = g_2^{(n)} = f_3^{(n)} = g_3^{(n)} = \frac{\beta^{(n)}}{m_{\Lambda_b}}. \tag{14}$$

Since all the contributions to these form factors will satisfy the above relations at any order in the $1/m_c$ expansion, and the overall matrix element is simply obtained as the sum of individual terms, the form factor relationships of eqn. (7) are satisfied to all orders in the $1/m_c$ expansion. In addition, it is clear that to all orders in this expansion, a total of two unknown, non-perturbative form factors are needed to describe the decay $\Lambda_b \to \Lambda_c$. However, each form factor will receive contributions from many different matrix elements, and it may be convenient to treat some of these as independent entities. Nevertheless, the relationships of eqn. (7) will still hold.

Somewhat surprisingly, much of what we have said above can be applied to the $1/m_b$ expansion as well. A typical matrix element in the $1/m_b$ expansion will have the form

$$\mathcal{M} = \bar{u}(v')\gamma_\mu(1-\gamma_5)\left(x^{(n)} + y^{(n)}\not{v}'\right) u(v). \tag{15}$$

Such a term is represented by figure 3. Here, we have used the spin symmetry of the charm quark to relate the vector and axial vector form factors as before. The contribution to the form factors of eqn. (3) from such a term is

$$f_1^{(n)} = g_1^{(n)} = x^{(n)} + \frac{m_{\Lambda_b}}{m_{\Lambda_c}} y^{(n)}, \quad f_2^{(n)} = -g_2^{(n)} = -f_3^{(n)} = g_3^{(n)} = \frac{y^{(n)}}{m_{\Lambda_c}}. \tag{16}$$

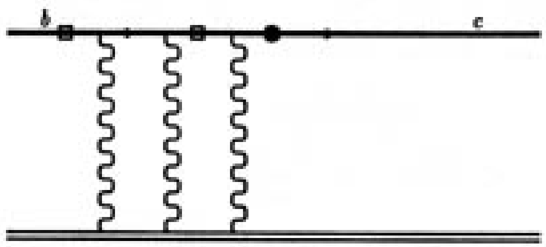

Figure 3: Diagrammatic representation of a typical term that may arise in the $1/m_b$ expansion.

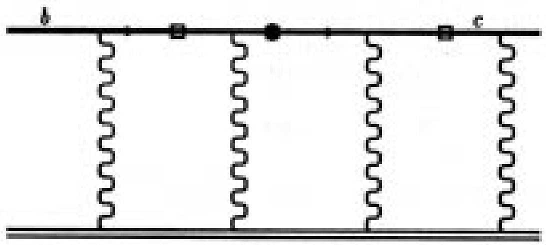

Figure 4: Diagrammatic representation of a typical crossed term that may arise in the $1/m_c$ and $1/m_b$ expansions.

We may again invoke the argument of additivity to claim that $f_1 = g_1$ to all orders in the $1/m_b$ expansion as well. The relationships among the other form factors remain as simple as before, but have been modified. Combining the results for the $1/m_b$ and $1/m_c$ expansions (with no crossed terms of the form $(1/m_b)^i(1/m_c)^j$, $i, j > 0$, (One may think of this as summing all diagrams of the form of figures 2 and 3.)), leads us to the conclusion that

$$f_1 = g_1 = \alpha + x + \beta \frac{m_{\Lambda_c}}{m_{\Lambda_b}} + y \frac{m_{\Lambda_b}}{m_{\Lambda_c}},$$

$$f_2 = g_3 = \frac{\beta}{m_{\Lambda_b}} + \frac{y}{m_{\Lambda_c}}, \quad f_3 = g_2 = \frac{\beta}{m_{\Lambda_b}} - \frac{y}{m_{\Lambda_c}}. \tag{17}$$

These relationships are explicitly broken by the so-called crossed terms such as the one represented in figure 4, which are of order $(1/m_b)^i(1/m_c)^j$.

