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The Continuous Electron Beam Accelerator Facility
Theory Group Preprint Series

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The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150

CEBAF-TH-92-06

Photoproduction amplitudes of P_{11} and P_{33} baryon resonances in the quark model

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Predictions are made for the photoproduction amplitudes of low-lying $N\frac{1}{2}^+$ (P_{11}) and $\Delta\frac{3}{2}^+$ (P_{33}) resonances, using a quark model with relativistic corrections to the transition operator, and mixed nonrelativistic wavefunctions which are correctly orthogonal to the ground states. These amplitudes are also calculated using relativized model wavefunctions. The results for the Roper resonance N(1440) are in marked disagreement with the data.

I. INTRODUCTION

Recently Close and Li [1] and Warns, Schröder, Pfeil, and Rollnik [2] have calculated the photoproduction (and electroproduction) amplitudes of the nucleon and delta baryon resonances in models which use the Isgur-Karl (IK) model [3–5] wavefunctions for the resonances, and which add relativistic corrections to the transition operator. One of these corrections had been applied to the calculation of photoproduction amplitudes with unmixed oscillator wavefunctions by Kubota and Ohta [6]. The effects of hyperfine mixings on these amplitudes, calculated with the non-relativistic operator, were calculated by Koniuk and Isgur [7]. Forsyth and Cutkosky [8] also performed a similar calculation using a more general operator, and Sartor and Stancu[9] have calculated with the non-relativistic operator but with a more sophisticated basis for the wavefunctions. Refs. [7–9] used wavefunctions for the ground states N(938) and $\Delta(1232)$ *and* their excited states which include mixings between the $N=0$ and $N=2$ bands brought about by color-hyperfine interactions. Note that the previously published Isgur-Karl model [4] wavefunctions for the $N=2$ band *excited* states with $J^P=\frac{1}{2}^+$ and $J^P=\frac{3}{2}^+$ do not include mixings with the $N=0$ band, and so are not orthogonal to the ground states. It is therefore incorrect to use the mixed ground states [5] and these excited states in the same calculation, as is done in Refs. [1, 2], and doing so leads to erroneous results for these excited states.

In this paper the Isgur-Karl model wavefunctions of the ground and excited $N\frac{1}{2}^+$ and $\Delta\frac{3}{2}^+$ states are formed by diagonalising an energy matrix which includes the hyperfine terms. The resulting corrected predictions with IK model wavefunctions and using Close and Li's transition operator are calculated. Another possible source of error is that this model ignores mixings between the ground states and the radially excited states, brought about by the presence of anharmonicities in the spin-independent potential. For example, the IK model Roper resonance has a large negative anharmonic perturbation on its mass, but its wavefunction is unaffected; this might lead to erroneous results. This possibility is examined here by estimating these couplings using the wavefunctions which result from a relativized spectroscopic model [10], along with Close and Li's

transition operator. In the relativized model the wavefunctions are expanded in a large oscillator basis and the resulting Hamiltonian matrix is diagonalized, with the result that (apart from basis truncation beyond $N=6$) the spin-independent potential is treated without wavefunction perturbation theory.

There has been a lot of interest in two of the states considered here, the Roper resonance N(1440) and the $\Delta(1600)$, because of their relatively poor description in spectroscopic models [4, 8–11]. There has also been some controversy about whether or not there are two states in the partial wave analyses [12] at the mass of the Roper resonance, which now appears unlikely [13]. Models exist which describe one or both of these states as hybrid baryons [14]. For this reason it is crucial to see whether or not their photocouplings calculated in the conventional quark model are compatible with the data.

Ohta [15] has calculated the photocouplings of the Roper resonance and the $\Delta(1600)$, along with those of the $\Delta(1232)$ to normalize some of the parameters, and gets good agreement with the data for the Roper and $\Delta(1232)$. The results for the $\Delta(1600)$ are large and incompatible with the recent data [16]. The model is similar to that of Ref. [1], except that it carries out an expansion to one higher order in p/m, and includes explicit contributions from the vector potential. The effects of interband mixing on the amplitudes are neglected. It has not been extended to the other measured photocouplings, so there remains a question about whether the agreement for the Roper resonance survives in a model confronted with all of the data. Gavela *et al.* [17] have also calculated the size of the two P_{11} resonance photocouplings using a $^3P_0$ quark-pair-creation model to create ρ and ω mesons, which then couple to the photon (vector dominance). Their model also gives good agreement for the Roper photocouplings, although their other prediction (for the N(1710)) is not as good when compared to recent data. Similar questions remain about the model's global applicability.

The next two sections describe the Isgur-Karl and relativized-model wavefunctions used here, and Close and Li's corrected transition operator. These are followed by a description of the results, and a section discussing these and drawing conclusions.

