CEBAF-TH 92-10

CEBAF
The Continuous Electron Beam Accelerator Facility
Theory Group Preprint Series

The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150

HEAVY QUARK SYMMETRY

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ABSTRACT

New symmetries of the strong interactions appear in heavy quark physics. They can be used to predict many properties of hadrons containing a single heavy quark. Some of these predictions are expected to play an important role in determining the values of elements of the Cabibbo–Kobayashi–Maskawa matrix.

1. INTRODUCTION

There are very few cases in which it is possible using analytic methods to make systematic predictions based on Quantum Chromodynamics (QCD) in the low-energy, nonperturbative regime. Indeed, this theory has proved so intractable to analytic methods that all such predictions are based not on dynamical calculations, but rather on some symmetry of QCD. Isospin symmetry was the first such symmetry discovered, and we now understand that this approximate symmetry arises because the light quark mass difference $m_d - m_u$ is much smaller than the masses associated with confinement which are set by the QCD scale Λ_{QCD}. Predictions based on isospin symmetry would, in a world with only strong interactions, be exact in the limit $m_d - m_u \to 0$; corrections to this limit can be studied systematically in an expansion in the small parameters $(m_d - m_u)/\Lambda_{QCD}$ and the electromagnetic fine structure constant α. $SU(3)$ flavor symmetry is similar, but the corrections are larger since $(m_s - m_d)/\Lambda_{QCD}$ is not small. Chiral symmetry $SU(2)_L \times SU(2)_R$ arises in QCD because *both* m_d and m_u are small compared to Λ_{QCD}; it is associated with the separate conservation of vector and axial vector currents. Although spontaneously broken in nature, the existence of this underlying symmetry allows the systematic expansion of chiral perturbation theory in which many low-energy properties of QCD are related to a few reduced matrix elements. If the strange quark mass is also treated as small compared with the QCD scale, then the chiral symmetry group becomes $SU(3)_L \times SU(3)_R$.

Over the last few years there has been progress in understanding systems containing a single heavy quark[1-10] (*i.e.*, a quark with mass m_Q much greater

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than the scale Λ_{QCD} of the strong interactions). It is now appreciated that there is a new symmetry of QCD, similar to isospin or chiral symmetry, in operation in such systems[6]. This symmetry arises because once a quark becomes sufficiently heavy, its mass becomes irrelevant to the nonperturbative dynamics of the light degrees of freedom of QCD. Consider, as an extreme example, two *very* heavy quarks of masses one and ten kilograms. Although these quarks will live in the usual hadronic "brown muck" of light quarks and glue, they will hardly notice it: their motion will fluctuate only slightly about that of a free heavy quark. Given that such quarks therefore define with great precision their own center-of-mass, we can study hadronic systems built on them in the frame where they act as static sources of color localized at the origin. The equations of QCD in the neighborhood of such an isolated heavy quark are therefore those of the light quark and gluonic degrees of freedom subject to the boundary condition that there is a static triplet source of color-electric field at the origin (*i.e.*, the heavy quark is treated as a Wilson line). Since this boundary condition is the same for both of our hypothetical heavy quarks (in the static approximation which is essentially perfect given their masses), the solutions for the states of the light degrees of freedom in their presence will be the same (see Fig. 1). Thus *the light degrees of freedom will be symmetric under an isospin-like rotation of the heavy quark flavors into one another* even though the heavy quark masses are *not* almost equal. In particular, the heavy meson and baryon excitation spectra built on any heavy quark will be the same, as will be all amplitudes for the scattering of light hadrons off any state built on the heavy quark.

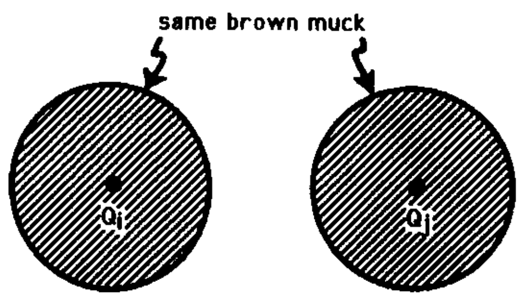

FIG. 1: Q_i and Q_j are surrounded by identical brown muck

The preceeding comments ignored the spin of the heavy quark. This is appropriate in QCD since the spin of a heavy quark decouples from the gluonic field: all heavy quarks look like scalar heavy quarks to the light degrees of freedom. Since the flavor *and* spin of the heavy quark are irrelevant, the static heavy quark symmetry is actually $SU(2N_h)$, where N_h is the number of heavy quarks (see Fig. 2). (We will see below that the full symmetry group is much larger since heavy quarks moving with different velocities cannot be scattered into each other by the strong interactions). At the spectroscopic level this additional symmetry means that each spectral level built on a heavy quark (unless it happens to have spin zero in its light degrees of freedom) will be a degenerate doublet in total spin.

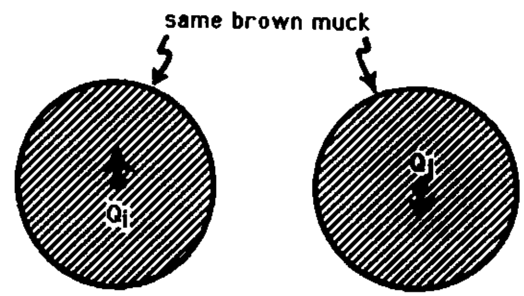

FIG. 2: ...even if the spin of Q_j is flipped

Heavy quark flavor symmetry is thus analogous to the fact that different isotopes of a given element have the same chemistry: their electronic structure is almost identical because they have the same nuclear charge. The spin symmetry is in turn analogous to the near degeneracy of hyperfine levels in atoms: the electronic structure of the states of a hyperfine multiplet are almost the same because nuclear magnetic moments are small.

In the situation described above where the light degrees of freedom (*i.e.*, light quarks and antiquarks and the gluons) typically have four-momenta small compared with the heavy quark mass, it is appropriate to go over to an effective theory where the heavy quark mass goes to infinity, with its four-velocity fixed [6,8] (see Fig. 3). This fixed-velocity approximation is similar to one often used in classical mechanics. When a baseball is thrown it churns up the air in its path, but, unless thrown by a major league pitcher, the air has a negligible impact on the baseball's path. Typically we just neglect the air and say that the ball falls under the influence of gravity in a way described by Newton's laws. The heavy quark is analogous to the baseball and the light quarks, antiquarks, and gluons are analogous to the air. In our case, the heavy quark path is a straight worldline described by a four-velocity v^μ satisfying $v^2 = 1$. There is another similarity between the heavy quark effective theory and the classical physics of a baseball: pair creation of heavy quark-antiquark pairs doesn't occur in the effective theory.

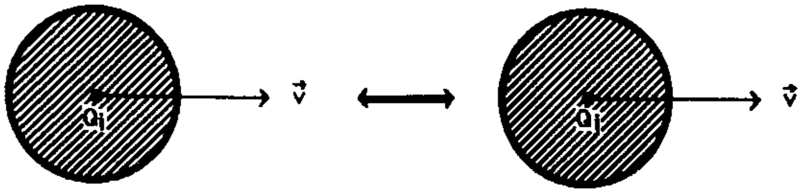

FIG. 3: $Q_i(\vec{v})$ is related by the symmetry to $Q_j(\vec{v})$

The $SU(2N_h)$ spin-flavor symmetry of the heavy quark effective theory is not manifest in the full theory of QCD; it only becomes apparent in the effective theory where the heavy quark masses are taken to infinity. This situation is familiar from our experience with the light quark flavor symmetries of QCD mentioned above. The strong interactions of light quarks q (with masses m_q that are much less than the QCD scale) are greatly simplified by going over to an effective theory where the light quark masses are taken to zero. For N light quarks this effective theory has an $SU(N)_L \times SU(N)_R$ chiral symmetry that is spontaneously broken to the vector $SU(N)_V$ subgroup. Again, the symmetry is not immediately apparent in the full theory of QCD. However, as long as the quark masses are small compared with the QCD scale, they have only a small impact on strong interaction dynamics. Thus the effective theory, where the light quark masses are set to zero, is a good approximation to QCD. We will see that the heavy quark flavor-spin symmetry endows us with predictive power much in the same way that light quark chiral symmetry does. For light quark chiral symmetry, it is possible to treat the small quark masses as perturbations and consider the corrections of order m_q/Λ_{QCD} to predictions based on the effective theory where $m_q \to 0$. Similarly, for heavy quark spin-flavor symmetry, it is possible to treat as perturbations the Λ_{QCD}/m_Q corrections to the predictions based on the effective heavy quark theory where $m_Q \to \infty$.