V. CORRECTIONS TO THE $(1/M_C)^N$ PROTECTED PREDICTIONS

QCD radiative corrections to the 'protected' predictions arise from one loop matching at the scale $\mu = m_b$. These matching contributions change the leading order relations between form factors since, at the one loop level, an additional operator appears. It has been shown in [4] that the one loop matching at $\mu = m_b$ requires replacing the arbitrary Γ matrix in (1) with

$$\Gamma \rightarrow \Gamma - \frac{\alpha_s(m_b)}{6\pi} \gamma_\mu \not{v} \Gamma \not{v} \gamma^\mu. \tag{18}$$

Using this expression, we may calculate the corrections to the vector current. We obtain

$$\begin{aligned}
&\langle \Lambda_c(p') \, | V_\mu | \, \Lambda_b(v) \rangle \\
&= \bar{u}(p') \Big[(F_1(v \cdot p') - F_2(v \cdot p')) \left(1 - \frac{\alpha_s(m_b)}{3\pi}\right) \gamma_\mu \\
&+ \left(F_2(v \cdot p') + \frac{\alpha_s(m_b)}{3\pi} F_1(v \cdot p') \right) v_\mu \Big] u(v).
\end{aligned} \tag{19}$$

The resulting corrections to the standard representations of the form factors are

$$\begin{aligned}
f_1 &= (F_1(v \cdot p') - F_2(v \cdot p')) \left(1 - \frac{\alpha_s(m_b)}{3\pi}\right) \\
&+ \left(1 + \frac{m_{\Lambda_c}}{m_{\Lambda_b}}\right) \left(F_2(v \cdot p') + \frac{\alpha_s(m_b)}{3\pi} F_1(v \cdot p') \right), \\
f_2 &= \frac{1}{m_{\Lambda_b}} \left(F_2(v \cdot p') + \frac{\alpha_s(m_b)}{3\pi} F_1(v \cdot p') \right) = f_3.
\end{aligned} \tag{20}$$

Similarly, the axial vector current form factors become

$$\begin{aligned}
g_1 &= (F_1(v \cdot p') + F_2(v \cdot p')) \left(1 - \frac{\alpha_s(m_b)}{3\pi}\right) \\
&- \left(1 - \frac{m_{\Lambda_c}}{m_{\Lambda_b}}\right) \left(F_2(v \cdot p') - \frac{\alpha_s(m_b)}{3\pi} F_1(v \cdot p') \right),
\end{aligned}$$

$$g_2 = \frac{1}{m_{\Lambda_b}} \left(F_2(v \cdot p') - \frac{\alpha_s(m_b)}{3\pi} F_1(v \cdot p') \right) = g_3. \quad (21)$$

The corrections to G_A/G_V now depend on the ratio of the form factors at zero momentum transfer,

$$r = \left. \frac{F_2}{F_1} \right|_{v \cdot p' = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2}{2m_{\Lambda_b}}} . \quad (22)$$

This leads to

$$\frac{G_A}{G_V} = 1 - 2\frac{\alpha_s(m_b)}{3\pi} \frac{r m_{\Lambda_b} + m_{\Lambda_c}}{m_{\Lambda_b} + r m_{\Lambda_c}} + \mathcal{O}(\alpha_s^2(m_b)). \quad (23)$$

Eq.(23) depends on the ratio r which is an unknown number. However, if we assume that r is positive we may obtain limits on the size of the corrections by using

$$\frac{m_{\Lambda_c}}{m_{\Lambda_b}} \le \frac{r m_{\Lambda_b} + m_{\Lambda_c}}{m_{\Lambda_b} + r m_{\Lambda_c}} \le \frac{m_{\Lambda_b}}{m_{\Lambda_c}}, \quad (24)$$

so that $0.85 \le G_A/G_V \le 0.98$ for positive r. If the heavy quark expansion is valid, one expects r to be small, so that G_A/G_V should lie near the upper end of this range. For negative r the QCD corrections vanish for $r = -m_{\Lambda_c}/m_{\Lambda_b}$ and may become very large if $r \sim -m_{\Lambda_b}/m_{\Lambda_c}$. However, in this limit, the leading order contributions to G_A and G_V vanish, and the expansion is rendered questionable at best.

As is obvious from the form of eq.(1) there are no QCD radiative corrections at the scale $\alpha_s(m_c)$, at any order in the $1/m_c$ expansion. An explicit calculation of these corrections has been performed in [15] for the form factors. However, as has been pointed out in [15], the relations between the form factors remain intact such that G_A/G_V is still unity.

It has been shown in [9, 15] that the leading order corrections in Λ_{QCD}/m_b vanish for G_A/G_V. Our arguments above suggest that such corrections vanish to all orders in the $1/m_b$ expansion. Recent results of Falk and Neubert [16] confirm this claim at order $1/m_b^2$, and show that the leading correction appears at order $\Lambda_{QCD}^2/(m_c m_b)$, which is small. Note, however, that in their treatment, the kinematic mass factors of eqn. (5) are also expanded in powers of the quark mass, so that $1/m_c^2$ and $1/m_b^2$ terms do appear. We emphasize that no such terms appear as a result of new operators in the heavy quark expansion.

