

What do we know about the Q^2 Evolution of the Gerasimov-Drell-Hearn Sum Rule?

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Abstract

We study contributions from baryon resonance excitations to the Q^2 dependence of the Gerasimov-Drell-Hearn sum rule. The results indicate that the sum rule at small Q^2 is largely saturated by contribution from the lower mass resonant states. We find a strong Q^2 dependence of the resonant contributions, leading even to a sign change of the sum rule integral at $Q^2 \simeq 0.8 \text{ GeV}^2$. The global Q^2 dependence indicates discrepancies to the interpretation of the EMC polarized structure function data on A_1^p in the resonance region, which can be largely attributed to the elimination of the $P_{33}(1232)$ resonance from the EMC analysis.

The results of the polarized proton structure function measurement of the EMC collaboration¹ have prompted numerous speculations about whether or not in the deep inelastic region the spin of the proton is carried by the quarks. This has led to renewed interest² in experimental tests of the sum rule of Gerasimov³, and Drell and Hearn⁴, and in measurements of its Q^2 evolution. The sum rule relates the difference in the total photo-absorption cross section on nucleons for photon-nucleon helicity $\lambda_{\gamma N} = 1/2$ and $\lambda_{\gamma N} = 3/2$ to the anomalous magnetic moment of the target nucleon:

$$\int_{\nu_{thr}}^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(\nu, 0) - \sigma_{3/2}(\nu, 0)] = -\frac{2\pi^2\alpha}{M^2} \kappa^2 \quad (1)$$

where ν is the photon energy, $\sigma_{1/2}$ and $\sigma_{3/2}$ are the absorption cross sections for total helicity $1/2$ and $3/2$, and κ is the anomalous magnetic moment of the target nucleon. The GDH sum rule has been derived on rather general grounds but has never been tested experimentally. However, there is evidence from the analysis of single pion photo production that the sum rule cannot be grossly violated^{5,6}. A recent calculation⁷ from extended current algebra indicated that the absolute value of the GDH integral may be even larger than -0.524 GeV^{-2} .

On the other hand, the interpretation of the EMC results on the deep inelastic polarized proton structure functions suggests the following behavior around $Q^2 = 10 \text{ GeV}^2$:

$$\int_{\nu_{thr}}^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)] = \frac{0.141 \pm 0.035}{Q^2} \quad (2)$$

where $\sigma_{1/2}$ and $\sigma_{3/2}$ are the transverse cross sections. A comparison of eqn. (1) and

(2) suggests that satisfaction of the GDH sum rule requires dramatic changes in the helicity structure of the γp coupling between the deep inelastic region and $Q^2 = 0$; e.g. the sum rule integral has to change its sign at some value of Q^2 .

In this work we study contributions to the sum rule for proton targets using empirical information from the electroproduction of nucleon resonances. The possibility that the GDH sum rule was being saturated by low lying resonances has previously been pointed out by Aznauryan⁸ and by Close⁹. A significant amount of pion and eta electroproduction data has been collected in the nucleon resonance region¹⁰. The analysis of these data in terms of resonant and non resonant contributions led to the extraction of the transverse resonance photo coupling amplitudes $A_{1/2}(Q^2)$ and $A_{3/2}(Q^2)$ for the most prominent resonant states in the range $0. \leq Q^2 \leq 3.0 \text{ GeV}^2$, where $Q^2 = -(p_e - p_{e'})^2$, and p_e and $p_{e'}$ are the four-momentum vectors of the incoming and scattered electron, respectively.

Inspection of eqn. (1) shows that the low mass states give the largest contributions to the sum rule. Therefore, knowledge of the photo coupling amplitudes of these states is most crucial for an accurate determination of the sum rule integral. In the following section we describe the parametrizations, and assumptions made in describing existing electro-production data.

1. $\gamma_p \rightarrow P_{33}(1232)$

At small Q^2 , the transition to the first isobar state is known to be dominantly a magnetic dipole transition M_{1+} , with only a small⁹ electric quadrupole multipole E_{1+} :

$$|E_{1+}/M_{1+}| < 0.05$$

The E_{1+} contribution to the total photoabsorption cross section is therefore negligible. The transition to the $P_{33}(1232)$ (or Δ) can thus be described by the magnetic transition formfactor G_M^Δ alone. We use the following empirical parametrization from a fit to the experimental data¹²:

$$G_M^\Delta(Q^2) = G_M^\Delta(0) \cdot G_D(Q^2) \cdot e^{-0.2 \cdot Q^2}$$
$$G_E^\Delta(Q^2) = 0, \tag{3}$$

where G_D is the usual dipole formfactor.

