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# ONCE MORE ON THE APPLICABILITY OF PERTURBATIVE QCD TO ELASTIC FORM FACTORS *

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**Abstract**

The long-standing problem of the applicability of perturbative QCD to hadronic elastic form factors is discussed. The basic ingredients both of the asymptotic large-$Q^2$ treatment and of the QCD sum rule approach are analyzed. The main conclusion is that for accessible energies and momentum transfers the soft (nonperturbative) contributions dominate over those due to the hard quark rescattering subprocesses.

For many years, elastic form factors have been a subject of very intensive studies – both experimental and theoretical. The reason is that they contain an important information about the internal structure of the "elementary" particles. In the non-relativistic quantum mechanics, *e.g.*, the form factor is directly given by the Fourier transform of the charge distribution inside a system. In the (light-cone) quantum field theory, the form factor of a two-body bound state is given by a convolution

$$F(q^2) \sim \int \psi_P(x,k_\perp)\, \psi_P(x,k_\perp + xq)\, d^2k_\perp dx \tag{1}$$

involving initial and final state wave functions. If $q$, the momentum transfer to the system, is large enough, studying the form factor one can extract information about the high-$k_\perp$ behavior of the bound state wave function. In the pre-QCD parton model [1], it was assumed that the wave functions $\psi_P(x,k_\perp)$ are strongly (*e.g.*, exponentially) suppressed in the high-$k_\perp$ region. In that case the form factor integral

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(1) is dominated by the $x \sim 1/q$ region, and the result is determined by the small-$x$ behaviour of the hadronic wave function. If $\psi_P(x, k_\perp)$ has a power-law behaviour $\psi(x) \sim x^\alpha$ for small $x$, the form factor is also power behaved: $F(q^2) \sim q^{-\alpha - const}$ for large $q$. Physically, this means that the large-$q$ behaviour of $F(q^2)$ is determined by the configuration when the active (quark) parton carries a bulk part of the hadron momentum while the spectators take a wee $\sim 1/q$ fraction of it. This is the essence of the mechanism formulated by Feynman in his 1972 book [1].

However, if $\psi_P(x, k_\perp)$ is only power suppressed for high transverse momenta $\psi_P(x, k_\perp) \sim \phi(x) k_\perp^{-\beta}$, then a power-law large-$q$ behaviour $F(q^2) \sim q^{-\beta - 2}$ is given by the $k_\perp$ integration. In fact, such a power-law term is generated in any quantum field theory model due to rescattering processes between constituents. This situation also has a simple physical interpretation: the initial state is composed of partons collinear to the momentum $P$ and the hard rescatterings just compensate the effect of the external momentum transfer $q$ and convert the system into a similar final state with all partons carrying finite fractions of the final momentum $P'$. This picture was used by Brodsky and Farrar [2] in 1973 who noticed that, in a theory with a dimensionless coupling constant, the resulting form factor behaviour satisfies the quark counting rule [3] : $F_n(Q^2) \sim (1/Q^2)^{n-1}$, with $n$ being the number of quarks inside a hadron. The quark counting rules are in good agreement with experimental data on hadronic form factors and other exclusive processes, and it was only natural to assert that the asymptotic behaviour of the hadronic form factors is basically understood. The only problem was to justify the hard rescattering picture within a reliable theory of hadrons.

Such a theory - QCD - was created in the same 1973 year, and at the end of the 70's it was finally demonstrated that the high-$k_\perp$ tail of the hadronic wave function (to be referred as $\psi_P^{hard}(x, k_\perp)$) can be calculated within the perturbative QCD approach in terms of a perturbative short-distance kernel $V$ and the nonperturbative "soft" wave function $\psi_P^{soft}(x, k_\perp)$:

$$\psi_P^{hard} \sim V \otimes \psi_P^{soft}. \tag{2}$$

The kernel just includes the hard gluon rescatterings. Displaying $\psi$ as a sum of the soft and hard components, one arrives at the QCD factorization expansion [4]. It states essentially that a hadronic form factor in QCD can be represented by a sum of terms of increasing complexity. The first term – a purely *soft* contribution – contains no short-distance (SD) subprocesses. Its large-$Q^2$ behaviour is determined by the Feynman mechanism, and in QCD it vanishes like $1/Q^4$ (or faster) for the pion, like $1/Q^6$ for the nucleons, *etc*. The *hard* rescattering terms are also present in this expansion and contribute $O(\alpha_s/Q^2)$ to the pion form factor and $O((\alpha_s/Q^2)^2)$ to the nucleon form factors. In addition, there are also corrections to the hard term: higher order corrections containing extra $\alpha_s$ factors and higher twist $O((\alpha_s/Q^2)(M^2/Q^2)^n)$ corrections. Thus, perturbative QCD supports the statement that the asymptotic behaviour of hadronic form factors is really described by the quark counting rules. Experimentally, however, these rules seem to work even for rather low $Q^2$ values of



the order of a few $GeV^2$.