II. WAVEFUNCTIONS

A. Nonrelativistic wavefunctions

To generate the mixed wavefunctions for the $\frac{1}{2}^+$ nucleon and $\frac{3}{2}^+$ delta resonances we must diagonalise the color hyperfine interaction matrix in these sectors. There are five nucleon states with $J^P = \frac{1}{2}^+$ up to $N = 2$ in the nonrelativistic model, which we label $N^2S_S\frac{1}{2}^+$ (the $N = 0$ state), $N^2S_{S'}\frac{1}{2}^+$, $N^2S_M\frac{1}{2}^+$, $N^4D_M\frac{1}{2}^+$, and $N^2P_A\frac{1}{2}^+$. Here the notation and conventions of Isgur and Karl's paper on the positive-parity excited states [4] are used. The hyperfine energy matrix which results [4, 5] is,

$$\begin{array}{cc} & \begin{array}{ccccc} N^2S_S\frac{1}{2}^+ & N^2S_{S'}\frac{1}{2}^+ & N^2S_M\frac{1}{2}^+ & N^4D_M\frac{1}{2}^+ & N^2P_A\frac{1}{2}^+ \end{array} \\ \begin{array}{c} N^2S_S\frac{1}{2}^+ \\ N^2S_{S'}\frac{1}{2}^+ \\ N^2S_M\frac{1}{2}^+ \\ N^4D_M\frac{1}{2}^+ \\ N^2P_A\frac{1}{2}^+ \end{array} & \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{4} & \frac{\sqrt{6}}{4} & \frac{\sqrt{15}}{20} & 0 \\ & -\frac{5}{8} & \frac{-\sqrt{2}}{8} & -\frac{\sqrt{5}}{40} & 0 \\ & & -\frac{5}{16} & \frac{\sqrt{10}}{16} & 0 \\ & & & \frac{1}{8}-\frac{9}{40} & -\frac{3\sqrt{10}}{40} \\ & & & & 0 \end{bmatrix} \end{array} \qquad (1)$$

where the matrix elements are in units of

$$\delta = \frac{4\alpha_s\alpha^3}{3\sqrt{2\pi}m_d^3}, \qquad (2)$$

and the matrix is symmetric. The off-diagonal matrix elements involving the $L = 2$ state $N^4D_M\frac{1}{2}^+$ (and the second part of its diagonal matrix element) are due to the tensor interaction, whereas all the others arise from the contact interaction. Similarly there are four delta states with $J^P = \frac{3}{2}^+$ up to $N = 2$, which are $\Delta^4S_S\frac{3}{2}^+$ (the $N = 0$ state), $\Delta^4S_{S'}\frac{3}{2}^+$, $\Delta^4D_S\frac{3}{2}^+$, and $\Delta^2D_M\frac{3}{2}^+$, with the hyperfine energy matrix

$$\begin{array}{cc} & \begin{array}{cccc} \Delta^4S_S\frac{3}{2}^+ & \Delta^4S_{S'}\frac{3}{2}^+ & \Delta^4D_S\frac{3}{2}^+ & \Delta^2D_M\frac{3}{2}^+ \end{array} \\ \begin{array}{c} \Delta^4S_S\frac{3}{2}^+ \\ \Delta^4S_{S'}\frac{3}{2}^+ \\ \Delta^4D_S\frac{3}{2}^+ \\ \Delta^2D_M\frac{3}{2}^+ \end{array} & \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{4} & \frac{\sqrt{30}}{20} & -\frac{\sqrt{15}}{20} \\ & \frac{5}{8} & -\frac{\sqrt{10}}{40} & \frac{\sqrt{5}}{40} \\ & & \frac{1}{4} & \frac{\sqrt{2}}{20} \\ & & & \frac{1}{8} \end{bmatrix} \end{array} \qquad (3)$$

Again the off-diagonal terms involving the D states are tensor interactions and all the others are contact interactions. In order to extract the wavefunctions for these states we must also add in the diagonal energies which result in the model from harmonic oscillator energies plus anharmonic perturbations. Since the original spectroscopy was done [4] without the hyperfine mixings between the $N = 0$ and $N = 2$ band states, the resulting energies would differ from the results of Isgur and Karl. In order to obtain the best estimate of the hyperfine mixings in the wavefunctions the following procedure is adopted here: the $N\frac{1}{2}^+$ matrix is diagonalised iteratively with floating diagonal entries for the first three states, subject to the requirement that the lowest energy eigenvalues turn out to be 938 MeV, 1440 MeV and 1710 MeV, the physical masses of the states they represent. The last two diagonal entries are set at $E(70, 2^+) + (1/8 - 9/40)\delta = 1905$ MeV and $E(20, 1^+) = 2020$ MeV, where $E(70, 2^+)$ and $E(20, 1^+)$ are the spin-independent model energies of these last two states which couple weakly to πN and so have no experimental counterparts. The result is the eigenvector matrix

