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TOWARDS THE CONTINUUM LIMIT OF STAGGERED WEAK MATRIX ELEMENTS

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Abstract

I review the status of calculations of weak matrix elements and the spectrum using staggered fermions. Particular emphasis is placed on the approach to the continuum limit.

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1 INTRODUCTION

In their dreams, lattice calculators of matrix elements imagine presenting their experimental colleagues with results having small and reliably known errors. While the progress towards this goal has been slow, some of the sources of error are small and more-or-less understood. In this talk I will focus on scaling violations, i.e. errors arising from the finite lattice spacing, a.

Two matrix elements of particular interest are the kaon B-parameter, B_K, and the B-meson decay constant, f_B. If we could calculate these with small errors then we could use existing data on CP violation in $\bar{K}-K$ mixing (ϵ), and on $\bar{B}-B$ mixing, to pin down the parameters of the Cabibbo-Kobayashi-Maskawa matrix. For a discussion of the present state of the phenomenological applications of lattice results see the talks of Lepage and Martinelli [1].

I will concentrate in this talk on B_K,

$$B_K(a) = \frac{\langle\overline{K}|\bar{s}\gamma_\mu(1+\gamma_5)d\ \bar{s}\gamma_\mu(1+\gamma_5)d|K\rangle}{\frac{8}{3}\langle\overline{K}|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K\rangle}\ . \tag{1}$$

It is worth stressing why we expend so much effort on this quantity. This is, first, because, with the methods we have developed, B_K has much smaller statistical errors than other dimensionless quantities. (The numerical values of the pion masses in lattice units are more accurate, but they are not, by themselves, useful.) Second, the calculation of B_K makes full use of the residual chiral symmetry of staggered fermions. This more than offsets the complications due to having four flavors of continuum fermion for every lattice staggered fermion. For more discussion of these points see Refs. [2, 3, 4] and particularly [5]. These references also explain technical details of the calculations which are not given here. Let me also mention that the Kyoto/Tsukuba group has begun calculating B_K using staggered fermions [6], and finds results in agreement with ours.

To date, there have been few calculations using staggered fermions of matrix elements involving heavy quarks, such as f_B. Here, chiral symmetry does not play an important role, so that there is nothing to offset the complications arising from the quadrupling of flavors. Given the tantalizing status of the calculation of this quantity [1], however, it is interesting to attempt the staggered calculation. We have begun to do so.

Our calculations are part of the weak matrix element Grand Challenge. Last year we shared about 12000 Cray-2 hours with Claude Bernard and Amarjit Soni, who use Wilson fermions.

2 FLAVOR SYMMETRY BREAKING

If we are to understand scaling violations in matrix elements, we had better first understand these violations in the spectrum itself. For quantities such as m_N/m_ρ this is, however, very difficult. We would have to observe the dependence of such ratios on the lattice spacing, but the large statistical errors mask any such dependence [7]. In addition, if we work in the quenched approximation, as we do here, then we do not know what the "correct" answer is in the continuum limit.

For staggered fermions, we can study scaling violations by making a virtue out of the vice of doubling. In the continuum limit, a single staggered fermion represents four degenerate flavors, and the spectrum has an $SU(4)$ flavor symmetry. At finite lattice spacing this flavor symmetry is broken down to a discrete subgroup. For example, the continuum pions lie in a $\mathbf{15}+\mathbf{1}$, while on the lattice the $\mathbf{15}$ breaks down into four 3-d and three 1-d representations [8]. The mass splitting between these lattice representations should vanish in the continuum limit. Since one can calculate all the pion masses with good accuracy, the approach to the continuum limit can be studied quantitatively.

In previous work [9] we calculated the masses of pions in four of the representations which are contained in the $\mathbf{15}$ in the continuum limit: the Goldstone pion π, a member of a 3-d representation π_3, and two others. We found that, to good approximation, all the non-Goldstone pions were roughly degenerate, all being somewhat heavier than the Goldstone pion. Thus the difference between the Goldstone pion mass, m_π, and that of the non-Goldstone π_3, m_3, is representative of flavor symmetry breaking, and I focus on this difference here. At this conference the Kyoto/Tsukuba group presented beautiful results for the masses of the pions in all representations [10]. They find that all but the Goldstone are close to degenerate. They have also calculated the masses of all rhos, nucleons and Deltas, but the statistical errors are too large to study flavor breaking using these states.