One can imagine two possible scenarios.

- pQCD scenario:
  - Soft contribution dominates the low-$Q^2$ region, but dies out very fast to become negligible for $Q^2 > 3GeV^2$ in the pion case and for $Q^2 > 15 - 20\,GeV^2$ in the nucleon case.
  - Hard contribution dominates in the above regions and is large enough to describe the data.
- nonpQCD scenario:
  - Soft contribution is large enough to describe the data at all accessible $Q^2$.
  - Hard contribution is numerically very small in this whole region.

It is instructive to see how the asymptotic QCD predictions can describe existing data. In the pion case, one has a simple formula [5, 6, 7]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\ \varphi(x,\mu)\ \varphi(y,\mu) \left\{ \frac{g^2(\mu)}{9xyQ^2} + \frac{g^2(\mu)}{9(1-x)(1-y)Q^2} \right\} \tag{3}$$

It corresponds to a parton-model type picture, in which the pion is described by a function characterizing the probability amplitude $\varphi(x,\mu)$ to find the pion in a state where quarks carry fractions $xP$ and $(1-x)P$ of its longitudinal momentum $P$. The dependence of $\varphi(x,\mu)$ on the factorization scale $\mu$ is given by the renormalization group. In particular, as $\mu \to \infty$, the pion wave function $\varphi(x,\mu)$ acquires a very simple and natural form [4, 7]

$$\varphi_\pi(x, \mu \to \infty) \to \varphi_\pi^{as}(x) = 6f_\pi x(1-x), \tag{4}$$

where $f_\pi = 133MeV$ is the pion decay constant setting the wave function normalization. Thus, in the formal $Q \to \infty$ limit, pQCD predicts the absolute normalization of the pion form factor: $Q^2 F_\pi^{as}(Q^2) = 8\pi f_\pi^2 \alpha_s(Q^2)$. The experimental value for this combination is between 0.3 and 0.4 $GeV^2$, and to get it one should take $\alpha_s \sim 0.7$ to 0.9, which is a bit large by modern standards (but in 1979 such values were considered as acceptable ones!). So, the only way out now is to assume that the pion wave function $\varphi(x, Q^2)$ for low $Q^2$ strongly differs from its asymptotic form $\varphi^{as}(x)$.

Chernyak and A.Zhitnitsky proposed [8] to use a double-humped wave function

$$\varphi^{CZ}(x) = 30 f_\pi x(1-x)(1-2x)^2. \tag{5}$$

Its use increases the result by factor 25/9 compared to the asymptotic prediction.

Thus, if one takes the CZ form for $\varphi(x)$ and $\alpha_s$ of order of 0.3, one can formally describe the data by eq. (3). Still, there is a highly disturbing observation concerning the above "success" of pQCD: the bulk part of the relevant contribution comes from the regions where the virtualities of the exchanged gluons ($xyQ^2$ in the pion case) are not very large [9, 10]. One can easily verify that, with the CZ wave function, 50% of the whole contribution is due to the regions where both $x$ and $y$ are smaller than 0.2 ($xyQ^2$ smaller than $Q^2/25$, *i.e.*, smaller than $0.15\,GeV^2$ for $Q^2 < 4\,GeV^2$!) and 40% is due to the regions where either $x$ or $y$ is smaller than 0.2. Only 1.5% of the total contribution comes from the region, where both $x$ and $y$ are larger than 1/2 and one can treat the exchanged gluon as sufficiently virtual to rely on asymptotic freedom. More than 90% is due to the regions of small virtualities where the applicability of pQCD is more than questionable.

In this region one can expect large nonperturbative effects, *e.g.*, one should expect that the "hard" gluon propagator is modified $1/k^2 \to 1/(k^2 - M^2)$ due to the confinement effects. Now, if one substitutes the denominator factors $xyQ^2$ by $(xyQ^2 + M^2)$, with the effective gluon mass $M \sim 300 - 500\,MeV$, one immediately observes that the resulting value for $Q^2 F_\pi(Q^2)$ is smaller than the experimental one by a factor of 10, both for the asymptotic and CZ wave functions.