2. $\gamma_p \rightarrow P_{11}(1440)$

Much less data are available for the transition to the Roper resonance $P_{11}(1440)$. We use the parametrization of Li et al.¹³ to describe the rapid fall off with Q^2 in accordance with the data¹³. It is normalized to the real photon limit:

$$A_{1/2} = -\frac{3}{4}k \times 0.35 \sqrt{\frac{2\pi}{k_0}} \mu_p \exp\left(-\frac{1}{6}\left(\frac{k}{\alpha}\right)^2\right), \tag{4}$$

where k and k_0 are the photon's 3-momentum and energy respectively. μ_p is the photon magnetic momentum, and $\alpha = 0.25$ is a parameter related to the charge radius of the Roper resonance in the model of Li¹³. A treatment adopted from Foster and Hughes¹⁴ is used in extending photoproduction to electroproduction. This parametrization is in agreement with existing exclusive data, as well as with the analysis of inclusive cross section data which found no indication of the $P_{11}(1440)$ excitation at high Q^2 .

3. $\gamma_v p \rightarrow S_{11}(1535), D_{13}(1520), F_{15}(1680)$

These resonances are the dominant states of the $[70, 1^-]_1$ and $[56, 2^+]_2$ super multiplets, respectively, assuming a $SU(6) \otimes O(3)$ symmetry scheme. These are the only higher mass states for which data are available over a significant range in Q^2 . The data have been compiled by Breuker et al.¹⁵ in terms of reduced quark electric and quark magnetic multipole moments. The relation between the photo coupling helicity amplitudes and the reduced quark multipole moments is given in appendix A. We use the following parametrization to describe the transition into the $[70, 1^-]_1$ super multiplet (in units of GeV):

$$\tilde{e}_1^{11} = 3.0$$

$$\tilde{m}_1^{11} = 3.8(0.4 - k_{EVF}^2)$$

$$\bar{m}_1^{12} = 5.0 \cdot k_{EVF}^2, \quad k_{EVF}^2 \leq 1.0 \text{ GeV}^2$$

$$\bar{m}_1^{12} = (6. - 1.0 \cdot k_{EVF}^2), \quad k_{EVF}^2 > 1.0 \text{ GeV}^2.$$

For the transition into the $[56, 2^+]_2$ super multiplet the parametrization

$$\bar{e}_2^{22} = 0.99$$

$$\bar{m}_2^{22} = 0.75\sqrt{5} - 1.5\sqrt{5}k_{EVF}^2$$

$$\bar{m}_2^{23} = 5.0k_{EVF}^2, \quad k_{EVF}^2 \leq 1.0\text{GeV}^2$$

$$\bar{m}_2^{23} = 5.7 - 0.7k_{EVF}^2, \quad k_{EVF}^2 > 1.0\text{GeV}^2$$

is used, which provides a good representation of the data¹⁵.

The set of parametrizations which is used to describe the experimentally measured states $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, and $F_{15}(1680)$, is referred to as set I, where no model assumptions have been made except for the $P_{11}(1440)$.

We will now make additional assumptions in order to obtain information about the experimentally not determined transition amplitudes belonging to the $[70, 1^-]_1$ or $[56, 2^+]_2$ supermultiplets.

4. $\gamma_n p \rightarrow S_{11}(1650), S_{31}(1620), D_{13}(1700), D_{33}(1700), D_{15}(1675)$

The least model dependent assumptions are based on $SU(6)_W$ algebraic symmetry relations between the transitions from the $[56, 0^+]_0$ ground state and members of these multiplets. It is assumed that only a single quark is affected in the transition (single quark transition assumption, SQT), then algebraic relations for the transition into the states within one supermultiplet can be derived^{16,17}. In the Q^2 range of our study, Warns et al.¹⁸ estimate the multiple quark transition (MQT) amplitudes to be typically less than 10% of the SQT amplitudes using a relativized quark model. The MQT contributions to the total absorption cross section will therefore be negligible. In set II, in addition to set I we assume the SQT algebraic relations to be valid for the transition into the $[70, 1^-]_1$ super multiplet (see appendix A). Using these relations we obtain predictions for the following states in $[70, 1^-]_1$: $S_{11}(1650)$, $S_{31}(1620)$, $D_{13}(1700)$, $D_{33}(1700)$, and $D_{15}(1675)$. The $D_{13}(1700)$ and the $D_{15}(1675)$ are assumed to be not mixed, their photocoupling amplitudes for proton targets will then be identical 0. The $S_{11}(1650)$ photocoupling amplitude is non-zero because this state it is a mixture of two $SU(6)$ states.

$$A_{1/2}^{S_{11}(1650)} = A_{1/2}^{S_{11}(1535)} \times \tan 38^\circ$$

The 38° mixing angle is in agreement with the experimental data. With the currently available data, the SQT prediction cannot be tested accurately. However, in the few cases where data are available, the algebraic relations are in reasonable agreement

with the data¹⁹. We will now make further assumptions about the higher mass states.