How rapidly is the continuum limit approached with staggered fermions? The standard lore is that the corrections are of $O(a^2)$, smaller than those for Wilson fermions which are of $O(a)$. (This is up to factors of $log(a)$, which I will ignore.) The issues were discussed in Refs. [11] and [12], but have not excited much interest for a decade. Since the question is now of practical importance for matrix elements, it is of interest to tighten up the arguments. The following discussion is presented in the hope that it will stimulate further work.

The best that one can hope to do is give a proof in perturbation theory. The goal would be to show that correlators having external quarks and gluons with physical momenta ($qa << 1$) differ from their continuum counterparts by terms starting at $O(a^2)$. A naive argument for this is as follows. It is easy to see that propagators and vertices differ from those of the continuum only by *even* powers of a [11, 12]. Thus our desired result is true at tree-level. In loops, factors of a^{2n} in vertices can be compensated by loop integrals, which can have arbitrarily high order of divergence, $\int^{\pi/a} d^4k k^{l-4}$. But since integrals of odd powers of k vanish by symmetry, l must be even, so the integrals produce only even (inverse) powers of a. All in all, one arrives at corrections proportional to powers a^2. By power counting these must be positive powers.

This argument leaves open the possibility that flavor breaking enters at $O(1)$ rather than $O(a^2)$. This might occur because in loops the gluons can have momenta of $O(1/a)$, and the vertices of such gluons break flavor at $O(1)$. It was shown in Ref. [11], however, that the flavor breaking "cancels", and is absent at $O(1)$. Since there are no terms of $O(a)$, flavor breaking (and other corrections) must be of $O(a^2)$.

Whatever one thinks of this argument, it is clearly of interest to do a numerical study to test it. We can now do this, for we have results from $\beta = 5.7-6.4$, a range over which a falls by roughly four. Our main emphasis over the last year has been to improve the statistics at $\beta = 6.2$ and 6.4; Table 1 shows our present sample of lattices. All results at $\beta = 6.2$ and 6.4 in this talk are preliminary; in particular the errors are likely to be underestimates. The new data at $\beta = 6.2$ supercedes our earlier results on a $18^3 \times 42$ lattice [9]. More details will

β	5.7	6.0	6.2	6.4
L_s	16	24	32	32
L_t	32	40	48	48
Sample	13	15	12+11	8+7+7
$a^{-1}(f_\pi)$	0.8(1)	1.7(1)	2.6(1)	3.4(2)
$a^{-1}(m_{\rho,N})$	1.0(1)	1.9(1)	2.5(1)	3.4(3)

Table 1: Quenched lattices. a^{-1} is given in GeV.

be given elsewhere [13]. Based on the comparison between 16^3 and 24^3 lattices at $\beta = 6$, we conclude that all the lattices listed in the Table are large enough ($L \geq 1.6fm$) that finite volume effects are smaller than our statistical errors [9].

In terms of the physical strange quark mass, the quark masses we use are roughly in the range $m_s/3 - m_s$ for all lattices. We also use much smaller masses at $\beta = 5.7$. Our results for the spectrum are included in the summary talk of Toussaint [7].

In order to study the approach to the continuum limit we need to know the lattice spacing for each β. We determine these by extrapolating f_π, m_ρ, and m_N to the chiral limit. We quote the values resulting from f_π, and from the average of m_ρ and m_N. In the quenched approximation there is no need for different determinations of a to agree. That they do agree, albeit within the rather large errors, shows that the quenched lattices are giving a reasonable approximation to the real world.