Since the essential gluon virtualities are much smaller than the total momentum transfer $Q^2$, the proper argument of the effective coupling constant must be much lower than $Q^2$. Otherwise, the perturbative expansion has large coefficients. At the next-to-leading order, this problem was studied more than 10 years ago [11]. It is convenient to present the results in terms of the coefficient $B(Q,\mu)$ characterizing the magnitude of the radiative correction for the pion form factor

$$F_\pi(Q^2) = F_0(Q^2)\left\{1 + \frac{\alpha_s(\mu)}{\pi} B(Q,\mu)\right\}. \tag{6}$$

Within the $\overline{MS}$-scheme and for $\mu = Q$ one has $B \approx 7$ for the asymptotic wave function and $B \approx 20$ for the CZ wave function. To get $B^{CZ} \approx 0$, one should take $\mu \sim 0.01\,Q$, *i.e.*, in this case the pQCD expansion is self-consistent only starting from $Q \sim 100\,GeV$. The origin of the large correction can be easily seen from the explicit expression for the hard scattering amplitude

$$\begin{aligned} T_\pi^{hard} = \frac{g^2(\mu)}{9xyQ^2}\Bigg\{1 + \frac{\alpha_s(\mu)}{\pi}\Bigg[C_F\left(\ln^2\frac{xyQ^2}{\mu^2} - \ln^2\frac{Q^2}{\mu^2}\right) + 4C_F \ln\left(xy\left(\frac{Q^2}{\mu^2}\right)^2\right) - \\ -2C_F \ln\frac{Q^2}{\mu^2} + 3C_F\ln(xy) - \left(11 - \frac{2}{3}N_f\right)\ln\left(\frac{xyQ^2}{\mu_R^2}\right) - \\ -2\left(C_F - \frac{N_c}{2}\right)\ln(xy) + f(x,y)\Bigg]\Bigg\} + \{x \to (1-x), y \to (1-y)\} \end{aligned} \tag{7}$$

where $f(x,y)$ is regular when $x = 0$ and/or $y = 0$. For a broad wave function, the most important term comes from the soft gluon radiation. It induces the Sudakov-type double logarithms. Though the double logarithms in $Q^2$ cancel (otherwise there would be no factorization of short and long distances), there remain double logs in $x$ and $y$. Another observation is that Sudakov terms will appear in higher orders as well, producing large corrections in higher loops. The lesson from these studies



is that the pQCD scenario for the pion form factor is internally inconsistent: to describe the data one should take a broad wave function, but a broad wave function enhances the contribution coming from the region of small $x$ and $y$, and, as a result, the expansion parameter is $\alpha_s(Q/k)$ rather than $\alpha_s(Q)$, with a very large $k$.

The QCD asymptotic predictions for the nucleon form factors are normalized by the proton "decay" constant which is experimentally unknown. However, one can estimate it using QCD sum rules. Using the asymptotic form for the nucleon wave function $\psi^{as}(x_1, x_2, x_3) \sim x_1 x_2 x_3$, and the pQCD leading twist formula one obtains a vanishing result for the proton magnetic form factor [12]. The neutron form factor calculated in the same way, with the proton "decay" constant estimated by Belyaev and Ioffe [13] is numerically small ( its magnitude is $\sim 1\%$ of the data) and has the sign opposite to data. If one takes a "nonrelativistic" wave function $\psi^{nr}(x_1, x_2, x_3) \sim \delta(x_1 - 1/3)\delta(x_2 - 1/3)\delta(x_3 - 1/3)$ then the pQCD results for both the proton and neutron magnetic form factors have wrong sign and also $\sim 1\%$ level of magnitude compared to data. The fact that the use of narrow wave functions in the pQCD formula gives a wrong sign for the nucleon magnetic form factors was first observed by the Yerevan group [14] in 1980. This observation, however, was ignored outside the USSR for almost 5 years until the paper by Chernyak and I.Zhitnitsky [15] appeared, where it was shown that if one uses a broad and highly asymmetric nucleon wave function, with a large fraction of the total momentum carried by one of the quarks, then the results of a perturbative calculation for the magnetic form factors has sign and magnitude in agreement with experimental data. However, this success of pQCD faces the same problems as in the pion case. Again, the bulk part comes from the regions where the "hard" gluons have very small virtualities. As a result, the factor of 100 magnification produced by the CZ wave functions is ruined by nonperturbative effects parametrized by an effective gluon mass