$$\begin{array}{cc} \begin{array}{ccccc} 938 & 1440 & 1710 & 1898 & 2058 \end{array} & \\ \begin{bmatrix} 0.9246 & 0.2958 & 0.2035 & 0.1218 & -0.0423 \\ -0.2901 & 0.9551 & -0.0491 & -0.0319 & 0.0114 \\ -0.2426 & -0.0147 & 0.9015 & 0.3445 & -0.0979 \\ -0.0459 & -0.0006 & -0.3697 & 0.8030 & -0.4651 \\ -0.0030 & -0.0001 & -0.0849 & 0.4697 & 0.8787 \end{bmatrix} & \begin{array}{c} N^2S_S\frac{1}{2}^+ \\ N^2S_{S'}\frac{1}{2}^+ \\ N^2S_M\frac{1}{2}^+ \\ N^4D_M\frac{1}{2}^+ \\ N^2P_A\frac{1}{2}^+ \end{array} \\ \begin{array}{ccccc} 1029 & 1399 & 1690 & (1905) & (2020) \end{array} & \end{array} \qquad (4)$$

where the first row gives the eigenvalues, the eigenvectors are listed by columns and the last row gives the final diagonal entries. The resulting masses for the heaviest two states, and the N(938) wavefunction, are quite similar to those of Refs. [4, 5]. The mixings of $N^2S_M\frac{1}{2}^+$ into N(1440) and of $N^2S_{S'}\frac{1}{2}^+$ into N(1710) have changed substantially (the former changing sign) relative to Ref. [4], due to interband mixings.

A similar process is carried out for the $\Delta\frac{3}{2}^+$ matrix, with a floating first diagonal entry constrained by the requirement that the lowest eigenvalue is 1232 MeV. Since quark model values for the mass of the first excited $\Delta\frac{3}{2}^+$ state are consistently higher [4, 8–10] than the mass of the two-star state $\Delta(1600)$, and it is not possible to constrain the third eigenvalue to its physical value (1920 MeV), the remaining diagonal entries are taken at their model values, $E(56', 0^+) + 5\delta/8 =$ 1788 MeV, $E(56, 2^+) + \delta/4 = 1925$ MeV, and $E(70, 2^+) + \delta/8 = 1972$ MeV. The corresponding eigenvector matrix is

$$\begin{array}{c} \begin{matrix} 1232 & 1799 & 1946 & 1983 \end{matrix} \\ \begin{bmatrix} 0.9667 & -0.1601 & 0.1844 & -0.0765 \\ 0.2192 & 0.9205 & -0.3043 & 0.1098 \\ -0.1094 & 0.3076 & 0.9238 & 0.2002 \\ 0.0740 & -0.1802 & -0.1416 & 0.9706 \end{bmatrix} \begin{matrix} \Delta^4 S_S \frac{3}{2}^+ \\ \Delta^4 S_{S'} \frac{3}{2}^+ \\ \Delta^4 D_S \frac{3}{2}^+ \\ \Delta^2 D_M \frac{3}{2}^+ \end{matrix} \\ \begin{matrix} 1275 & (1788) & (1925) & (1972) \end{matrix} \end{array} \qquad (5)$$

with masses and the $\Delta(1232)$ wavefunction similar to those of Refs. [4, 5].

B. Relativized model wavefunctions

Details of determining the wavefunctions for these states from the relativized model, and a description of the model, can be found in Ref. [10]. The important difference between this model and the Isgur-Karl model, for the purposes of this work, is the more realistic treatment of the spin-independent potential. The successful spectroscopy of the nonrelativistic model in light quark systems can be rationalized (given the obvious importance of relativistic effects in systems with $p/m \simeq 1$) by noting that the model uses effective values of the parameters, which are able to make up for some of the deficiencies of a nonrelativistic treatment. An example is the (constituent) quark mass which in the nonrelativistic model is an effective mass containing some of the kinetic energy of the quark (as well as 'dressing' of the current quark mass from QCD). Another is the strong coupling constant, which has a large size to compensate for deficiencies in the treatment of the relativistic dynamics of bound spinors and the perturbative solution of the dynamical problem. In the IK model [4] the spectroscopy of the states considered here is driven mainly by the spin-independent potential. This splits the $N = 2$ band states into a pattern independent of the form of the potential, in first order in its anharmonic part. The next most important effect is the hyperfine interaction.

The effects on the wavefunction from the anharmonic terms are not included in the IK model, although the effects of the hyperfine interaction (as calculated above) are. The size of the anharmonic 'perturbations' on the spectroscopy, which for the Roper resonance, for example, are larger than the zeroth-order oscillator splitting, should warn us that the effects on the wavefunctions may be large. The relativized model deals with these effects by diagonalizing the full potential in a large harmonic oscillator basis (three bands above the ground state). The spin-independent potential is that which results from adding the lengths of a minimum-length Y-shaped string between the quarks, and multiplying by (roughly) the meson string tension. The hyperfine interaction is also dealt with differently. For details see Ref. [10].