Figure 1 shows our results for flavor symmetry restoration, using $a^{-1}(f_\pi)$. According to the arguments given above, the dimensionless quantity $R = (m_3^2 - m_\pi^2)/m_\pi^2$ should vanish like a^2, if one works at fixed physical m_π. Thus R/a^2 should be a universal function of the physical m_π^2, up to corrections of $O(a^2)$. This is tested in Fig. 1, and the results confirm the expectations reasonably well. The points at $\beta = 6.2$ lie slightly above the others, but this could easily be due to an incorrect choice of a. A scale of $1/a = 2.4$GeV at $\beta = 6.2$ brings all the curves together. Since a varies by a factor of four, it is quite clear that a dominant correction proportional to a rather than a^2 is ruled out. Thus the lore is supported by our numerical results.

The shape of the curves can be understood as follows. Chiral symmetry predicts

$$(m_\pi a)^2 = (A_\pi a)(m_q a) + O(m_q^2)\,, \tag{2}$$

while we expect for a non-Goldstone that

$$(m_3 a)^2 = \delta_0 (fa)^2 + (1+\delta_1)(A_\pi a)(m_q a) + O(m_q^2), \tag{3}$$

where all masses and amplitudes are in physical units, and the dimensions of the intercept of m_3^2 have been arbitrarily set by $f = f_\pi(m_q = 0)$. The dimensionless quantities δ_0 and δ_1 characterize flavor breaking, and should thus vanish like a^2

$$\delta_0 = (\Lambda_0 a)^2 + O(a^4)\,;\ \delta_1 = (\Lambda_1 a)^2 + O(a^4)\,, \tag{4}$$

In terms of these, the quantity we plot is

$$R/a^2 = \Lambda_0^2 f^2/m_\pi^2 + \Lambda_1^2 + O(m_q, a^2)\,. \tag{5}$$

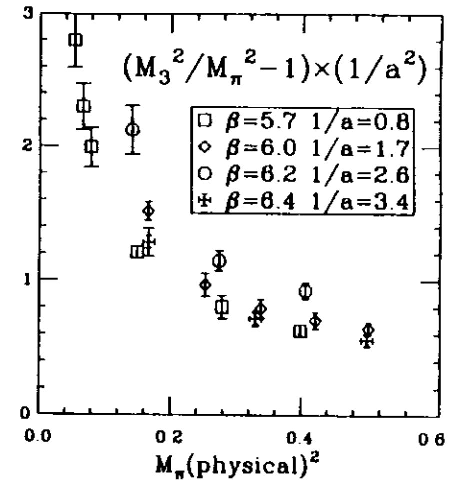

Figure 1: Flavor symmetry restoration. $1/a$ is given in GeV.

This is a function which blows up as $m_\pi^2 \to 0$, as is seen in Fig. 1.

It is interesting to extract the scales characterizing the $O(a^2)$ terms. From the figure, we find $\Lambda_0 \approx 2.3$ GeV and $\Lambda_1 \approx 0.6$ GeV. If we set the scale of the intercept of m_3^2 using m_ρ instead of f_π, we would find $\Lambda_0 \approx 0.4$ GeV. These are typical hadronic scales, so the violations of scaling are of the expected size.

3 SCALING OF QUENCHED B_K

Armed with this understanding of scaling violations in the staggered spectrum, we now turn to B_K. The results we presented at last year's meeting [4] are shown again in Fig. 2, using the new values for a. Here and below we do not include perturbative corrections in B_K. They are small ($1-2\%$), with an uncertainty of about the same size. The figure shows a significant drop in B_K from $\beta = 6$ to 6.4, suggesting large scaling violations. This is not conclusive, for there are various unsatisfactory features of the data. First, the results at $\beta = 6.2$ and 6.4 have large errors. Second, the 6.2 data is not smooth as a function of m_K^2. And, third, the results at $\beta = 5.7$ have the opposite slope from those at higher β.

To clarify the situation, we have followed the plan suggested in last year's talk. The present status (based on the sample in Table 1) is shown in Fig. 3. There are some significant changes from last year, which I describe in turn.