The relationship between operators involving the heavy quarks (e.g., $\bar{Q}\gamma_\mu q$) in the full theory of QCD and operators in the effective theory where the heavy quark masses go to infinity involves some interesting applications of perturbative QCD. Contributions to matrix elements of these operators from loop graphs with virtual momenta comparable to or greater than the heavy quark mass are clearly not correctly reproduced by the effective theory. However, because of asymptotic freedom these differences can be handled by perturbative QCD.

Most of the physics underlying heavy quark symmetry has been understood for a long time and has, to some extent, been incorporated into phenomenological models used to predict properties of hadrons containing a single heavy quark.[11,12] What is new is that we now understand that this physics arises from symmetries of an effective theory that is a systematic limit of QCD. Consequently, model-independent predictions are now possible. The most important predictions are for semileptonic B-meson decay form factors. These are expected to play an important role in the accurate determination of the values of the Cabibbo–Kobayashi–Maskawa matrix elements V_{cb} and V_{ub} from experimental data.

2. THE EFFECTIVE THEORY

We are interested in the physical situation where a heavy quark Q is interacting with light degrees of freedom which carry four momenta much smaller than the heavy quark mass m_Q. For such a situation it is appropriate to go over to an effective theory where the heavy quark mass m_Q goes to infinity with its four velocity v^μ fixed. We can derive the Feynman rules for the effective theory by taking the above limit of the Feynman rules for QCD. In the full theory of QCD the heavy quark propagator is

$$\frac{i(\not{p}_Q + m_Q)}{p_Q^2 - m_Q^2} . \tag{1}$$

To get the propagator in the effective theory, write

$$p_Q^\mu = m_Q v^\mu + k^\mu , \tag{2}$$

where k^μ is a residual momentum that is small compared to the heavy quark mass. In the numerator the residual momentum can be neglected, but in the denominator of the propagator it cannot be. In the limit $m_Q \to \infty$ the heavy quark propagator becomes

$$\frac{i(\not{v} + 1)}{2v \cdot k} . \tag{3}$$

In the full theory of QCD the vertex for heavy quark gluon interactions is*

$$-ig\gamma_\mu T^a , \tag{4}$$

* In dimensional regularization with minimal subtraction, an additional factor of $\mu^{\epsilon/2}$ (where $n = 4 - \epsilon$ is the number of space-time dimensions), appears in eqs. (4) and (7).

where g is the strong coupling and T^a is an $SU(3)$ color generator. Given the form of the propagator (3), the vertex always appears sandwiched between factors of $\frac{(1+\slashed{v})}{2}$, so the vertex in the effective theory can be taken as

$$-ig\frac{(1+\slashed{v})}{2}\gamma_\mu\frac{(1+\slashed{v})}{2}T^a = -igv_\mu T^a\frac{(1+\slashed{v})}{2} . \tag{5}$$

With the vertex of the form (5), factors of $(\slashed{v}+1)/2$ in the numerators of propagators and in vertices can be moved to the outside of any Feynman graph where they give unity on hitting on-shell spinors $u(v,s)$. As a result, we may take the propagator for a heavy quark in the effective theory to be

$$\frac{i}{v\cdot k} , \tag{6}$$

and the vertex for gluon-heavy quark interactions is

$$-igT^a v_\mu . \tag{7}$$

Equations (6) and (7) can be taken as defining the effective heavy quark theory. They are a momentum space realization of the fact that the heavy quark propagates in a manner described by a Wilson line.[4,5]

We can get the same result from field theory[7,8] without referring to the Feynman rules of the full theory of QCD. The part of the QCD Lagrangian density involving the heavy quark field is

$$\mathcal{L} = \bar{Q}(i\slashed{D} - m_Q)Q . \tag{8}$$

(Here we omit counter terms. For a discussion of renormalization in QCD and the effective theory, see Section 5.) Write[8] for a heavy quark with velocity v (as an approximation for $m_Q \to \infty$)

$$Q = e^{-im_Q v\cdot x}h_v^{(Q)} , \tag{9}$$

where the field $h_v^{(Q)}$ is constrained to satisfy

$$\slashed{v}h_v^{(Q)} = h_v^{(Q)} . \tag{10}$$

Putting eq. (9) into the QCD Lagrangian density gives

$$\mathcal{L}_v = \bar{h}_v^{(Q)}[m_Q(\slashed{v}-1) + i\slashed{D}]h_v^{(Q)} .$$

Using eq. (10), this becomes

$$\mathcal{L}_v = \bar{h}_v^{(Q)}i\slashed{D}h_v^{(Q)} , \tag{11a}$$

which can be further simplified to

$$\begin{aligned}\mathcal{L}_v &= \bar{h}_v^{(Q)}\frac{(1+\slashed{v})}{2}i\slashed{D}\frac{(1+\slashed{v})}{2}h_v^{(Q)} \\ &= \bar{h}_v^{(Q)}iv\cdot Dh_v^{(Q)} .\end{aligned} \tag{11b}$$

This effective Lagrangian density (11b) reproduces the Feynman rules in eqs. (6) and (7). Note that when a derivative acts on $h_v^{(Q)}$ it produces a factor of the residual momentum, since the large part of momentum was scaled out of the field. It is important to remember that eq. (9) is an approximation because eq. (10) constrains $h_v^{(Q)}$. In general we have, for a heavy quark,

$$Q = e^{-im_Q v\cdot x}[h_v^{(Q)} + \mathcal{X}_v^{(Q)}] , \tag{12}$$

where

$$\slashed{v}h_v^{(Q)} = h_v^{(Q)}, \quad \slashed{v}\mathcal{X}_v^{(Q)} = -\mathcal{X}_v^{(Q)} . \tag{13}$$

The $\mathcal{X}_v^{(Q)}$ part of Q (which is of order Λ_{QCD}/m_Q) arises because the heavy quark is not quite on shell as it propagates.

The effective theory is *not* a nonrelativistic approximation for the heavy quark (it is in some sense a classical approximation). Nothing prevents the spatial components of the four velocity v^μ from being of order unity. Of course, for a single heavy quark, this is not a very significant statement, since it is always possible to go to its rest frame. However, if there are two heavy quarks moving with different velocities then one cannot go to a frame where they are both at rest, and there is nothing in the effective theory that restricts the second heavy quark to be moving nonrelativistically in the rest frame of the first heavy quark. In particular, we will be interested here in the situation where an external current acts and changes a heavy quark's velocity (and possibly its flavor). Even though interactions with gluons don't change the heavy quark's four-velocity, external currents (from couplings to a W-boson, for example) can. Then it is not possible to go to the rest frame of both the initial and final heavy quark.

Note that while the field $h_v^{(Q)}$ destroys a heavy quark of four-velocity v, it does not create an antiquark. In the effective theory the field for the antiquark is an independent degree of freedom because pair creation is not present.

In the effective theory the equation of motion for $h_v^{(Q)}$ is

$$v\cdot Dh_v^{(Q)} = 0 . \tag{14}$$

and the ϵ_a are arbitrary infinitesimal parameters. Note that since $[\not v, S_a] = 0$, transforming by this group preserves the constraint, $\not v h_v^{(Q)} = h_v^{(Q)}$.