The self-consistency of the perturbative expansion is also doubtful. To the best of my knowledge, nobody calculated yet the radiative corrections for the nucleon form factors case. However, the structure of the one-loop corrections for the pion form factors unambiguosly indicates that the Sudakov effects will dominate in the nucleon case as well. Since the Sudakov terms persist in higher orders, a natural idea is to sum them up. Such an attempt was undertaken recently by Li and Sterman [16]. They observed also that the Sudakov effects suppress the contribution from the soft region $x, y \sim 0$. This effect can be also understood using our formula (7). Indeed, the most important term $\left[\ln^2\left(xy\frac{Q^2}{\mu^2}\right) - \ln^2\left(\frac{Q^2}{\mu^2}\right)\right]$ vanishes when $x, y \to 1$ and is negative (and large, if $Q^2 >> \mu^2$) when $xyQ^2 \sim \mu^2$. Thus, the contribution from the region of small $x$ and $y$ is perturbatively suppressed, and the claim is that pQCD analysis may become self-consistent for lower $Q^2$. Note, however, that if the soft region is suppressed, then the total pQCD contribution is smaller than the original lowest-order term, and the "agreement" with data, produced by using broad wave functions, is lost. For the time being, it is also unclear what portion of the exact next-to-leading result is reproduced within the Li-Sterman approximation.

Let us now consider the nonperturbative aspects of the pQCD scenario and analyze to what extent one can justify the assumptions that the hadronic wave functions at accessible energies are very broad, humpy functions of CZ type, but, at the same time, the soft nonperturbative contributions to the hadronic form factors rapidly vanish with $Q^2$.

The standard argument in favor of the CZ wave functions is that they are dictated by the QCD sum rules. In particular, the moments $\langle\xi^N\rangle \equiv \int_{-1}^{1}\varphi\left(\frac{1+\xi}{2}\right)\xi^N\,d\xi$ of the pion wave function are given by the SR [8]

$$f_\pi^2\langle\xi^N\rangle = \frac{3M^2}{4\pi^2}\frac{(1-e^{-s_0^{(N)}/M^2})}{(N+1)(N+3)} + \frac{\alpha_s\langle GG\rangle}{12\pi M^2} + \frac{16}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}(11+4N) \qquad (8)$$

where $s_0^{(N)}$ is the effective threshold characterising the position of higher states and $M^2$ is the so-called Borel parameter. The values of $\langle\xi^N\rangle$ are extracted from the sum rule using the requirement of the best agreement between its left- and right-hand sides. The $N=0$ case was considered in the pioneering SVZ paper: if one takes $s_0 \approx 0.7\,GeV^2$, the right-hand side is fairly constant for all $M^2 > 0.6\,GeV^2$, with the output value $f_\pi \approx 130\,MeV$, in a good agreement with experiment. The correlation between the $s_0$ and $f_\pi$ values (obtained from a fitting procedure) is well reproduced by the local duality relation $f_\pi^2 = s_0/(4\pi^2)$ that follows from the SR in the formal $M^2 \to \infty$ limit.

The same strategy was used by CZ for the higher moments $N=2$ and $N=4$. Note, however, that the nonperturbative terms in their sum rule have a completely different $N$-dependence compared to the perturbative one: the perturbative term decreases like $1/N^2$ for higher moments while the condensate terms are either constant or even increasing with $N$. Thus, the effective scale in the channel (settled by the ratio of the condensate terms to the perturbative one) substantially increases for higher $N$: $s_0^{(2)} \approx 2s_0$, $s_0^{(4)} \approx 3s_0$, *etc.* Again, the fitted values of $\langle\xi^N\rangle^{as}$ are well reproduced by the local duality relation:

$$\langle\xi^N\rangle^{as} \approx \frac{s_0^{(N)}}{4\pi^2 f_\pi^2}\frac{3}{(N+1)(N+3)} \approx \frac{s_0^{(N)}}{s_0^{(0)}}\langle\xi^N\rangle^{as}.$$

As a result, the value $\langle\xi^2\rangle^{CZ} = 0.43$ found by CZ is by factor 2 larger than $\langle\xi^2\rangle^{as} = 1/5$. Such a large value can be attributed only to a wave function concentrated in the region $|\xi| \sim 1$. This is how the CZ wave function was extracted from the QCD sum rules. A crucial implicit assumption in this derivation is that one can neglect higher power corrections, and it is sufficient to take into account only the lowest condensates.