The spectroscopy that results from this process is comparable to that of the IK model, which is encouraging given the lack of freedom to fit band centers of mass, *etc.* The spectroscopy of two of the states considered here (compared to other non-strange states in this band) is perhaps the most problematic in both nonrelativistic and relativized models. The Roper resonance is predicted to be about 100 MeV heavier than experiment, although it fits quite well into the pattern of splitting of the states in its band (whose centre of mass is about 40 MeV too high). Also the mass of the lightest excited $\Delta\frac{3}{2}^+$ is about 190 MeV too high (or it needs to drop another 150 MeV out of its band), as in nonrelativistic models [4, 8]. Recent partial wave analyses by Arndt, Li, Roper, Workman, and Ford at VPI [18] and Manley and Saleski at Kent State [19] have confirmed the existence of the latter state, so it remains a problem for the quark model. This work is motivated, in part, by noting that the study of the photocouplings of these states is a particularly sensitive way to determine if new physics is needed to explain their nature.

III. TRANSITION OPERATOR

The photon transition operator used here builds upon the nonrelativistic operator by the addition of terms which must be included in an expansion to $O(p/m)^2$, which supplement the usual orbit-flip (or convection) plus spin-flip terms with spin-orbit [20, 6] and two-body[20, 15, 1] terms. The convection term is also rewritten in order to avoid explicit dependence of the transition operator on the vector-exchange part of the binding potential [21, 1]. Similarly, explicit dependence on the scalar part of the binding potential can be avoided by including kinetic and scalar-binding energies in the effective quark mass m^*. The resulting transition operator

$$H=\sum_{i=1}^{3}\Big\{-e_i\mathbf{r}_i\cdot\mathbf{E}_i+i\frac{e_i}{2m^*}\left(\mathbf{p}_i\cdot\mathbf{k}_i\mathbf{r}_i\cdot\mathbf{A}_i+\mathbf{r}_i\cdot\mathbf{A}_i\mathbf{p}_i\cdot\mathbf{k}_i\right)-\mu_i\boldsymbol{\sigma}_i\cdot\mathbf{B}_i$$
$$-\frac{1}{2m^*}\left(2\mu_i-\frac{e_i}{2m^*}\right)\frac{\boldsymbol{\sigma}_i}{2}\cdot[\mathbf{E}_i\times\mathbf{p}_i-\mathbf{p}_i\times\mathbf{E}_i]\Big\}$$
$$+\sum_{i<j}\frac{1}{2M_Tm^*}\left(\frac{\boldsymbol{\sigma}_i}{2}-\frac{\boldsymbol{\sigma}_j}{2}\right)\cdot[e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j] \tag{6}$$

is then expanded to the same order as the spectroscopic Hamiltonian in the IK model. Here m^* is the the effective light-quark mass, e_i, $\sigma_i/2$, and $\mu_i=ge_i/2m^*$ are the charge, spin, and magnetic moment of the quark i, and $\mathbf{A}_i:=\mathbf{A}(\mathbf{r}_i)$. The baryon system recoils with mass M_T. Close and Li argue that it is necessary to add these terms if one also considers the mixings in the wavefunctions brought about by the strong-hyperfine interaction, which is itself of $O(p/m)^2$. They are also necessary [20] if the electromagnetic interaction is to satisfy the low-energy theorems in Compton scattering, and the Drell-Hearn-Gerasimov sum rule.

For ease of calculation it is useful to write the transition operators to be used between quark model states in a slightly different form from those of Ref. [1]. By insertion of the usual radiation field for the absorption of a photon into Eq. 6, and then integrating over the baryon center of mass coordinate, the transverse photo-excitation amplitudes can be written as simple expectation values over flavor, spin, and spatial internal coordinates

$$A_\lambda^N=3\langle X;J\lambda|H_3|N;\tfrac{1}{2}\,\lambda-1\rangle. \tag{7}$$

Here the initial photon has a momentum $\mathbf{k}||\hat{\mathbf{z}}$, the initial nucleon has a momentum $\mathbf{P}_i||\hat{\mathbf{z}}$, and the angular momenta are quantized along $\hat{\mathbf{z}}$. The exchange symmetry of the IK model wavefunctions has been used to replace a sum over quarks with three times the third-quark expectation value [22]. The operator H_3 can be written in the form $H_3=H_3^{\rm nr}+H_3^{\rm vp}+H_3^{\rm so}+H_{12}^{2b}$, with the nonrelativistic (nr) operator

$$H_3^{\rm nr}=-\frac{e_3}{m^*}\frac{1}{\sqrt{2}}\sqrt{\frac{2\pi}{k_0}}\left(\sqrt{\frac{2}{3}}p_{\lambda+}-gk\frac{\sigma_{3+}}{2}\right)e^{-ik\sqrt{\frac{2}{3}}\lambda_z}. \tag{8}$$

Here k_0 is the 0-component of the photon four-momentum (equal to $k=|\mathbf{k}|$ for real photons), and the momenta $\mathbf{p}_\rho$ and $\mathbf{p}_\lambda$ are conjugate to the Jacobi three-body coordinates $\boldsymbol{\rho}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$ and $\boldsymbol{\lambda}=(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)/\sqrt{6}$. The derivative operator here is that which arises from the usual convection Hamiltonian

$$H^{\rm conv}=-\sum_{i=1}^{3}\frac{e_i}{2m^*}\left(\mathbf{p}_i\cdot\mathbf{A}_i+\mathbf{A}_i\cdot\mathbf{p}_i\right). \tag{9}$$