If there are N_h heavy quarks $Q_1, \ldots Q_{N_h}$, moving with the same four velocity v, then, denoting the corresponding fields in the effective theory by $h_v^{(j)}$, the Lagrangian density becomes

$$\mathcal{L}_v = \sum_{j=1}^{N_h} \bar{h}_v^{(j)} i v \cdot D h_v^{(j)} \ . \tag{22}$$

This Lagrangian density is completely independent of the heavy quark masses, so the $SU(2)$ spin symmetry of (11b) becomes a $SU(2N_h)$ spin-flavor symmetry of (22). Note that because the heavy quark masses can be very different, this symmetry, which relates quarks of the same velocity, generally relates quarks of very different momentum. This is one of the unusual aspects of heavy quark symmetry.

At this point it is worth reviewing the continuous global (approximate) symmetries of the strong interactions. The six quarks u, d, s, c, b, t naturally break into two groups of three. The light u, d and s quarks have masses that are much less than the QCD scale* ($m_u \sim 0.005\ GeV, m_d \simeq 0.01\ GeV, m_s \simeq 0.15\ GeV$). Because of this it is appropriate to go over to an effective theory where the light quark masses are set to zero. This effective theory has a $SU(3)_L \times SU(3)_R$ chiral symmetry that is spontaneously broken to the vector $SU(3)_V$ subgroup. Note that the $SU(3)_V$ symmetry does not arise because the light quark masses are nearly equal ($m_s/(m_u + m_d) \simeq 10$), but because the light quark masses are small compared with the QCD scale. It is important to bear in mind the origin of the symmetry. For example, even though the pion and kaon are in the same multiplet (an octet) of the $SU(3)_V$ symmetry group, it is a mistake to use the light quark flavor symmetry to deduce that their masses should be almost equal (recall $m_\pi \simeq 0.14\ GeV$ and $m_K \simeq 0.49\ GeV$). The pion and kaon are pseudo-Goldstone bosons associated with the spontaneous breakdown of the $SU(3)_L \times SU(3)_R$ chiral symmetry; their masses go to zero as the light quark masses go to zero. One should apply the $SU(3)_V$ symmetry to quantities that go to a constant in this limit. (For example, a legitimate application would be to deduce that the proton to cascade mass ratio is close to unity.) The remaining three quarks c, b and t have masses large compared with the QCD scale ($m_c \simeq 1.8\ GeV, m_b \simeq 5.2\ GeV$ and $m_t > 90\ GeV$). Because of this it is useful to go over to an effective theory

* The success of chiral perturbation theory implies that these quarks have small masses. Mass ratios are derived from the pseudo-Goldstone masses. The values of the masses quoted here follow from a phenomenological model and can be valid at, at most, one particular subtraction point.

To derive this, one should, in principle, introduce a Lagrange multiplier for the constraint (10).* Alternatively one can work in the rest frame of the heavy quark and take $h_v^{(Q)}$ to be a two component object,[7] since in this frame the constraint just restricts the lower two components of $h_v^{(Q)}$ to vanish. After deriving the equation of motion in the rest frame, boosting back to a general frame yields eq. (14).

The heavy quark effective theory has symmetries not manifest in the Lagrangian of QCD. Since there is no pair creation in the effective theory, there is a U(1) symmetry of the effective Lagrangian (11b) associated with heavy quark conservation. Under an infinitesimal U(1) transformation of this type

$$h_v^{(Q)} \to h_v^{(Q)} + \delta h_v^{(Q)} \ , \tag{15}$$

with

$$\delta h_v^{(Q)} = i \epsilon_o h_v^{(Q)} \ . \tag{16}$$

Here ϵ_o is an arbitrary infinitesimal parameter. Since gamma matrices no longer appear in the gluon-heavy quark interaction (see eq. (7)), the spin of the heavy quark is conserved. Associated with this is an $SU(2)$ symmetry group of the Lagrangian in eq. (11b). To define the action of the $SU(2)$ group on the heavy quark fields we introduce three orthonormal four-vectors, $e_{a\mu}, a = 1, 2, 3$, that are orthogonal to the heavy quark's four-velocity

$$e_{a\mu} e_b^\mu = -\delta_{ab} \ , \tag{17}$$

$$v_\mu e_a^\mu = 0 \ . \tag{18}$$

Then the three matrices

$$S_a = i \sum_{b,c} \epsilon_{abc} [\not e_b, \not e_c] \ , \tag{19}$$

have the same commutation relations as the generators of $SU(2)$ and in the rest frame of the heavy quark are the usual spin-matrices. The Lagrangian (11b) is invariant under the $SU(2)$ group of (infinitesimal) transformations

$$h_v^{(Q)} \to h_v^{(Q)} + \delta h_v^{(Q)} \ , \tag{20}$$

where

$$\delta h^{(Q)} = i \sum_a \epsilon_a S_a h_v^{(Q)} \ , \tag{21}$$

* The two forms $\mathcal{L}_v = \bar{h}_v^{(Q)} i \not D h_v^{(Q)}$ and $\mathcal{L}_v = \bar{h}_v^{(Q)} i v \cdot D h_v^{(Q)}$ yield the same equation of motion for $h_v^{(Q)}$.

where the heavy quark masses go to infinity. This effective theory has a $SU(6)$ spin-flavor symmetry. The heavy quark flavor symmetry arises not because the heavy quarks are almost degenerate in mass, but because their masses are all large compared with the QCD scale. Again, it is important to bear in mind the physical origin of the symmetry. For example, even though the $\bar{B}$ and D mesons are in the same multiplet of the heavy quark flavor symmetry, it would be a mistake to use this symmetry to deduce that their masses should be almost equal (recall that experimentally $m_B \simeq 5.3\ GeV$ and $m_D \simeq 1.9\ GeV$). These are quantities that go to infinity as the heavy quark masses go to infinity. The heavy quark flavor symmetry should be applied to quantities that go to a constant in this limit.

There is another respect in which the $SU(3)_L \times SU(3)_R$ chiral symmetry of the light quarks is similar to the $SU(6)$ spin-flavor symmetry of the heavy quarks. Since the strange quark mass is not very small compared with the QCD scale, there are sizable (typically $\simeq 20\%$) corrections to predictions based on $SU(3)_L \times SU(3)_R$ chiral symmetry. Similarly, because the charm quark mass is not very large compared with the QCD scale, we expect sizable (typically* $\sim 20\%$) corrections to predictions based on $SU(6)$ spin-flavor symmetry.

Because the top quark is very heavy, it will probably not have a long enough lifetime to form a hadron. If it is heavy enough it will decay rapidly (compared with a strong interaction time scale) through the mode $t \to b + W$. Ironically, for the heaviest of the quarks, it is unlikely that heavy quark symmetry will play a useful role.

3. SPECTROSCOPIC APPLICATIONS OF HEAVY QUARK SYMMETRY[13]

In the limit $m_Q \to \infty$ the spin of the heavy quark $\vec{S}_Q$ and the spin of the light degrees of freedom (*i.e.*, the angular momentum of the light degrees of freedom in the heavy quark's rest frame)

$$\vec{S}_\ell = \vec{S} - \vec{S}_Q \ , \tag{23}$$

are separately conserved by the strong interactions (here $\vec{S}$ is the angular momentum of both the heavy quark and the light degrees of freedom in the heavy quark's rest frame, *i.e.*, the total spin). Therefore, in this limit, s_Q, m_Q, s_ℓ, m_ℓ are good quantum numbers. Since the dynamics are completely independent of the mass and spin of the heavy quark Q it is convenient to classify states containing a single heavy quark by s_ℓ. Then associated with each such state for the light degrees of freedom will be a degenerate doublet of hadrons with total spins (formed from combining the spin of the heavy quark $s_Q = 1/2$ with the spin of the light degrees of freedom s_ℓ)

$$s_\pm = s_\ell \pm 1/2 \ , \tag{24}$$

(unless $s_\ell = 0$, in which case a single $s = 1/2$ state is obtained). The flavor symmetry ensures that the spectrum is identical for each flavor Q up to an overall constant mass shift associated with the mass of the heavy quark. Of course, states are also labeled by their parity π (which is the same as the parity of the light degrees of freedom, π_ℓ, since the heavy quark has positive parity) and by other "radial" quantum numbers (see Fig. 4).