The soft contribution to the pion form factor can be also estimated within the framework of the QCD sum rules. The relevant sum rule

$$\begin{aligned} f_\pi^2 F_\pi(Q^2) \;=\; & \frac{1}{\pi^2}\int_0^{s_0} ds_1 \int_0^{s_0} ds_2\, \rho^{pert}(s_1,s_2,q^2)\exp\left(-\frac{s_1+s_2}{M^2}\right) \\ & + \frac{\alpha_s\langle GG\rangle}{12\pi M^2} + \frac{16}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}\left(13+\frac{2Q^2}{M^2}\right), \end{aligned} \qquad (9)$$



in fact, has a striking similarity to the pion wave function sum rule (just take $Q^2/M^2 \sim N$): the perturbative term vanishes like $1/(Q^2)^2$ for large $Q^2$, while the $\langle\bar{q}q\rangle$- and $\langle GG\rangle$-terms are constant or linearly increasing with $Q^2$, – even the numerical values of the coefficients are almost identical. The ratios of the nonperturbative terms to the perturbative term grow with $Q^2$ and, as a result, the parameter $s_0^{(Q^2)}$ straightforwardly extracted from the SR (9) increases with $Q^2$. In particular, for $Q^2 = 1\,GeV^2$ the fitting gives the $s_0$-value very close to that obtained from the SR for $f_\pi$: $s_0^{(Q^2\sim 1\,GeV^2)} \approx s_0^{(0)} \approx 0.7\,GeV^2$, just demonstrating the self-consistency of the physical interpretation of $s_0$ and of the whole SR approach. However, for $Q^2 = 6\,GeV^2$, a formal fitting gives the value $s_0(Q^2 = 6\,GeV^2) \approx 1.5\,GeV^2$ - by factor of 2 larger than $s_0^{(0)}$ [17]. This is precisely the same effect that produced the growth of $s_0^{(N)}$ and $\langle\xi^N\rangle$ in the CZ SR. Using the local duality relation for the pion form factor

$$F_\pi^{soft}(Q^2) \approx \frac{s_0}{4\pi^2 f_\pi^2}\left(1 - \frac{1+6s_0/Q^2}{(1+4s_0/Q^2)^{3/2}}\right), \qquad (10)$$

one can see that one gets larger $F_\pi^{soft}(Q^2)$ if one takes larger $s_0$. It should be emphasized that the soft contribution, estimated by local duality with a constant duality interval $s_0 \approx 0.7\,GeV^2$, is sufficiently large to describe existing data. If $s_0$ increases with $Q^2$, then the soft contribution is larger than the data [17].

Main lesson is that, within the QCD sum rule approach, the form of the pion wave function and the magnitude of the soft nonperturbative contribution to the pion form factor are strongly correlated : the sum rules for $\langle\xi^N\rangle$ and for $F_\pi(Q^2)$ have an essentially identical structure (with $N \leftrightarrow Q^2$), and it is impossible to increase $\langle\xi^2\rangle$ compared to $\langle\xi^2\rangle^{as}$ and at the same time get a rapidly decreasing soft contribution. In full accordance with the results obtained (in a wave function formalism) by Isgur and Llewellyn Smith, a big perturbative contribution is always accompanied by a big nonperturbative term, and the basic assumption of the pQCD scenario fails. A pragmatic observation is that taking $s_0 = const \approx 4\pi^2 f_\pi^2$ one gets $F_\pi^{soft}$ close to data and a small one-gluon-exchange contribution, the addition of which improves the agreement with data. The same statement is true for the nucleons: taking the nucleon duality parameter $S_0 = const \approx 2.3\,GeV^2$ (the numerical value extracted from the QCD sum rule for the nucleon mass [13]) in the local duality formula for the proton magnetic form factor

$$G_M^p(Q^2) = \frac{8}{3}\sqrt{T^2-1}\left\{(4T^2-1)(T^2-1) + (4T^2-3)T\sqrt{T^2-1}\right\}^{-1}, \qquad (11)$$

(where $T = 1 + Q^2/2S_0$) one gets a curve describing the data in a wide region $3\,GeV^2 < Q^2 < 20\,GeV^2$. Asymptotically, this soft contribution goes like $O(s_0^3/Q^6)$.