The difference $H^{\rm vp}$ between the rewritten convection term, (the first two terms in Eq. 6) and $H^{\rm conv}$, is a relativistic correction to the transition Hamiltonian due to the presence of the vector-exchange part of the binding potential; it can be written in the form

$$H_3^{\rm vp}=e_3\sqrt{\frac{2\pi}{k_0}}\sqrt{\frac{2}{3}}\Big\{\left(\frac{-i}{\sqrt{2}}\lambda_+\right)\left[k_0+\frac{k}{m^*}\left(\sqrt{\frac{2}{3}}p_{\lambda z}+\frac{k}{6}-\frac{P_i}{3}\right)\right]$$
$$-\frac{1}{m^*}\left(\frac{-p_{\lambda+}}{\sqrt{2}}\right)\Big\}e^{-ik\sqrt{\frac{2}{3}}\lambda_z}, \tag{10}$$

where $P_i=|\mathbf{P}_i|$. Similarly the spin-orbit (so) operator may be written

$$H_3^{\rm so}=(2g-1)\frac{e_3\sqrt{\pi k_0}}{2m^{*2}}\left\{\frac{\sigma_{3+}}{2}\left(\sqrt{\frac{2}{3}}p_{\lambda z}+\frac{k}{6}-\frac{P_i}{3}\right)-\frac{\sigma_{3z}}{2}\sqrt{\frac{2}{3}}p_{\lambda+}\right\}e^{-ik\sqrt{\frac{2}{3}}\lambda_z}, \tag{11}$$

and the two-body (2b) operator has the form

$$H_{13}^{\rm 2b}=\frac{e_3\sqrt{\pi k_0}}{2M_Tm^*}\Big\{\frac{\sigma_{\rho+}}{2}p_{\rho z}-\frac{\sigma_{\rho z}}{2}p_{\rho+}$$
$$+\frac{\sigma_{\lambda+}}{2}\left(p_{\lambda z}+\sqrt{\frac{2}{3}}[k+P_i]\right)-\frac{\sigma_{\lambda z}}{2}p_{\lambda+}\Big\}e^{-ik\sqrt{\frac{2}{3}}\lambda_z}, \tag{12}$$

where $\sigma_\rho = (\sigma_1 - \sigma_2)/\sqrt{2}$, and $\sigma_\lambda = (\sigma_1 + \sigma_2 - 2\sigma_3)/\sqrt{6}$.

The explicit appearance of P_i in Eq. 10, Eq. 11 and Eq. 12, and the nonrelativistic kinematics (the simple dependence on $k = |\mathbf{k}|$) demonstrate that amplitudes caclulated with these terms are frame dependent. Accordingly they are calculated in both the centre-of-momentum (cm) frame (where $P_i = -k$) and in the Breit (Br) frame (where $P_i = -k/2$) to estimate the error introduced by this lack of relativistic invariance, although the results in the Breit frame are theoretically preferable. For photoproduction we have

$$k_0 = k = \begin{cases} \frac{M_X^2 - M_N^2}{2M_X} & \text{cm frame} \\ \frac{M_X^2 - M_N^2}{\sqrt{2(M_X^2 + M_N^2)}} & \text{Breit frame.} \end{cases} \tag{13}$$

IV. RESULTS

Table I shows the results of this calculation using Isgur-Karl model wavefunctions, in both the centre of momentum and Breit frames. In all cases the results are for mixed initial and final state wavefunctions from Eqs. 4 and 5. The Breit frame results are shown decomposed into the pieces arising from $H^{\rm nr}$ (expressed as the sum of two terms: the convection term and the spin-flip term), $H^{\rm rp}$, $H^{\rm so}$, and $H^{\rm 2b}$. The signs of the photoproduction amplitudes for each resonance are determined by taking the product of the sign of the amplitude in Eq. 7 and the sign of the $X \to N\pi$ decay amplitude. There is also an extra conventional sign in the experimental amplitudes [16, 7] of -1(+1) for the photoproduction of an $N^*(\Delta^*)$. The simplest way to avoid miscalculating these signs is to calculate the πN amplitudes in some model using exactly the same wavefunctions used as input to the photoproduction calculation. This calculation has been done [23] here using the $^3P_0$ (quark-pair-creation) model and the above wavefunctions. The signs of the resulting πN amplitudes are stable to changes in the parameters away from those which best fit the pion partial-wave data. Table I also includes predictions for the photocouplings of states in Eq. 4 and 5 predicted by the quark model, which are unseen in the πN and photoproduction analyses ('missing' states). It is possible that evidence for them will be discovered at CEBAF. The results here indicate that they have rather small photocouplings and so should be difficult to

see in single-pion photoproduction (given their implicitly small πN couplings), but they may be visible in multi-pion final states like $\pi\Delta$, ρN, and ωN.