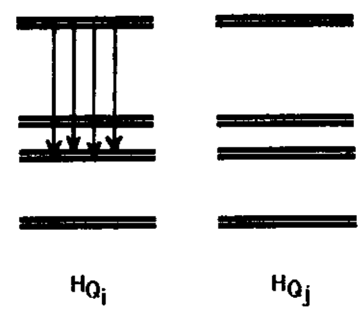

FIG. 4: the spectra and transitions of the hadrons built on Q_i and Q_j

To get a better picture of how this works let's consider the mesons with $Q\bar{q}$ flavor quantum numbers. (Note that although we use the *language* of the constituent quark model, our conclusions will be completely general.) It is reasonable to assume that the ground state mesons with these flavor quantum numbers have $s_\ell = 1/2$ and negative parity, forming a doublet consisting of a spin zero state ($s_- = 0$) which we denote by P_Q and a spin one state ($s_+ = 1$) which we denote by P_Q^*. In the case $Q = c$, these are the D and D^* mesons, and in the case $Q = b$, these are the $\bar{B}$ and $\bar{B}^*$ mesons. In terms of the spin of the heavy quark and the spin of the light degrees of freedom, the states (at rest) are

$$|P_Q\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \ , \tag{25a}$$

* This is based on a comparison with phenomenological models like the nonrelativistic constituent quark model. In the case of $SU(3)_L \times SU(3)_R$ symmetry, experience tells us to expect about 20% corrections. As the experimental situation in heavy quark physics improves, a better estimate of expected deviations from the predictions of $SU(6)$ spin-flavor symmetry will be possible.

$$|P_Q^*\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\}\ , \tag{25b}$$

where the state $|P_Q^* >$ in eq. (25b) has zero component of total spin along the quantization axis $\hat{z}$. In eqs. (25) the first arrow in a ket refers to the spin of the heavy quark along the z-axis, while the second arrow in a ket refers to that of the light degrees of freedom. Acting with the z-component of the heavy quark spin then gives

$$S_Q^z|P_Q >= \frac{1}{2}|P_Q^* > \ . \tag{25c}$$

Since $\vec{S}_Q$ commutes with the Hamiltonian, the P_Q and P_Q^* states are degenerate in mass.

Experimental tests on the spectroscopic consequences of heavy quark symmetry are, for the moment, very limited. The $D^* - D$ splitting of ~ 145 MeV is reduced to ~ 45 MeV in the $B^* - B$ multiplet. This is consistent with the expected "$1/m_Q$" approach to the symmetry limit. There is, at present, no data on excited systems containing a b quark, so the predicted equality between flavor sectors in the splittings cannot be compared with experiment. As we will now discuss, the little information available on excited charmed-hadron spectroscopy is also consistent with the expected doublet degeneracy.

Since the heavy quark flavor symmetry applies to the complete set of n-point functions of the theory, not only mass splittings, but also all strong decay amplitudes arising from the emission of light quanta like $\pi, \eta, \rho, \pi\pi$, etc., are independent of heavy quark flavor. For a given heavy quark flavor the spin symmetry ensures that two states with spins $s_\pm$ must have the same total widths. This equality between total widths typically arises in a nontrivial way. The two states of a given multiplet can decay to both states of every available multiplet with distinct partial widths whose sum must be identical. The spin symmetry determines ratios of these partial widths (see Fig. 4).

Consider the decays

$$H_Q \to [H_Q' h]_{LJ_h}\ ,$$

where h is a light hadronic system with orbital angular momentum L (with respect to H_Q') in a state of total angular momentum J_h, where $\vec{J}_h = \vec{L} + \vec{S}_h$ and $\vec{S}_h$ is the spin of h. The heavy quark acts as a static color source about which the reaction $s_\ell \to s_\ell' + h$ occurs. Since the spin of the heavy quark decouples from the light degrees of freedom, each such allowed partial wave amplitude for the light degrees of freedom will determine the amplitudes for the four hadronic level processes $s_+ \to s_+' + h, s_+ \to s_-' + h, s_- \to s_+' + h$ and $s_- \to s_-' + h$. The amplitudes for these four transitions will be of the form[13]

$$\begin{aligned} A(H_Q \to [H_Q' h]_{LJ_h}) \sim \sum_{m_\ell, m_\ell'} & C(J_h, m_\ell - m_\ell'; s', m_s + m_\ell' - m_\ell | s, m_s)^* \\ & \cdot C(s_\ell', m_\ell'; s_Q, m_s - m_\ell | s', m_s + m_\ell' - m_\ell)^* \\ & \cdot C(J_h, m_\ell - m_\ell'; s_\ell', m_\ell' | s_\ell, m_\ell) \cdot C(s_\ell, m_\ell; s_Q, m_s - m_\ell | s, m_s) \end{aligned} \tag{26}$$

where (s, m_s) and (s', m_s') are the $\vec{S}^2$ and S^z quantum numbers of H_Q and H_Q' and the C's are Clebsch–Gordan coefficients. Note that although the amplitude may depend on L via a reduced matrix element, the Clebsch–Gordon sum depends only on J_h.

It is simple to understand the origin of the four Clebsch–Gordon factors in eq. (26). The first Clebsch–Gordon coefficient arises from total angular momentum conservation, and the third from conservation of the spin of the light degrees of freedom. The second arises from a decomposition of the spin of H_Q' into the spin of the light degrees of freedom and the spin of the heavy quark. The fourth Clebsch–Gordon coefficient arises from an analogous decomposition of the spin of H_Q.

The heavy quark symmetry cannot, of course, tell us anything about the spectroscopy of the light degrees of freedom. It can only predict relationships between heavy quark systems involving given states of these degrees of freedom. As we have mentioned for mesons with $Q\bar{q}$ flavor quantum numbers, both the constituent quark model and experiment suggest that the ground states have $s_\ell^{\pi_\ell} = 1/2^-$ giving the $s_-^\pi = 0^-$ and $s_+^\pi = 1^-$ states P_Q and P_Q^*. The constituent quark model also suggests that the lowest lying excited states are likely to be those which correspond to giving the spin 1/2 constituent anti-quark a unit of orbital angular momentum resulting in $s_\ell^{\pi_\ell} = 1/2^+$ and $3/2^+$ multiplets.

As an application of eq. (26), consider the decay of the $s_\ell^{\pi_\ell} = 1/2^+$ and $3/2^+$ multiplets to the states P_Q, P_Q^* via emission of a pion. In this case $J_h = L$, so the partial wave amplitude can be given a single subscript. Parity conservation implies that L is even. Eq. (26) gives that the $s_+^\pi = 2^+$ state of the $s_\ell^{\pi_\ell} = 3/2^+$ multiplet has decay amplitudes in the proportions $\sqrt{(2/5)} : \sqrt{(3/5)}$ to the states $[P_Q\pi]_{L=2}$ and $[P_Q^*\pi]_{L=2}$ respectively. Its multiplet partner, with $s_-^\pi = 1^+$, decays at the same total rate exclusively to $[P_Q^*\pi]_{L=2}$. Note that the $s_-^\pi = 1^+$ state does not decay to $[P_Q^*\pi]_{L=0}$ even though this is an allowed channel. Eq. (26) also implies that the $s_+^\pi = 1^+$ state of the $s_\ell^{\pi_\ell} = 1/2^+$ multiplet decays exclusively to $[P_Q^*\pi]_{L=0}$, and does not decay to $[P_Q^*\pi]_{L=2}$. Its $s_-^\pi = 0^+$ state decays to $[P_Q\pi]_{L=0}$ with the same total rate.* These predictions are compatible with existing experimental information on mesons containing a charm quark. If one interprets the two confirmed states D_2^* (2460) and D_1 (2420) as members of the $s_\ell^{\pi_\ell} = 3/2^+$ multiplet, then their mass difference is consistent with being