One may ask, however: what is wrong with the original CZ-derivation that produced a broad wave function? This sum rule, taken at face value, definitely requires a drastic increase of $s_0^{(N)}$ for $N = 2, 4, \ldots$. To better understand the nature of the approximations made, it is instructive to rewrite the sum rule (8) for the wave function itself:

$$f_\pi^2\varphi_\pi(x) = \frac{3M^2}{2\pi^2}x(1-x)(1-e^{-s_0/M^2}) + \frac{\alpha_s\langle GG\rangle}{24\pi M^2}[\delta(x)+\delta(1-x)] + \frac{8}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}\{11[\delta(x)+\delta(1-x)]+2[\delta'(x)+\delta'(1-x)]\}. \quad (12)$$

The $O(1)$ and $O(N)$ terms in eq.(8) correspond to $\delta(x)$ and $\delta'(x)$ terms in eq.(12). In its turn, the presence of the $\delta(x)$ functions is evidently indicating that the vacuum fields are treated as carrying zero fraction of the pion momentum. This can be easily understood by observing that the operator product expansion (underlying any QCD sum rule) is a power series expansion over small momenta $k$ of vacuum quarks and gluons. Retaining only the $\langle\bar{q}q\rangle$ and $\langle GG\rangle$ terms (like in eqs.(8), (12)) is just equivalent to the assumption that $k$ is not simply small but exactly zero. However, it is much more reasonable to expect that the vacuum quanta have a smooth distribution with a finite width $\mu$. In configuration space, this means that vacuum fluctuations have a finite correlation length of the order of $1/\mu$, so that the two-point condensates like $\langle\bar{q}(0)q(z)\rangle$ die away when $|z|$ is large compared to $1/\mu$. Of course, one can always expand $\langle\bar{q}(0)q(z)\rangle$ in powers of $z$ starting with the local condensate $\langle\bar{q}(0)q(0)\rangle$ that produces eventually the $\delta(x)$ term. The question is, whether it is reasonable to do this, since the expansion resulting from such a Taylor series will not necessarily behave well.

According to the standard estimate [13], the average virtuality of the vacuum quarks $\lambda_q^2 \equiv \langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle \approx 0.4\,GeV^2$ is not small compared to the relevant hadronic scale $s_0 \approx 0.7\,GeV^2$. Thus, one cannot say that the correlation length of vacuum fluctuations is much larger than the hadronic size, and the constant-field approximation for the vacuum fields might not work, *i.e.*, the higher-power corrections might well ruin the conclusions derived from the original CZ sum rule. In particular, using a Gaussian model $\langle\bar{q}(z)q(0)\rangle = \langle\bar{q}q\rangle\exp(z^2\lambda_q^2/8)$, we obtained a modified sum rule for $\varphi_\pi(x)$, with the $\delta(x)$–functions substituted by terms like $x\theta(x < \lambda_q^2/2M^2)/\lambda_q^4$. For small $x$, the latter have the same behavior as the perturbative contribution. The value $\langle\xi^2\rangle = 0.25$ extracted from this sum rule [18] is much closer to the asymptotic value $\langle\xi^2\rangle^{as} = 0.2$ than to that of Chernyak and Zhitnitsky $\langle\xi^2\rangle^{CZ} = 0.43$. Fitting the modified sum rules produces a very mild variation of the threshold parameters $s_0^{(N)}, s_0^{(Q^2)}$ with $N$ or $Q^2$, respectively.

The value of $\langle\xi^2\rangle$ was also estimated within lattice QCD. A recent result [19] $\langle\xi^2\rangle^{lattice_2} = 0.11 \pm 0.01$ corresponds to a very narrow wave function, much more narrower than the asymptotic one (recall that $\langle\xi^2\rangle^{as} = 0.2$). An earlier estimate $\langle\xi^2\rangle^{lattice_1} = 0.34 \pm 0.17$ is not so restrictive. The "world average" of these lattice data seems to agree with the asymptotic value. The lattice estimates for the nucleon wave function do not confirm the large assymetry between different $x_i$ moments predicted by CZ-approach: no asymmetry was observed [21].

Our final conclusion is that only the nonpQCD scenario is a viable and a self-noncontradicting possibility to understand the dynamics of hadronic form factors.



Soft terms estimated within the framework of QCD sum rules and local duality (with constant duality interval $s_0$) is sufficiently large to describe the data and dominates at accessible energies. The hadronic wave functions obtained using the QCD local duality are close to their asymptotic forms and the "hard" contributions to form factors, calculated with this functions, are small at accessible $Q^2$. This picture is self-consistent and it is supported by an analysis of the modified QCD sum rules with nonlocal condensates. There was a criticism by Chernyak [22] that the results of such an approach are sensitive to the models one uses for the nonlocal condensates. This means, however, that studying the form factors one can eventually fix the form of the nonlocal condensates - the distribution functions of vacuum quarks and gluons.

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