5. $\gamma_v p \rightarrow P_{13}(1720), P_{31}(1910), P_{33}(1920), F_{35}(1905), F_{37}(1950)$

We will assume that the SQT algebraic relations be valid for the transition from the $[56, 0^+]_0$ ground state to the $[56, 2^+]_2$ supermultiplet (set III). The transition operator into this supermultiplet contains four independent terms, two of which are related to the usual quark orbit flip and quark spin flip, and two to a simultaneous spin-orbit flip with $\Delta L_z = \pm 1$, and $\Delta L_z = \pm 2$. Since there is only experimental information available for the $A_{1/2}$ and $A_{3/2}$ amplitudes of the $F_{15}(1680)$, additional assumptions are necessary to extract the four terms in the transition operator. The helicity amplitudes for the $F_{15}(1680)$ transition are related to the quark multipole moments:

$$A_{1/2} = F \cdot \left(\sqrt{\frac{1}{5}} e_1^{22} + \sqrt{\frac{2}{15}} m_1^{22} - \sqrt{\frac{2}{3}} m_1^{23} \right)$$

$$A_{3/2} = F \cdot \left(\sqrt{\frac{2}{5}} e_1^{22} + \sqrt{\frac{4}{15}} m_1^{22} + \sqrt{\frac{1}{3}} m_1^{23} \right).$$

We assume that, in analogy to the $[70, 1^-]_1$ transition, the electric term \bar{e}_1^{22} is independent of Q^2 . The \bar{m}_1^{23} and \bar{m}_1^{22} can then be determined, using the value of \bar{e}_1^{22} as determined at the pion photoproduction point²⁰. There is no information on \bar{m}_1^{21} for electroproduction. However, the $P_{31}(1910)$ is directly determined by \bar{m}_1^{21} (table 2). Since this state has negligible photocouplings, we assume them to be negligible in electroproduction as well. Predictions for the transition into the $P_{13}(1720)$, $P_{31}(1910)$, $P_{33}(1920)$, $F_{35}(1905)$, and $F_{37}(1950)$ states can then be obtained.

The following table illustrate the resonance states used in the three sets of calculations:

	Experiments	SQTM for $[70, 1^-]$	SQTM for $[56, 2^+]$
	$P_{33}, P_{11}, S_{11}, D_{13}, F_{15}$	$S'_{11}, S_{31}, D'_{13}, D_{15}, D_{33}$	$P_{13}, F_{35}, F_{37}, P_{31}, P'_{33}$
set I	X		
set II	X	X	
set III	X	X	X

From the known photon coupling helicity amplitudes discussed above, the contributions of individual resonances to the total transverse cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ at resonance position can be calculated by:

$$\sigma_{1/2, 3/2}^{Res} = \frac{2M}{W_0\Gamma} A_{1/2, 3/2}^2, \quad (5)$$

where M and W_0 are the nucleon and resonance mass and Γ is the total width of the resonance. The total resonant transverse absorption cross section can be calculated from the photon coupling helicity amplitudes. To describe the energy-dependent structure of resonances, the relativistic Breit-Wigner parametrization of Walker²⁰ is used. The helicity amplitudes at resonance position for a pion nucleon final state in Walker's notation are related to the photon couplings by:

$$A_{l\pm} = \mp f C_{\pi N}^I \cdot A_{1/2},$$

$$B_{l\pm} = \pm f \sqrt{\frac{16}{(2j-1)(2j+3)}} \cdot C_{\pi N}^I \cdot A_{3/2}$$

with

$$f = \sqrt{\frac{1}{(2j+1)\pi} \frac{k}{q_\pi} \frac{M\Gamma_\pi}{W_0\Gamma^2}}, \quad (6)$$

where and $j = l \pm 1/2$ is the spin of the resonance; $C_{\pi N}^I$ are the isospin Clebsch-Gordon coefficients; k and q_π are the 3-momentum of the photon and outgoing pion, respectively. The resonance widths were taken from the Review of Particle Properties 1990²¹

Assuming the same relations for all of the other final states, the total branching ratio from all final states will add up to 1. The total inclusive transverse cross sections from resonance contributions can then be calculated. In this calculation the non-resonance contributions are neglected except for the single pion Born term at low energies. The comparison with the analyses of Karliner⁵ at $Q^2 = 0$, as well as Workman and Arndt⁶ gives good agreement as shown in the following table. Note that in both of the Karliner and Workman & Arndt analyses the same inelastic contributions estimated by Karliner were used.