* These results were first noted by Rosner[14] who obtained them by taking the $m_Q \to \infty$ limit of a quark model calculation. Heavy quark symmetry allows us to see that they are model independent consequences of QCD in that limit.

a Λ_{QCD}/m_c correction to the limiting theory. Moreover, the $D_2^* \to D\pi$ and $D_2^* \to D^*\pi$ decay amplitudes are in the ratio 0.8 ± 0.1. (Here and in what follows we quote amplitudes with a phase space and typical barrier penetration factor of $[p_\pi^{2L+1} exp(-p_\pi^2/1\ GeV^2)]^{1/2}$ removed.*) This is very near the $m_c \to \infty$ prediction of $\sqrt{2/3}$. The ratio of $D_1 \to D^*\pi$ and $D_2^* \to D^*\pi$ decay amplitudes is 2.3 ± 0.6. Although the error is large, this is not particularly close to the $m_c \to \infty$ prediction of $\sqrt{5/3}$. It is important to remember, however, that the D_1 decay may be contaminated by an S-wave admixture from Λ_{QCD}/m_c corrections. Even though this S-wave amplitude vanishes as $m_c \to \infty$ it might be comparable in size to the D-wave amplitude, because they are very different objects: a small grapefruit can be larger than a typical apple. In fact, the quark model and light hadron data suggest that the S-wave decays of the $s_\ell^{\pi_\ell} = 1/2^+$ multiplet are much stronger than the D-wave decays of the $s_\ell^{\pi_\ell} = 3/2^+$ multiplet.

There are obviously many other possible applications of eq. (26). It will, of course, help in determining the s_ℓ quantum numbers of resonances containing a single heavy quark. It is also amusing to note that it can in principle be used to determine the total spin of a heavy quark state without measuring any angular distributions.

4. TRANSITION MATRIX ELEMENTS

In this section we consider some matrix elements of operators in the effective heavy quark theory. The matrix elements we focus on are those that are likely to play an important role in determining the Cabibbo–Kobayashi–Maskawa matrix elements V_{cb} and V_{ub}.

Consider first the heavy meson-heavy meson transition matrix element**

$$\frac{\langle P_{Q_j}(v')|\bar{h}_{v'}^{(j)}\gamma_\mu h_v^{(i)}|P_{Q_i}(v)\rangle}{\sqrt{m_{P_{Q_j}} m_{P_{Q_i}}}} = \tilde{f}_+(v+v')_\mu + \tilde{f}_-(v-v')_\mu . \quad (27)$$

Meson states in the full theory of QCD, denoted $|M(p,s)\rangle$, are conventionally normalized by

$$\langle M(p',s')|M(p,s)\rangle = 2E\delta_{ss'}(2\pi)^3\delta^3(\vec{p}-\vec{p}') . \quad (28)$$

* This amounts to including the kinematic effects of some $1/m_c$ corrections (see Ref. 13 for details). In particular, the importance of corrections to mass differences is amplified because, for example, the $D^* - D$ mass difference is not negligible compared with p_π. This effect is more significant the greater the value of L.

** Here we have labeled the states by their velocity rather than momentum. This is convenient because the heavy quark flavor symmetry relates heavy quarks with equal velocities. Also, it is understood that the operator $\bar{h}_{v'}^{(j)}\gamma_\mu h_v^{(i)}$ is evaluated at the origin of space-time.

The factor of energy in the normalization leads to the $\sqrt{m_{P_{Q_j}} m_{P_{Q_i}}}$ in the denominator of (27) if one is to have a quantity that is independent of the heavy quark mass in the effective theory. It is appropriate to use the effective theory for this matrix element. In the initial hadron the light degrees of freedom have a momentum of order $\Lambda_{QCD}v$ and in the final hadron the light degrees of freedom have a momentum of order $\Lambda_{QCD}v'$, so the squared invariant momentum transfer felt by the light degrees of freedom is only of order

$$\Lambda_{QCD}^2(v\cdot v' - 1) . \quad (29)$$

For $v \cdot v'$ of order unity, this is much less than the heavy quark masses.

In eq. (27) $\tilde{f}_\pm$ are functions of $v\cdot v'$. They have a tilde on them because the more usual definition of form factors for transitions of this type is

$$\langle P_{Q_j}(v')|\bar{h}_{v'}^{(j)}\gamma_\mu h_v^{(i)}|P_{Q_i}(v)\rangle = f_+(p+p')_\mu + f_-(p-p')_\mu , \quad (30)$$

where $p' = m_{P_{Q_j}} v'$ and $p = m_{P_{Q_i}} v$. A comparison of eqs. (27) and (30) implies

$$f_+ = \frac{1}{2}\left(\sqrt{\frac{m_{P_i}}{m_{P_j}}} + \sqrt{\frac{m_{P_j}}{m_{P_i}}}\right)\tilde{f}_+ - \frac{1}{2}\left(\sqrt{\frac{m_{P_i}}{m_{P_j}}} - \sqrt{\frac{m_{P_j}}{m_{P_i}}}\right)\tilde{f}_- , \quad (31)$$

$$f_- = \frac{-1}{2}\left(\sqrt{\frac{m_{P_i}}{m_{P_j}}} - \sqrt{\frac{m_{P_j}}{m_{P_i}}}\right)\tilde{f}_+ + \frac{1}{2}\left(\sqrt{\frac{m_{P_i}}{m_{P_j}}} + \sqrt{\frac{m_{P_j}}{m_{P_i}}}\right)\tilde{f}_- . \quad (32)$$

Since $\not{v} h_v^{(i)} = h_v^{(i)}$ and $\bar{h}_{v'}^{(j)}\not{v}' = \bar{h}_{v'}^{(j)}$ contracting $(v-v')^\mu$ with both sides of eq. (27) yields

$$\tilde{f}_- = 0 . \quad (33)$$

Finally, we note that at $v = v'$ the left side of eq. (27) is the matrix element of a conserved current associated with heavy quark flavor symmetry. The $\mu = 0$ component is related to a generator for the symmetry and its matrix element is known. This gives[1,6]

$$\tilde{f}_+(1) = 1 . \quad (34)$$

The basic physics underlying these formulas can be visualized using Fig. 5.

Similar manipulations determine the other form factors. Thus, the form factors characterizing the $P_{Q_i} \to P_{Q_j}$ and $P_{Q_i} \to P^*_{Q_j}$ matrix elements of the vector and axial vector currents are expressible in terms of a single universal function $\xi(v \cdot v')$ that is normalized to unity at zero recoil:

$$\xi(1) = 1 \ . \tag{43}$$

Explicitly,[6)]

$$\tilde{f}_+ = \xi \ , \quad \tilde{f}_- = 0 \ , \tag{44a}$$

$$\tilde{f} = (1 + v \cdot v')\xi \ , \tag{44b}$$

$$(\tilde{a}_+ - \tilde{a}_-) = -\xi \ , \quad (\tilde{a}_+ + \tilde{a}_-) = 0 \ , \tag{44c}$$

$$\tilde{g} = \xi \ . \tag{44d}$$

The function ξ is truly universal: it doesn't depend on the heavy quark's mass or spin, nor does it depend on the current which causes the $Q_i \to Q_j$ transition. The same function would even play a role in the physics of hadrons containing other heavy color triplet particles. Many extensions of the standard model, (e.g., supersymmetry and technicolor), contain such heavy spin zero color triplets.