(A) As explained in Ref. [4], we use slightly different methods on the $32^3 \times 48$ lattices than in our previous work. In particular we use wall sources in Landau rather than Coulomb gauge. In order to check that the new methods are not responsible for the difference between the

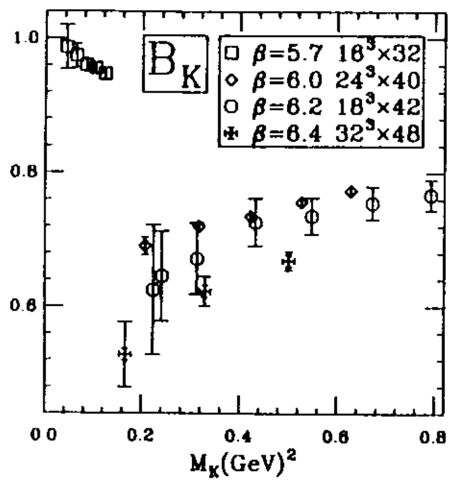

Figure 2: Results for B_K at Lattice 90, using $a^{-1}(m_\rho, N)$.

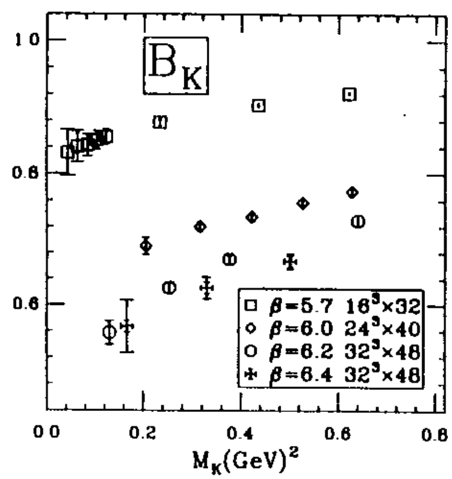

Figure 3: Present status of quenched B_K, using $a^{-1}(m_\rho, N)$.

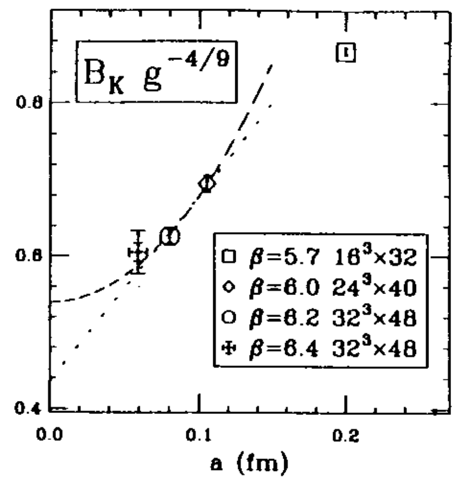

Figure 4: $B_K g^{-4/9}$ vs. a. Scales from $m_{\rho,N}$.

results at $\beta = 6$ and 6.4, we have repeated the calculation at $\beta = 6$ using the new methods. This we have done on $16^3 \times 24$ lattices, 40 in all, a size chosen to have physical dimensions close to that of the lattices at $\beta = 6.4$. The new and old results are in good agreement, giving us confidence in the new methods.

(B) We have doubled the sample at $\beta = 6.4$. The central values moved up by nearly 1 standard deviation, and the error is now a little smaller. Thus the scaling violations are slightly reduced in size, but have about the same significance.

(C) We have repeated the analysis on a newly generated sample of 32^3 lattices at $\beta = 6.2$. As at $\beta = 6.4$, these are separated by 800 overrelaxed and 200 Metropolis sweeps. These are almost the same physical size as the 24^3 lattices at $\beta = 6$, and are much larger than our old 18^3 lattices at $\beta = 6.2$. There are small differences in the results for pions between large and small lattices. The pion masses are smaller on the large lattices, and, as can be seen by comparing Figs. 2 and 3, B_K is smaller too. These differences are of marginal significance, and need further study. I will simply use the results from the large lattices, which have much smaller errors. Figure 3 shows that these results confirm the scaling violations, and considerably strengthen their significance.