As a test of the computer program used to calculate these amplitudes, the calculation of Ref. [1] has been reproduced, for the case of *unmixed* IK model wavefunctions in the cm frame, for every resonance for which there exists data. The results agree in magnitude for all states, and in sign (apart from the overall sign of the $N\frac{3}{2}^+(1720)$ amplitudes [24]) in the case of all states *not* considered here. The signs of the N(1440), N(1710), $\Delta(1600)$ and $\Delta(1920)$ unmixed amplitudes all differ [17, 25] from those of Koniuk and Isgur. In their work [7] the signs of three of these amplitudes are fit by the sign of a reduced (πN) amplitude P_{0i}'; in Ref. [23] there is no such freedom, and the signs are predicted. In the case of the $\Delta(1600)$ the situation is complicated by a πN amplitude which changes sign (in the $^3P_0$ model of Ref. [23]) when the initial and final states are mixed as in Eqs. 4 and 5. None of the other signs arising from the $N\pi$ vertex change because of mixing.

The amplitudes for the Roper resonance N(1440) show a cancellation between a small nonrelativistic term and the sum of the spin-orbit and two-body terms. The resulting small amplitudes are quite insensitive to changes in the parameters of the model, and significantly far from the data. Kubota and Ohta's [6] calculation shows a similar cancellation, as does that of Ohta [15] [in his amplitudes calculated to $O(m^{-2})$], although all of his amplitudes are larger due to a large quark anomalous moment. Similar small results (with the opposite sign) for $A^p_{\frac{1}{2}}$ have been found in a light-cone model at $Q^2 = 0$ for radial excitations by Weber [26]. The situation for the N(1710) is somewhat better, given the errors in the data. However the N(1710) amplitudes are sensitive to changes in the wavefunction due to the mixings of higher bands into the wavefunctions, and are in better agreement with the data in the relativized model (as shown below).

The amplitudes for the ground state $\Delta(1232)$ are essentially unchanged from Ref. [1], as expected; those of the first excited state $\Delta(1600)$ are larger and in poorer agreement with the data. This is due to a large nonrelativistic term from mixing [9] with the $\Delta(1232)$ (not calculated in Ref. [7]), and spin-orbit and two-body terms which approximately cancel. The situation for the $\Delta(1920)$ is improved, and agrees well with the data, for which no errors are quoted [16].

Table II shows the amplitudes for these states calculated in the Breit frame with relativised model wavefunctions [27]. The process of relativisation, generally speaking, replaces quark masses in operators with simple functions of their kinetic energies, and smears the quark coordinates. Since the photocouplings dealt with here are all for light-quark states which are not highly excited, it should be a reasonable approximation to replace the quark kinetic energy with a constant. This corresponds, up to the addition of an average scalar-binding energy [1], to the effective mass m^* in Eq. 6. There is no reason, however, for this effective mass to be the same as that which appears in the model [7] with IK wavefunctions. Quark smearing has the effect of multiplying the photocoupling amplitudes by a nonrelativistic form factor which falls off as the three-momentum transferred to the quark increases. With the light quarks smeared with a Gaussian distribution of the same size as that used in the relativized model spectroscopy [10], this form factor falls off only a few percent over the range of M_X values considered here, and so will not affect the photocouplings. Accordingly the measured photocouplings have been fit [28] by varying the quark effective mass. The quark magnetic moments are maintained at the values needed for a simple additive explanation of the nucleon moments by co-variation of g, and the recoil mass in Eq. 12 is kept as $M_T = 3m^*$. The amplitudes in Table II are calculated with a modestly increased value of m^* which yields the best global fit to all of the measured photocouplings.

The cancellation which leads to small couplings for the N(1440) persists. The N(1710) amplitudes maintain their signs and are reduced, and are now in quite good agreement with the (rather uncertain) data. Since this simple fit does not change the quark magnetic moments, the amplitudes for the $\Delta(1232)$ are largely unchanged, although there has been some improvement arising from the relativized model wavefunctions [29]. The $\Delta(1600)$ amplitudes have been reduced in size, and (due to a change in sign of the πN amplitude) have changed sign. Because of the obvious sensitivity of the magnitude and sign of the $\Delta(1600)$ amplitudes to mixings, those arising from the relativized wavefunctions may be

the most trustworthy as they allow the wavefunctions of the initial and final states the most freedom to mix *via* the interactions. The photocouplings of the $\Delta(1920)$ are smaller but still positive. These results are summarised in Fig. 1, where $A_{\rm nr}$ and the total amplitudes from Table I (in the Breit frame) are plotted, along with the total amplitudes from Table II, and the data.