Transition matrix elements involving the ground state (isospin-zero) baryons with $Q_i ud$ flavor quantum numbers are even easier to deduce than those involving the ground state mesons. These baryons are denoted by Λ_{Q_i} and we assume that they have $s^{\pi_\ell} = 0^+$ (this is suggested by the constituent quark model and in the case $Q = c$ is consistent with experiment). The "interpolating field" that destroys one of these baryons is

$$\Lambda_{Q_i}(v, s) = \phi_v \bar{u}(v, s) h_v^{(i)} \ , \tag{45}$$

where ϕ_v is a scalar field that destroys the light degrees of freedom. Note there is no factor of $\sqrt{m_{\Lambda_{Q_i}}}$ because in the full theory of QCD, the conventional normalization for baryon states $|B(p, s)\rangle$ is

$$\langle B(p', s')|B(p, s)\rangle = \left(\frac{E}{m_B}\right) \delta_{ss'} (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \ . \tag{46}$$

With the "interpolating field" in eq. (57) it is straightforward to see that[15,16)]

$$\langle \Lambda_{Q_j}(v', s')|\bar{h}_{v'}^{(j)} \Gamma h_v^{(i)}|\Lambda_{Q_i}(v, s)\rangle = \eta \bar{u}(v', s') \Gamma u(v, s) \ , \tag{47}$$

where η is a universal function of $v \cdot v'$ independent of th e heavy quark masses. The heavy quark flavor symmetry implies that at zero recoil

$$\eta(1) = 1 \ . \tag{48}$$

Apart from the overall factor of η, eq. (59) shows that the hadronic matrix element is like a heavy quark matrix element. This occurs because in a Λ_Q state the spin of the hadron is carried by the heavy quark.

Transition matrix elements between heavy and light states can also be considered. Here, the heavy quark flavor symmetry can be used to relate matrix elements involving different heavy quarks. For example,[2,5)]

$$\frac{\langle 0|\bar{q}\gamma_\mu\gamma_5 h_v^{(i)}|P_{Q_i}(v)\rangle}{\sqrt{m_{P_{Q_i}}}} = \frac{\langle 0|\bar{q}\gamma_\mu\gamma_5 h_v^{(j)}|P_{Q_j}(v)\rangle}{\sqrt{m_{P_{Q_j}}}} \ . \tag{49}$$

Similar relations hold to light final states like π and ρ provided, in the rest frame of the heavy quark, the final states have four-momenta small compared with the heavy quark masses[6)] (see below).

5. RELATIONSHIP BETWEEN OPERATORS IN QCD AND OPERATORS IN THE EFFECTIVE THEORY

In the previous section we wrote down matrix elements of operators in the effective theory. Before we can relate these to physical matrix elements, we must establish the relation between operators in the effective theory and full QCD. This relationship is of course nontrivial because in the low energy effective theory we have ignored high momentum virtual states of the heavy quark in its rest frame. Consider for example the relationship between the vector current in the full theory

$$V_\nu = \bar{q}_j \gamma_\nu Q_i \ , \tag{50}$$

and the renormalized vector current in the effective theory

$$O_{\gamma_\nu} = \bar{q}_j \gamma_\nu h_v^{(i)} + \text{counter term} \ . \tag{51}$$

In eqs. (50) and (51) the operators are evaluated at the origin of space-time. Naively the relationship between the two is the tree level one, $V_\nu = O_{\gamma_\nu}$. However, this relationship cannot survive beyond the tree level, because O_{γ_ν} depends on the subtraction point μ while V_ν doesn't. The subtraction point μ determines how much of a matrix element is put in the infinite part and subtracted away and how much remains in the finite value of the matrix element. Crudely speaking, virtual loop momenta greater than μ are subtracted away, and virtual loop momenta less than μ remain in the finite part. Consequently, if we choose $\mu = m_{Q_i}$ then the relationship

$$V_\nu = O_{\gamma_\nu}(m_{Q_i}) + \mathcal{O}(\alpha_s(m_{Q_i})) \tag{52}$$

holds. The finite parts of matrix elements have logarithmic dependence on the subtraction point. Consequently at $\mu = m_{Q_i}$, the operator $O_{\gamma_\nu}(m_{Q_i})$ has large

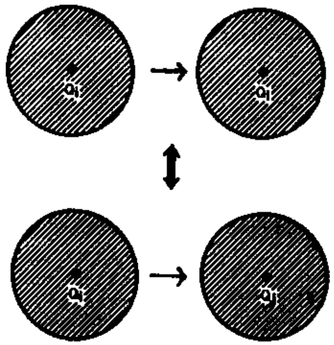

FIG. 5: $Q_i \to Q_j$ is related to the elastic transition $Q_i \to Q_i$ by the symmetry

The manipulations leading to eqs. (33) and (34) bear a striking resemblance to the method for deriving the predictions of light quark $SU(3)_V$ symmetry for form factors in the $K \to \pi$ matrix element of the current $\bar{s}\gamma_\mu d$. However, we see from eqs. (31) and (32) that heavy quark symmetry does not predict f_- to be zero and it provides a normalization, not at $q^2 \equiv (p'-p)^2 = 0$, but rather at the maximum value $q^2_{max} = (m_{P_i} - m_{P_j})^2$ where both the initial and final hadrons have the same four-velocity. We call this kinematic point zero recoil, since in the rest frame of the initial hadron the final hadron is also at rest.

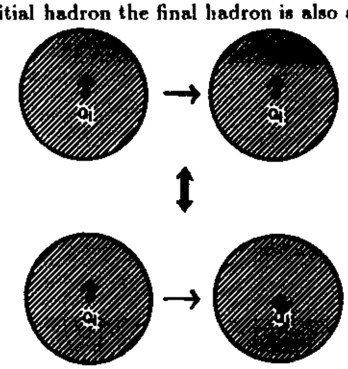

FIG. 6: ...even if a spin flips

The P_{Q_j} and $P^*_{Q_j}$ states are related by the spin symmetry (see Fig. 6). This allows us to get information on the transition matrix elements

$$\frac{\langle P^*_{Q_j}(v',\varepsilon)|\bar{h}^{(j)}_{v'}\gamma_\mu\gamma_5 h^{(i)}_v|P_{Q_i}(v)\rangle}{\sqrt{m_{P^*_{Q_j}} m_{P_{Q_i}}}} = \tilde{f}\varepsilon^*_\mu + (\varepsilon^*\cdot v)[\tilde{a}_+(v+v')_\mu + \tilde{a}_-(v-v')_\mu] \ , \quad (35)$$

and

$$\frac{\langle P^*_{Q_j}(v',\varepsilon)|\bar{h}^{(j)}_{v'}\gamma_\mu h^{(i)}_v|P_{Q_i}(v)\rangle}{\sqrt{m_{P^*_{Q_j}} m_{P_{Q_i}}}} = i\tilde{g}\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}v'^\lambda v^\sigma \ . \quad (36)$$

In eqs. (35) and (36), ε is the polarization vector for $P^*_{Q_j}$. Eqs. (35) and (36) give the most general form of these matrix elements consistent with Lorentz invariance and the parity conservation of the strong interactions. Since we are in the effective theory $\tilde{f}, \tilde{a}_\pm$ and $\tilde{g}$ are functions of $v\cdot v'$, independent of the heavy quark masses. Using the spin symmetry, all of these form factors can be related to $\tilde{f}_+$. Consider, for example, the relation (see eq. (25c))

$$\langle P^*_{Q_j}|\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v|P_{Q_i}(v)\rangle = 2\langle S^z_{Q_j}P_{Q_j}|\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v|P_{Q_i}(v)\rangle \ . \quad (37)$$

In eq. (37) the final states are at rest and $P^*_{Q_j}$ has zero component of spin along the z axis (this corresponds to a polarization vector $\varepsilon^\mu = (0,0,0,1)$); by omitting the four-velocity subscript on $\bar{h}^{(j)}_{v'}$ (and the velocity argument on the states) we are implying that $v'^\mu = (1,\vec{0})$. Since $S^z_{Q_j}$ is a hermitian operator and the initial state contains no heavy quarks of type Q_j,

$$\langle P^*_{Q_j}|\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v|P_{Q_i}(v)\rangle = 2\langle P_{Q_j}|[S^z_{Q_j},\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v]|P_{Q_i}(v)\rangle \ . \quad (38)$$