(D) We extended the calculation at $\beta = 5.7$ to larger quark masses. In the process we found an error in our analysis, which, fortunately, only affects the results at $\beta = 5.7$. The $\beta = 5.7$ result quoted in [2] is thus wrong. The correct results, shown in Fig. 3, do have the same slope as at larger β, but still show very large scaling violations.

Figure 4 shows a vertical slice through Fig. 3 at the physical kaon mass, plotted against lattice spacing. We include the factor $g^{-4/9}$ to make a quantity which, for small enough g,

Fit	Linear	Quadratic
$B_K g^{-4/9}$	0.44(4)	0.54(2)
$\hat{B}_K$	0.66(6)	0.78(3)
Λ_B(GeV)	1.1	1.0

Table 2: Extrapolated results for quenched B_K. Scales from $m_{\rho,N}$.

is independent of a, and thus has a smooth continuum limit. The value we wish to present to the experimentalists is that extrapolated to $a = 0$. We thus need to know whether the dependence on a is linear or quadratic. To investigate this, we make linear and quadratic fits to the first three points, i.e. we fit to

$$B_K g^{-4/9} = X(1 + (a\Lambda_B)^n); \quad n = 1, 2. \tag{6}$$

We do not include the point at $\beta = 5.7$, since the correction there is so large (80 – 100%) that it makes no sense to truncate the series in a. The results are shown in the figure. The quadratic fit is better ($\chi^2 = 0.5$ vs. $\chi^2 = 1.2$), but both fits are reasonable. Thus our data are not good enough to decide between linear and quadratic dependence on a.

We quote in Table 2 the extrapolated results for $B_K g^{-4/9}$. (Very similar results are obtained using $a^{-1}(f_\pi)$.) These results are for lattice kaons composed of two degenerate quarks of mass $m_s/2$. In the physical kaon, however, the quarks are far from degenerate. From our results at $\beta = 6$ we estimate that this increases B_K by about 3% (Ref. [15, 4]).

Phenomenological analyses use $\hat{B}_K = B_K \alpha_s^{-2/9}$, which we also quote in Table 2. These values are for physical kaons, i.e. they include the 3% increase in $\hat{B}_K$ due to the extrapolation from degenerate to non-degenerate quarks. Following Ref. [14] we take $\alpha_s^{-2/9} = 1.34$ at $\beta = 6$.

There are various uncertainties in $\hat{B}_K$. That due to the extrapolation to the continuum limit is roughly ± 0.06, and is perhaps the largest. Others are: (A) Perturbative corrections, which introduce a 1–2% uncertainty [5]; (B) Uncertainty in the extrapolation to the chiral limit, perhaps half of the 3% upward shift. (C) Extrapolation to infinite volume. Any effect appears to be smaller than the statistical errors [2]; (D) Quenching. Dynamical fermions of mass $\sim m_s/3$ have little effect on B_K [16], but this needs further study. (E) Finally, there is roughly a 10% uncertainty in the value of α_s to use. This is a problem common to all model calculations of $\hat{B}_K$.

To reduce the uncertainty in the extrapolation to $a = 0$ requires, at the very least, accurate results up to $\beta = 6.4$. It is very important to know whether B_K is typical, i.e. whether all matrix elements will have scaling violations of this size, and require calculations out to $\beta = 6.4$ to obtain few percent accuracy. Martinelli [1] has suggested that B_K, calculated using our methods, might have abnormally large corrections. This is because we use quasi-local operators, with quark and antiquark fields at different points in a 2^4 hypercube, which we make gauge invariant by fixing to Landau gauge [3, 5]. Thus there are gauge artifacts (related to the lack of a transfer matrix, and to Gribov copies), which vanish when $a \to 0$. To test Martinelli's suggestion we must repeat the calculation with operators made gauge invariant by the introduction of gauge links. It is by no means clear, however, that his suggestion is correct, for the scales characterizing the corrections (given in Table 2) are not abnormally large. They are of similar size to those obtained from the flavor breaking in the spectrum.

4 IMPROVED OPERATORS

I close this talk with a brief description of work in progress to reduce the extrapolation error.