GDH integral	S_{BL}	S_{KA}	S_{WA}
πN channel only	-0.490	-0.491	-0.496
Total	-0.572	-0.655	-0.660

S_{BL} =this analysis; S_{KA} =Karliner analysis; S_{WA} =Workman and Arndt analysis.

Each of the integrations were carried out over the photon lab energy from threshold to 1.8 GeV. Our results from the three sets of photon coupling amplitudes are shown

in figure 1. The dash, dotdash and solid lines represent the set I, set II and set III calculations respectively. It is clear from the difference between set I and set III that there are some contributions to the sum rule integral from the intermediate resonance states; while the contributions from higher mass resonance states are insignificant as evidenced by the very small difference from set II and set III.

All three sets of the GDH sum rule calculation show very strong Q^2 -dependence below $Q^2 = 1.0 \text{ GeV}^2$ and a sign flip around $Q^2 = 0.8 \text{ GeV}^2$. Very small Q^2 -dependence is observed above $Q^2 = 1.5 \text{ GeV}^2$. Note that, except for the $P_{11}(1440)$, the set I calculation resulted from a direct fit to the photon coupling constants extracted from the pion electroproduction experimental data.

The strong Q^2 -dependence below $Q^2 = 1 \text{ GeV}^2$ is rather striking. The $P_{33}(1232)$ excitation is known to be strongly excited at $Q^2 = 0$, and its dominant magnetic transition formfactor decreases rapidly as Q^2 increases. To understand the $P_{33}(1232)$ contribution to the strong Q^2 -dependence in the GDH integral, the GDH sum rule integral was evaluated when the $P_{33}(1232)$ was excluded. The results are shown in figure 2. The solid and dash lines represent the set III calculation with and without the $P_{33}(1232)$ resonance contributions, respectively. Obviously, the Q^2 -dependence is dramatically reduced without the $P_{33}(1232)$ resonance.

In conclusion, the information on the Q^2 evolution of the GDH sum rule integral is crucial for understanding the nucleon spin structure in the non-perturbative QCD domain. Our analysis provides information on the Q^2 evolution of the GDH inte-

gral using existing experimental data in the nucleon resonance region. Excitation of nucleon resonances, especially of the $P_{33}(1232)$ largely saturate the GDH sum rule at $Q^2 = 0$, and also provide important contributions in the Q^2 range of our study. Our analysis shows that the behaviour in the deep inelastic region cannot simply be extended to small Q^2 and into the resonance region. Approximate agreement with the extrapolated EMC analysis at smaller Q^2 is achieved only when the dominant contribution from the $P_{33}(1232)$ resonance is neglected. An inclusive measurements of the polarized structure functions in the resonance region is pending at CEBAF²². From this experiment, more direct information on the Q^2 evolution of the GDH sum rule will be obtained.

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1 Appendix A

It has been shown¹⁵ that applying the SQTm, the transition amplitudes in $[56, 0^+]_0 \rightarrow [70, 1^-]_1$ group can be expressed by the quark electric and quark magnetic multipoles of e_λ^{LL} , m_λ^{LL} and $m_\lambda^{L,L+1}$ and the transition amplitudes in $[56, 0^+]_0 \rightarrow [56, 2^+]_2$ group can be expressed by the quark electric and quark magnetic multipoles of e_λ^{LL} , m_λ^{LL} , $m_\lambda^{L,L+1}$ and $m_\lambda^{L,L-1}$ respectively,

$$A_{1/2} = (-1)^{J-1/2} \frac{e}{2} (M(W_R^2 - M^2))^{-1/2} \left[\left(\frac{2J+3}{4J+2} \right)^{1/2} G_\lambda^{J+1/2} - \left(\frac{2J-1}{4J+2} \right)^{1/2} G_\lambda^{J-1/2} \right]$$

$$A_{3/2} = (-1)^{J-1/2} \frac{e}{2} (M(W_R^2 - M^2))^{-1/2} \left[\left(\frac{2J-1}{4J+2} \right)^{1/2} G_\lambda^{J+1/2} - \left(\frac{2J+3}{4J+2} \right)^{1/2} G_\lambda^{J-1/2} \right].$$

G_λ^L 's are functions of the quark multipoles¹⁵. The G_1^L 's which give the transverse transition amplitudes are listed in Table 1 for $[70, 1^-]_1$ group and table 2 for $[56, 2^+]_2$ group respectively.