QCD radiative corrections from the running between m_b and m_c will induce contributions of order $(\alpha_s/\pi)(m_c/m_b)\ln(m_c/m_b)$ which are calculable in a systematic way. The lowest order calculation has been performed in [4]; using the results of this paper the corrections of this type lead to a change in G_A/G_V of -2%.

One may also integrate out the b and the c quark simultaneously at some intermediate scale $m_b \ge \mu \ge m_c$ [5]. This method has the advantage that one may include all powers of (m_c/m_b) at the price of a scale ambiguity in the strong coupling constant. Using the results of [17] one may calculate the form factor ratio g_1/f_1 for $q^2 = 0$ which corresponds to $v \cdot v' = (x^2+1)/(2x)$ where x denotes the mass ratio $x = m_c/m_b$. Inserting this we obtain

$$\frac{G_A}{G_V} = 1 - \frac{4\alpha_s(\mu)}{3\pi} \frac{2x}{x^2-1} \ln x - \frac{2\alpha_s(\mu)}{3\pi} \left(3 + \frac{4(1+x^2)}{1-x^2} \ln x \right). \quad (25)$$

The first order α_s term originates from a mass independent term (i.e. a term of order $\alpha_s(\mu)$) that arises due to the different one loop matching of the vector and the axial vector currents. The second term originates from corrections of order $(\alpha_s(\mu)/\pi)(m_c/m_b)$ and $(\alpha_s(\mu)/\pi)(m_c/m_b)\ln(m_c/m_b)$. Both terms yield sizable corrections but with opposite signs. For $\mu = m_c$ ($\alpha_s(m_c) = 0.40$), the first term gives a correction of 14% to G_A/G_V, while the second one gives -20%. For $\mu = m_b$ ($\alpha_s(m_b) = 0.25$) we have 9% from the first and -12% from the second term. The corrections from this approach therefore give

$$\frac{G_A}{G_V} = 1 + (3-6)\%, \quad (26)$$

where the actual sizes of the corrections depend on the choice of the scale μ.

VI. IMPLICATIONS FOR NONLEPTONIC DECAYS

As an application of these considerations, let us examine their implications for the two-body nonleptonic decays $\Lambda_b \to \Lambda_c \pi$ and $\Lambda_b \to \Lambda_c D_s$, where we are using the lightest and one of the heaviest pseudoscalars allowed. We may write the decay amplitude as

$$\begin{aligned} M_{\Lambda_b \to \Lambda_c P} &= c\bar{u}(p') \left[A + B\gamma_5 \right] u(p) \\ &= c'\bar{u}(p') \left[f_1 \not{p}_P + f_3 m_P^2 - g_1 \not{p}_P \gamma_5 - g_3 p_P^2 \gamma_5 \right] u(p), \end{aligned} \quad (27)$$

where p_P is the momentum of the pseudoscalar meson, m_P is its mass, and c and c' contain all other non-essential factors like G_F and V_{bc}. The second equality assumes that the amplitude can be factorized in the usual way. From the above discussion, if we consider all orders in the $1/m_c$ expansion, this may be rewritten as

$$M_{\Lambda_b \to \Lambda_c P} = c\bar{u}(p')\left(f_1 \not{p}_P + f_3 m_P^2\right)(1-\gamma_5)\, u(p). \tag{28}$$

If we ignore the corrections we have discussed earlier in this article, and treat the Λ_c as a light baryon, then the relationships of eqn. (7) apply, and we obtain

$$\begin{aligned} A &= (m_{\Lambda_c} - m_{\Lambda_b})\left(F_1 + \frac{m_{\Lambda_c}}{m_{\Lambda_b}} F_2\right) - \frac{m_P^2}{m_{\Lambda_b}} F_2, \\ B &= (m_{\Lambda_c} + m_{\Lambda_b})\left(F_1 + \frac{m_{\Lambda_c}}{m_{\Lambda_b}} F_2\right) - \frac{m_P^2}{m_{\Lambda_b}} F_2. \end{aligned} \tag{29}$$