Since the data cannot tell us how to extrapolate to the continuum limit, we are forced to see if we can predict the dependence analytically. Although I have argued that corrections to quantities calculated with the staggered action alone (such as the spectrum) are of $O(a^2)$, there can be $O(a)$ corrections in matrix elements induced by the external operators. Indeed, in perturbation theory, our four-fermion operators do have $O(a)$ terms in their matrix elements, starting at tree level [5]. It is not yet clear, however, whether or not these terms contribute to B_K. It might be that the projection onto external states of a definite flavor, here that of the Goldstone pion, removes the $O(a)$ term. This is true at tree-level in perturbation theory.

In the absence of an analytic prediction, a fall-back position is to remove the $O(a)$ term whether it is present or not! More precisely, we can improve the operator so that it has no $O(a)$ terms in its tree-level matrix elements. It will still have terms of $O(a^2)$, which, when combined with loop divergences, can give rise to $O(g^2 a)$ corrections. But typically such corrections will be smaller by $g^2/16\pi^2$, i.e. an order of magnitude. Thus either (A) the $O(a)$ corrections to our matrix elements are large, in which case they should be greatly reduced by the improvement of the operators; or (B) the $O(a)$ corrections are small or absent, in which case the improvement will have little effect. This improvement program is similar to that with Wilson fermions, except that we do not need to improve the staggered action.

It turns out to be simple to improve our operators at tree level [5]: each field is replaced by a smeared field, the net effect being that the entire operator lives on a 4^4 hypercube. Furthermore, since we use wall sources, it is simple to repeat the analysis using the new operator, for we need no additional propagators. The sub-sample that has been improved to date is shown in Table 3.

We have not, however, completed the calculation of the finite perturbative corrections to the relation between our improved operators and the original operators. Nevertheless, we can still glean something from the uncorrected results, because the perturbative corrections should vary very little in our range of β. The results are shown (for the physical kaon mass, but not including the extrapolation to the chiral limit) in Table 3. Smearing has caused a significant increase in B_K, but this could be due to the lack of perturbative corrections. What is perhaps more significant is that the drop in B_K from $\beta = 6$ to 6.2 is smaller. This might indicate the improvement of an $O(a)$ term.

Clearly much more work, both theoretical and numerical is needed. But I think we can take heart from the level of detail under study.

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β	5.7	6.0	6.2
Sample	13	13	2+11
Original	0.869(5)	0.698(13)	0.627(11)
Improved	0.881(3)	0.715(7)	0.668(8)

Table 3: Results for quenched $B_K g^{-4/9}$. Scales from $m_{\rho,N}$.

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References

[1] G. Lepage, G. Martinelli, these proceedings

[2] G. Kilcup, S. Sharpe, R. Gupta and A. Patel, Phys. Rev. Lett. 64 (1990) 25

[3] G. Kilcup, Nucl. Phys. B (Proc. Suppl.) 17 (1990) 533

[4] S. Sharpe, Nucl. Phys. B (Proc. Suppl.) 20 (1991) 429

[5] S. Sharpe, in "Standard Model, Hadron Phenomenology and Weak Decays on the Lattice", Ed. G. Martinelli, to be published by World Scientific

[6] H. Mino, these proceedings

[7] D. Toussaint, these proceedings

[8] M. Golterman and J. Smit, Nucl. Phys. B262 (1985) 328

[9] R. Gupta, G. Guralnik, G. Kilcup and S. Sharpe, Phys. Rev. D43 (1991) 2003

[10] N. Ishizuka, these proceedings

[11] H. Sharatchandra, H. Thun and P. Weisz, Nucl. Phys. B192 (1981) 205

[12] M. Golterman and J. Smit, Nucl. Phys. B245 (1984) 61

[13] R. Gupta, G. Kilcup and S. Sharpe, in preparation.

[14] M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Rome preprint 792 (3/91).

[15] G. Kilcup, talk at DPF 91, to appear.

[16] G. Kilcup, Nucl. Phys. B (Proc. Suppl.) 20 (1991) 417

[17] S. Sharpe, A. Patel, R. Gupta, G. Guralnik and G. Kilcup, Nucl. Phys. B286 (1987) 253