Using the commutation relation

$$[S^z_{Q_j},\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v] = -\frac{1}{2}\bar{h}^{(j)}\gamma_0 h^{(i)}_v \quad (39)$$

we have that

$$\langle P^*_{Q_j}|\bar{h}^{(j)}\gamma_3\gamma_5 h^{(i)}_v|P_{Q_i}(v)\rangle = -\langle P_{Q_j}|\bar{h}^{(j)}\gamma_0 h^{(i)}_v|P_{Q_i}(v)\rangle \ . \quad (40)$$

Eqs. (27), (33), (35), and (40) imply

$$\tilde{f} + (\tilde{a}_+ + \tilde{a}_-)(v^3)^2 = \tilde{f}_+(1+v^0) \ , \quad (41)$$

in the frame where $v' = (1,\vec{0})$. Eq. (41) yields, in a general frame,

$$\tilde{a}_+ + \tilde{a}_- = 0 \ , \quad \tilde{f} = (1+v\cdot v')\tilde{f}_+ \ . \quad (42)$$

logarithms of (m_{Q_i}/Λ_{QCD}) in its matrix elements. These can be transferred from the matrix elements of the operator to a coefficient by scaling μ down from m_{Q_i} to the QCD scale. In the leading logarithmic approximation, the relationship between V_ν and $O_{\gamma_\nu}(\mu)$ has the form

$$V_\nu = C_i(\mu) O_{\gamma_\nu}(\mu) \ . \tag{53}$$

where the μ dependence of C_i cancels that of O_{γ_ν}. According to eq. (52) , C_i obeys the matching condition

$$C_i(m_{Q_i}) = 1 + O(\alpha_s(m_{Q_i})) \ . \tag{54}$$

Detailed calculations[2,5] give that

$$C_i(\mu) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(\mu)}\right]^{-6/(33-2N)} \ . \tag{55}$$

where N is the number of light flavors appropriate to the momentum interval between m_{Q_i} and μ. Similar relationships hold for other operators $\bar{q}_j \Gamma Q_i$.

As another example, relevant to the matrix elements discussed in Section 4, we consider the relation between the operator $V_\nu = \bar{Q}_j \gamma_\nu Q_i$ in the full theory of QCD, and the operator $T_{\gamma_\nu} = \bar{h}_{v'}^{(j)} \gamma_\nu h_v^{(i)} +$ counter term in the effective theory. We imagine that $m_{Q_i} >> m_{Q_j}$. In this case one finds[17,18] that, for $\mu < m_{Q_j}$

$$\bar{Q}_j \gamma_\nu Q_i = C_{ji}(\mu) T_{\gamma_\nu}(\mu) \ , \tag{56}$$

where $T_{\gamma_\nu} = \bar{h}_{v'}^{(j)} \gamma_\nu h_v^{(i)} +$ counter term, and

$$C_{ji}(\mu) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(m_{Q_j})}\right]^{-6/(33-2N)} \left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(\mu)}\right]^{a_L(v\cdot v')} \ , \tag{57}$$

where

$$a_L(v \cdot v') = \frac{8}{(33-2N)} [v \cdot v' r(v \cdot v') - 1] \ . \tag{58}$$

In eq. (58), N is the number of light quarks appropriate to the momentum interval between m_{Q_j} and μ. An identical formula, with the same coefficient, holds for the relationship between the axial current $\bar{Q}_j \gamma_\nu \gamma_5 Q_i$ and $T_{\gamma_\nu \gamma_5}$.

The results of this section and those of Section 4 can be combined to give matrix elements that are relevant for experiment. The exclusive decays $\bar{B} \to D e \bar{\nu}_e$ and $\bar{B} \to D^* e \bar{\nu}_e$ are determined by the V_{cb} element of the Cabibbo–Kobayashi–Maskawa matrix and by the hadronic matrix elements

$$\frac{\langle D(v')|\bar{c}\gamma_\mu b|\bar{B}(v)\rangle}{\sqrt{m_B m_D}} = C_{cb} \xi(v \cdot v')(v + v')_\mu \ , \tag{59}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu \gamma_5 b|\bar{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = C_{cb} \xi(v \cdot v')[(1 + v \cdot v')\epsilon^*_\mu - (\epsilon^* \cdot v) v'_\mu], \tag{60}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|\bar{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = i C_{cb} \xi(v \cdot v') \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta \ . \tag{61}$$

The subtraction point dependence of ξ cancels that of C_{cb}. At zero recoil

$$\xi(1) = 1 \ . \tag{62}$$

The present data on semileptonic $\bar{B} \to D e \bar{\nu}_e$ and $\bar{B} \to D^* e \bar{\nu}_e$ decays are consistent with the predictions in eqs. (59)-(62). The exclusive decay $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ is determined by the V_{cb} element of the Cabibbo–Kobayashi–Maskawa matrix and the matrix element

$$\langle \Lambda_c(v',s')|\bar{c}\gamma_\mu(1-\gamma_5)b|\Lambda_b(v,s)\rangle = C_{cb} \eta(v \cdot v') \bar{u}(v',s)\gamma_\mu(1-\gamma_5)u(v,s) \ . \tag{63}$$

Again, the subtraction point dependence of η cancels that of C_{cb}, and at zero recoil the flavor symmetry of the effective theory implies the normalization

$$\eta(1) = 1 \ . \tag{64}$$

The predictions in eqs. (59)-(64) can be used to extract the value of $|V_{cb}|$ from experimental data on exclusive semileptonic B and Λ_b decay.

The decay constant $f_{P_{Q_i}}$ of the heavy $Q_i \bar{q}$ meson P_{Q_i} is defined by

$$\langle 0|\bar{q}\gamma_\nu \gamma_5 Q_i|P_{Q_i}(p)\rangle = f_{P_{Q_i}} p_\nu \ . \tag{65}$$

Using eqs. (49) and (55) one has the relationship

$$f_B = \sqrt{\frac{m_D}{m_B}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} f_D \ . \tag{66}$$

The decay constant of the D meson, f_D, should be measured in the future from the leptonic D decay, $D \to \bar{\mu}\nu_\mu$. At present the limit is $f_D \le 290$ MeV.

The heavy quark methods open an interesting avenue for determining the magnitude of the V_{ub} element of the Cabibbo–Kobayashi–Maskawa matrix.[6] Ordinary isospin symmetry plus heavy quark symmetry implies that, for example,

$$\langle \rho(k,\epsilon)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(v)\rangle$$

$$= \left(\frac{m_B}{m_D}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{6/25}\langle\rho(k,\epsilon)|\bar{d}\gamma_\mu(1-\gamma_5)c|D(v)\rangle\ . \qquad (67)$$

Eq. (67) is valid in the rest frame of the $\bar{B}$ and D for momentum k small compared with the heavy quark masses.* Since in the Cabibbo-suppressed semileptonic decay $D \to \rho\bar{e}\nu_e$ the weak mixing angles are known, the right side of eq. (67) can be determined experimentally. With this information, experimental data on $\bar{B} \to \rho e\bar{\nu}_e$ will allow a determination of $|V_{ub}|$ (see Fig. 7). If one uses light quark $SU(3)_V$ flavor symmetry instead of isospin, then the Cabibbo allowed semileptonic decay $D \to K^*\bar{e}\nu_e$ can be used. The form factors for this decay have already been determined experimentally.[20] Of course this strategy can be used for any convenient light hadronic final state.

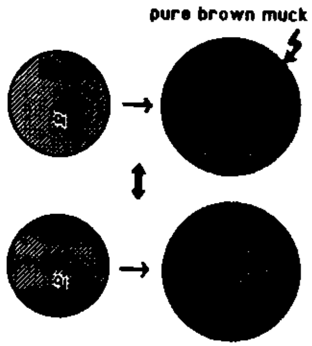

FIG. 7: heavy-to-light transitions are also related by the symmetry

It is possible to systematically improve order by order in $\alpha_s(m_c)$ and $\alpha_s(m_b)$ the matching between operators in the full theory of QCD and operators in the effective theory.[17,21] This gives calculable perturbative corrections and does not cause any loss of predictive power. (For example, the matrix elements for the decays $\bar{B} \to De\bar{\nu}_e$ and $\bar{B} \to D^*e\bar{\nu}_e$ are still expressed in terms of a single universal function, $\xi(v{\cdot}v')$, that is normalized to unity at zero recoil: $\xi(1) = 1$). The leading perturbative corrections have been calculated and are typically of order 10%. In this Section we have for pedagogical reasons treated m_c/m_b as a small quantity. It has been argued[22] that it is more accurate to go over to the effective theory in one step in which both the b and c quarks are treated as heavy, keeping in the perturbative $\alpha_s(m_b)$ corrections to the matching conditions the full dependence on m_c/m_b.