V. DISCUSSION AND CONCLUSIONS

It appears that this model is incapable of explaining the measured photocouplings of the Roper resonance N(1440). Comparison of Tables I and II show that the cancellation between the $H^{\rm nr}$ terms and the sum of the $H^{\rm so}$ and $H^{\rm 2b}$ terms is independent of details of initial and final wavefunctions (as long as they remain orthogonal). The size of these relativistic corrections relative to the $H^{\rm nr}$ terms in this case might call into question the convergence of such a p/m expansion. However the average size of the expectation of $H^{\rm nr}$ is significantly larger than that of $H^{\rm vp} + H^{\rm so} + H^{\rm 2b}$, when the photocouplings of *all* of the resonances are considered. The point of view taken here is that Eq. 6 represents a minimum set of tensor terms required by gauge invariance [20, 1], and so if their coefficients are viewed as free (subject to other constraints like the nucleon magnetic moments) the physics of the transition operator will have been efficiently parametrised. This appears to be borne out by an improvement of the fit to *all* of the photocoupling data upon addition of these terms with suitably adjusted parameters [1, 28]. As a consequence it would appear that, in this model, a conventional picture of the Roper resonance and its photocouplings is incompatible with the current data.

For the other states dealt with here the situation is less clear. The $\Delta(1600)$ amplitudes appear to be poorly described in the model with Isgur-Karl model wavefunctions, but the discrepancy diminishes when the relativised model wavefunctions are used. The other states for which which data exists seem to fit fairly well, given the rather uncertain data.

It is significant that, like the $\Delta(1232)$, the Roper resonance is a light state with a large [13] coupling to πN. It is therefore possible that virtual nucleon-pion loops [30] renormalise the photocouplings of these states. The absolute sizes of the error in the two cases are similar. It seems rather unlikely, given the lack of many such discrepant states below 2 GeV, that the Roper resonance and the $\Delta(1600)$ are pure hybrids. The next highest *experimental* states with these quantum numbers [N(1710) and $\Delta(1920)$] are quite well described in the conventional quark model and are too heavy to be assigned the lightest quark-model states. They could, however, be mixed [14] with relatively light hybrid states with these quantum numbers. More precise data on the photo- and electroproduction of the resonances considered here will be crucial in deciding among these possibilities.

VI. ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-76ER03066 and Contract No. DE-AC05-84ER40150. The author wishes to thank Richard Cutkosky and Zhenping Li for helpful discussions, Arun Gupta for his help with iterative diagonalizations, and Winston Roberts for his help with the calculation of signs.

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[29] In Capstick and Karl in Ref. [22], a solution of this problem is reported based on the small quark mass in the relativised model (m=0.22 GeV). In the light of the global fit of Ref. [28] this solution becomes less attractive. Moreover, current conservation with the nonrelativistic transition operator also requires an m^* of the order of 300-400 MeV; our calculation found the electromagnetic current roughly conserved at low Q^2 (with relativised model wavefunctions) using an inconsistent value of 336 MeV for the quark mass.

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TABLES

TABLE I. Photoproduction amplitudes with the mixed Isgur-Karl model wavefunctions of Eqs. 4 and 5 calculated with the transition operators H^{nr}, H^{vp}, H^{so}, and H^{2b}, with $g = 1$, $m^* = 336$ MeV, $M_T = 3m^*$, and $\alpha = 410$ MeV. Amplitudes are in units of 10^{-3} GeV$^{-\frac{1}{2}}$.

State (Energy)	A_λ^N	A_{nr} (Br)	A_{vp} (Br)	A_{so} (Br)	A_{2b} (Br)	total (Br)	expt.	total (cm)
$N\frac{1}{2}^+(1440)$	$A^p_{\frac{1}{2}}$	0+26	0	-32	11	5	-69±7	4
	$A^n_{\frac{1}{2}}$	0-15	0	22	-11	-4	37±19	-3
$N\frac{1}{2}^+(1710)$	$A^p_{\frac{1}{2}}$	0+67	0	-29	6	43	5±16	53
	$A^n_{\frac{1}{2}}$	1-45	0	16	-7	-35	-5±23	-46
$N\frac{1}{2}^+(1898)$	$A^p_{\frac{1}{2}}$	2+42	0	-10	-13	22		34
	$A^n_{\frac{1}{2}}$	-4-18	0	-11	12	-21		-32
$N\frac{1}{2}^+(2058)$	$A^p_{\frac{1}{2}}$	-8+16	0	-6	-19	-17		-15
	$A^n_{\frac{1}{2}}$	8-4	0	-9	20	14		12
$\Delta\frac{3}{2}^+(1232)$	$A^{p,n}_{\frac{1}{2}}$	-1-90	1	-6	3	-91	-141±5	-98
	$A^{p,n}_{\frac{3}{2}}$	-1-155	0	-7	5	-159	-258±19	-170
$\Delta\frac{3}{2}^+(1600)$	$A^{p,n}_{\frac{1}{2}}$	7-58	-8	15	-11	-55	-22±29	-69
	$A^{p,n}_{\frac{3}{2}}$	-5-112	4	43	-21	-91	1±22	-114
$\Delta\frac{3}{2}^+(1920)$	$A^{p,n}_{\frac{1}{2}}$	5+48	0	-15	2	41	43±?	55
	$A^{p,n}_{\frac{3}{2}}$	-5+30	0	41	-22	44	23±?	69
$\Delta\frac{3}{2}^+(1983)$	$A^{p,n}_{\frac{1}{2}}$	36+9	-6	20	-34	24		33
	$A^{p,n}_{\frac{3}{2}}$	-21+27	4	-7	-3	-1		8

TABLE II. Photoproduction amplitudes with the relativized model wavefunctions of Ref. [10] calculated in the Breit frame with the transition operators H^{nr}, H^{vp}, H^{so}, and H^{2b} with $g = 1.3$, $m^* = 437$ MeV, $M_T = 3m^*$, and $\alpha = 0.5$ GeV. Units are as in Table I.