When the meson is the pion, we may safely ignore its mass, and the above amplitude becomes even simpler. One immediate consequence of this simplicity is that one can predict the values of the spin parameters to be $\alpha = -1$, $\beta = \gamma = 0$, to all orders in $1/m_c$. Note that $\beta = 0$ because the form factors are real, and we are ignoring final state interactions. Including the mass of the pion changes these predictions, but only at the 0.1% level. This means that, should these parameters be measured for the decay $\Lambda_b \to \Lambda_c \pi$, any deviation from these predictions must be due to some source other than $1/m_c$ terms. Such sources may include terms in the $1/m_b$ expansion, or possible corrections to the factorization approximation. From the discussion of the previous sections, we expect the largest corrections of the former type to be of order $\Lambda_{QCD}^2/(m_c m_b)$, which is expected to be small. At the moment, we are unable to estimate the sizes of any possible factorization corrections.

As noted previously for the $\Lambda_c \to \Lambda\pi$ decay [18], the prediction of $\alpha = -1$ holds as long as the ratio of form factors, $r = F_2/F_1$, is such that we are far away from the possible zeroes in the expressions for A and B above. These zeroes occur near $r = -m_b/m_c$, and from the discussion above, the leading order contributions to f_1, g_1, A and B all vanish near this value of r. It would be surprising if this value were adopted by Nature since, if the charm quark is heavy, $r = 0$ at zeroth order in $1/m_c$. If the heavy quark expansion holds, one expects that F_2 should receive 'small' contributions, and that the ratio r should still be small. Thus, one intuitively expects that for the $\Lambda_b \to \Lambda_c \pi$ transitions, we should always be far away from the 'critical' value of r. For $\Lambda_b \to \Lambda_c \pi$, one therefore expects that, modulo assumptions about factorization, the prediction $\alpha = -1$ is valid to all orders in $1/m_c$.

The situation appears to become less straightforward when we consider the D_s. Clearly, ignoring its mass should lead to large errors in any predictions that can be made. However, if we now assume that $r = |F_2/F_1| \leq 1$, an assumption

that is based on the arguments of the previous paragraph, we obtain the results shown in figure 5. This figure shows that despite the large mass of the D_s meson, one can predict that $\alpha < -0.9$. More importantly, these results again show that large deviations from the leading order predictions will not reflect problems that arise in the $1/m_c$ expansion, but rather hint at the importance of other effects, such as $1/m_b$ terms, or corrections to factorization.

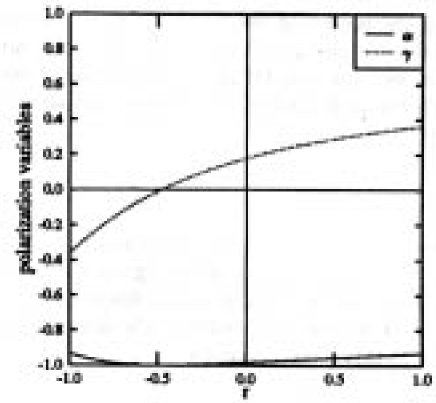

Figure 5: α and γ for the process $\Lambda_b \to \Lambda_c D_s$, as a function of the ratio $r = F_2/F_1$.

VII. CONCLUSIONS

In the previous sections, we have shown how we can use the the description of matrix elements for heavy to light transitions, in HQET, to make statements for the corresponding heavy to heavy transitions. In particular, we have shown that some leading order results are in fact valid to all orders in the $1/m_c$ expansion, where m_c here denotes the mass of the lighter of the two heavy quarks involved in the transition. We find that some of the relationships are preserved to all orders in the $1/m_b$ expansion as well.

All of the predictions we have analyzed have been for the heavy Λ type baryons, as these are the simplest objects in HQET. In particular, one of the crucial features is that each heavy Λ is alone its spin-symmetry multiplet. Clearly, one may attempt to arrive at similar predictions for other hadronic states. For instance, such predictions would be of prime interest for the corresponding decays of heavy mesons. The complication that arises is that in the heavy quark limit, the spin symmetry multiplets of the mesons, for instance, contain mesons of differing spin, which must be treated differently when the 'heavy' quark is no longer heavy.

Despite the complication with the spin-symmetry multiplets, some useful relations may still be obtained if one considers other baryon decays. In the $1/m_c$ expansion, the discussion is very much the same as in section III, so that no new v' terms carrying the indices of the Γ can arise. For mesons, on the other hand, it is not clear whether any protected predictions can arise. Indeed, the trace formalism that is used for calculating matrix elements between mesons suggests that it is unlikely for such predictions to exist. However, the question certainly warrants further study.

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