* Model calculations suggest that eq. (67) holds, even for k comparable with the heavy charm quark mass.[19]

6. STATUS AND PROSPECTS

In this brief introduction, I have not had time to mention many of the important developments that have taken place in this subject in the last year. Foremost among these must be the study of $1/m_Q$ effects[23–28], which has, among other things, revealed that some predictions of heavy quark symmetry receive *no* $1/m_Q$ corrections ("Luke's Theorem"). Bjorken's sum rule[10,29] for the slope of ξ and the work of Dugan and Grinstein on factorization [30,31] should also be singled out for mention. In addition, the recent literature contains applications of the heavy quark methods to inclusive semileptonic decays,[10,33] semileptonic Λ_b decays to excited charmed baryons[33], nonleptonic $\bar{B}$ (and Λ_b) decays[34,30,31,35], semileptonic Λ_c decay[36], heavy meson pair production in e^+e^- annihilation[37], and rare $\bar{B}$ meson decays[38].

The ultimate utility of the ideas presented here will depend largely on the size of the Λ_{QCD}/m_c corrections *. The experimental successes we have found suggest that, in at least some cases, they are not anomalously large. Theoretically, we know that certain zero recoil predictions are uncorrected in leading order in Λ_{QCD}/m_c. In the case of $\Lambda_b \to \Lambda_c e\bar{\nu}_e$ decay, it is remarkable that most of the predictive power over all momentum transfers is retained when Λ_{QCD}/m_c corrections are included. Nevertheless, we must expect that, as was the case for light flavor $SU(3)$, it will require some experience before we can gauge the reliability and range of applicability of heavy quark symmetry.

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* Some lattice Monte Carlo calculations suggest that there are large Λ_{QCD}/m_c corrections to the relation between decay constants in eq. (66) . For a review see Ref. 39.

logarithms of (m_{Q_i}/Λ_{QCD}) in its matrix elements. These can be transferred from the matrix elements of the operator to a coefficient by scaling μ down from m_{Q_i} to the QCD scale. In the leading logarithmic approximation, the relationship between V_ν and $O_{\gamma_\nu}(\mu)$ has the form

$$V_\nu = C_i(\mu) O_{\gamma_\nu}(\mu) \ , \tag{53}$$

where the μ dependence of C_i cancels that of O_{γ_ν}. According to eq. (52) , C_i obeys the matching condition

$$C_i(m_{Q_i}) = 1 + \mathcal{O}(\alpha_s(m_{Q_i})) \ . \tag{54}$$

Detailed calculations[2,5] give that

$$C_i(\mu) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(\mu)}\right]^{-6/(33-2N)} \ . \tag{55}$$

where N is the number of light flavors appropriate to the momentum interval between m_{Q_i} and μ. Similar relationships hold for other operators $\bar{q}_j \Gamma Q_i$.

As another example, relevant to the matrix elements discussed in Section 4, we consider the relation between the operator $V_\nu = \bar{Q}_j \gamma_\nu Q_i$ in the full theory of QCD, and the operator $T_{\gamma_\nu} = \bar{h}_{v'}^{(j)} \gamma_\nu h_v^{(i)} +$ counter term in the effective theory. We imagine that $m_{Q_i} >> m_{Q_j}$. In this case one finds[17,18] that, for $\mu < m_{Q_j}$

$$\bar{Q}_j \gamma_\nu Q_i = C_{ji}(\mu) T_{\gamma_\nu}(\mu) \ , \tag{56}$$

where $T_{\gamma_\nu} = \bar{h}_{v'}^{(j)} \gamma_\nu h_v^{(i)} +$ counter term, and

$$C_{ji}(\mu) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(m_{Q_j})}\right]^{-6/(33-2N)} \left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(\mu)}\right]^{a_L(v\cdot v')} \ , \tag{57}$$

where

$$a_L(v \cdot v') = \frac{8}{(33-2N)} [v \cdot v' r(v \cdot v') - 1] \ . \tag{58}$$

In eq. (58), N is the number of light quarks appropriate to the momentum interval between m_{Q_j} and μ. An identical formula, with the same coefficient, holds for the relationship between the axial current $\bar{Q}_j \gamma_\nu \gamma_5 Q_i$ and $T_{\gamma_\nu \gamma_5}$.

The results of this section and those of Section 4 can be combined to give matrix elements that are relevant for experiment. The exclusive decays $\bar{B} \to D e \bar{\nu}_e$ and $\bar{B} \to D^* e \bar{\nu}_e$ are determined by the V_{cb} element of the Cabibbo–Kobayashi–Maskawa matrix and by the hadronic matrix elements

$$\frac{\langle D(v')|\bar{c}\gamma_\mu b|\bar{B}(v)\rangle}{\sqrt{m_B m_D}} = C_{cb} \xi(v \cdot v')(v + v')_\mu \ , \tag{59}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu \gamma_5 b|\bar{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = C_{cb} \xi(v \cdot v')[(1 + v \cdot v')\epsilon^*_\mu - (\epsilon^* \cdot v) v'_\mu], \tag{60}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|\bar{B}(v)\rangle}{\sqrt{m_B m_{D^*}}} = i C_{cb} \xi(v \cdot v') \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta \ . \tag{61}$$

The subtraction point dependence of ξ cancels that of C_{cb}. At zero recoil

$$\xi(1) = 1 \ . \tag{62}$$

The present data on semileptonic $\bar{B} \to D e \bar{\nu}_e$ and $\bar{B} \to D^* e \bar{\nu}_e$ decays are consistent with the predictions in eqs. (59)-(62). The exclusive decay $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ is determined by the V_{cb} element of the Cabibbo–Kobayashi–Maskawa matrix and the matrix element

$$\langle \Lambda_c(v', s')|\bar{c}\gamma_\mu(1-\gamma_5) b|\Lambda_b(v,s)\rangle = C_{cb} \eta(v \cdot v') \bar{u}(v', s)\gamma_\mu(1-\gamma_5) u(v,s) \ . \tag{63}$$

Again, the subtraction point dependence of η cancels that of C_{cb}, and at zero recoil the flavor symmetry of the effective theory implies the normalization

$$\eta(1) = 1 \ . \tag{64}$$

The predictions in eqs. (59)-(64) can be used to extract the value of $|V_{cb}|$ from experimental data on exclusive semileptonic B and Λ_b decay.

The decay constant $f_{P_{Q_i}}$ of the heavy $Q_i \bar{q}$ meson P_{Q_i} is defined by

$$\langle 0|\bar{q}\gamma_\nu \gamma_5 Q_i|P_{Q_i}(p)\rangle = f_{P_{Q_i}} p_\nu \ . \tag{65}$$

Using eqs. (49) and (55) one has the relationship

$$f_B = \sqrt{\frac{m_D}{m_B}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} f_D \ . \tag{66}$$

The decay constant of the D meson, f_D, should be measured in the future from the leptonic D decay, $D \to \bar{\mu}\nu_\mu$. At present the limit is $f_D \le 290$ MeV.

The heavy quark methods open an interesting avenue for determining the magnitude of the V_{ub} element of the Cabibbo–Kobayashi–Maskawa matrix.[4] Ordinary isospin symmetry plus heavy quark symmetry implies that, for example,

$$\langle \rho(k,\epsilon)|\bar{u}\gamma_\mu(1-\gamma_5) b|\bar{B}(v)\rangle$$

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