State (Energy)	A_λ^N	A_{nr}	A_{vp}	A_{so}	A_{2b}	total	expt.
$N\frac{1}{2}^+(1440)$	$A^p_{\frac{1}{2}}$	0+31	0	-33	6	4	-69± 7
	$A^n_{\frac{1}{2}}$	0-20	0	20	-6	-6	37± 19
$N\frac{1}{2}^+(1710)$	$A^p_{\frac{1}{2}}$	1+32	0	-22	1	13	5± 16
	$A^n_{\frac{1}{2}}$	0-16	0	8	-2	-11	-5± 23
$N\frac{1}{2}^+(1880)$	$A^p_{\frac{1}{2}}$	-1+16	0	-3	-11	0	
	$A^n_{\frac{1}{2}}$	-1+1	0	-20	12	-9	
$N\frac{1}{2}^+(1975)$	$A^p_{\frac{1}{2}}$	-4+13	0	-9	-12	-12	
	$A^n_{\frac{1}{2}}$	3-4	0	-3	12	8	
$\Delta\frac{3}{2}^+(1232)$	$A^{p,n}_{\frac{1}{2}}$	0-107	0	-2	0	-108	-141± 5
	$A^{p,n}_{\frac{3}{2}}$	0-185	0	-1	1	-186	-258± 19
$\Delta\frac{3}{2}^+(1600)$	$A^{p,n}_{\frac{1}{2}}$	0+41	0	-15	5	30	-22± 29
	$A^{p,n}_{\frac{3}{2}}$	0+71	0	-28	8	51	1± 22
$\Delta\frac{3}{2}^+(1920)$	$A^{p,n}_{\frac{1}{2}}$	-14+26	9	-23	14	13	43± ?
	$A^{p,n}_{\frac{3}{2}}$	6-24	-5	50	-14	14	23± ?
$\Delta\frac{3}{2}^+(1985)$	$A^{p,n}_{\frac{1}{2}}$	29+4	-17	12	-22	6	
	$A^{p,n}_{\frac{3}{2}}$	-18-7	10	31	-14	3	

FIGURES

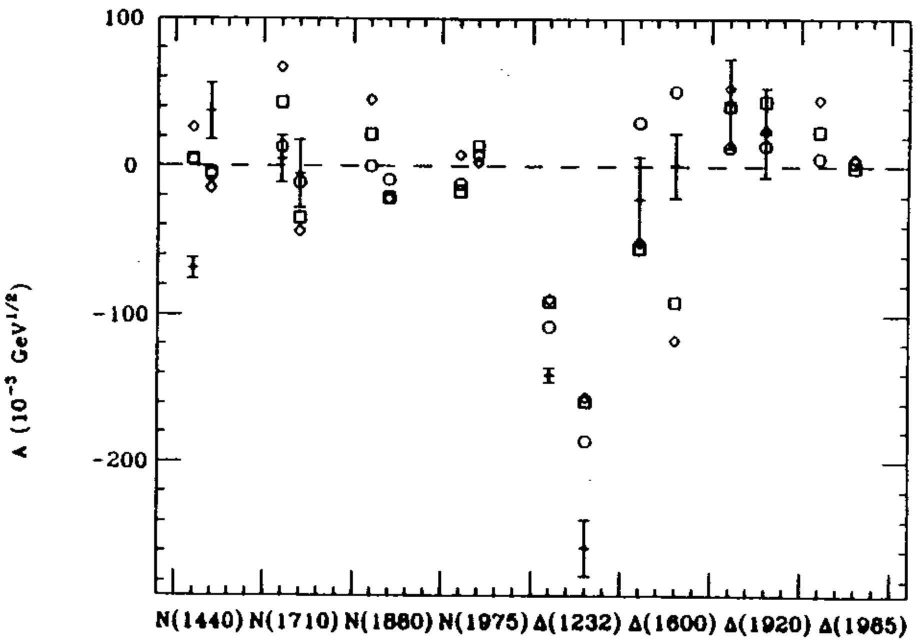

FIG. 1. Breit frame photoproduction amplitudes with mixed-nonrelativistic and relativized-model wavefunctions. Diamonds and squares are the A_{nr} and 'total' amplitudes from Table I, respectively; octagons are the 'total' amplitudes from Table II. Amplitudes are plotted in the order $A^p_{\frac{1}{2}}$, $A^n_{\frac{1}{2}}$, $A^p_{\frac{3}{2}}$, and $A^n_{\frac{3}{2}}$.