The equal-velocity frame (EVF) was chosen to minimize relativistic effects. The set of reduced quark multipoles are defined by factoring out a common dipole form factor ($F(\vec{k}_{EVF}^2)$):

for $[70, 1^-]_1$ states :

$$e_1^{11} = \bar{e}_1^{11} F(\vec{k}_{EVF}^2)$$

$$m_1^{11} = \bar{m}_1^{11} F(\vec{k}_{EVF}^2)$$

$$m_1^{12} = \bar{m}_1^{12} F(\vec{k}_{EVF}^2)$$

for $[56, 2^+]_2$ states :

$$e_1^{22} = \bar{e}_1^{22} |\vec{k}_{EVF}\rangle F(\vec{k}_{EVF}^2)$$

$$m_1^{21} = \bar{m}_1^{21} |\vec{k}_{EVF}\rangle F(\vec{k}_{EVF}^2)$$

$$m_1^{22} = \bar{m}_1^{22} |\vec{k}_{EVF}\rangle F(\vec{k}_{EVF}^2)$$

$$m_1^{23} = \bar{m}_1^{23} |\vec{k}_{EVF}\rangle F(\vec{k}_{EVF}^2).$$

Table 1: The G_1^L 's as a function of the quark multipoles for the transition from ground states (p,n) to the resonance states in $[56, 2^+]_2$ super multiplet

$[56, 2^+]_2$	m_1^{21}		e_1^{22}		m_1^{22}		m_1^{23}	
	p	n	p	n	p	n	p	n
$F_{15}(1680)$			$-\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{8}{45}}$	-1	$\frac{2}{3}$
$P_{13}(1810)$	1	$-\frac{2}{3}$	$-\sqrt{\frac{2}{5}}$	0	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{4}{15}}$		
$F_{37}(1950)$							$-\sqrt{\frac{16}{21}}$	$-\sqrt{\frac{16}{21}}$
$F_{35}(1890)$			0	0	$-\sqrt{\frac{28}{45}}$	$-\sqrt{\frac{28}{45}}$	$\sqrt{\frac{8}{63}}$	$\sqrt{\frac{8}{63}}$
$P_{33}(1920)$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0	$\sqrt{\frac{4}{15}}$	$\sqrt{\frac{4}{15}}$		
$P_{31}(1910)$	$-\frac{2}{3}$	$-\frac{2}{3}$						

Table 2: The G_1^L 's as a function of the quark multipoles for the transition from ground states (p,n) to the resonance states in $[70, 1^-]_1$ super multiplet

$[70, 1^-]_1$	\tilde{e}_1^{11}		\tilde{m}_1^{11}		\tilde{m}_1^{12}	
	p	n	p	n	p	n
$D_{13}(1520)$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{27}}$	1	$-\frac{1}{3}$
$S_{11}(1535)$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{27}}$		
$D_{33}(1700)$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{27}}$	$-\sqrt{\frac{1}{27}}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$S_{31}(1635)$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{27}}$	$\sqrt{\frac{2}{27}}$		
$D_{15}(1675)$					0	$\sqrt{\frac{2}{3}}$
$D_{13}(1700)$	0	0	0	$-\sqrt{\frac{10}{27}}$	0	$-\frac{1}{3\sqrt{10}}$
$S_{11}(1650)$	0	0	0	$-\sqrt{\frac{2}{27}}$		

Figure Caption:

Figure 1. The GDH integral from $Q^2 = 0$ to $Q^2 = 2.5 \text{ GeV}^2$ from three sets of calculations. The dashed, dot-dashed and solid lines represent the calculation from set I, II and II respectively as described in the text. The short dashed lines represent equation (2)

Figure 2. The GDH integral from $Q^2 = 0$ to $Q^2 = 2.5 \text{ GeV}^2$ with (solid) and without (dashed) $P_{33}(1232)$ resonance contributions respectively. The short dashed lines represent equation (2)

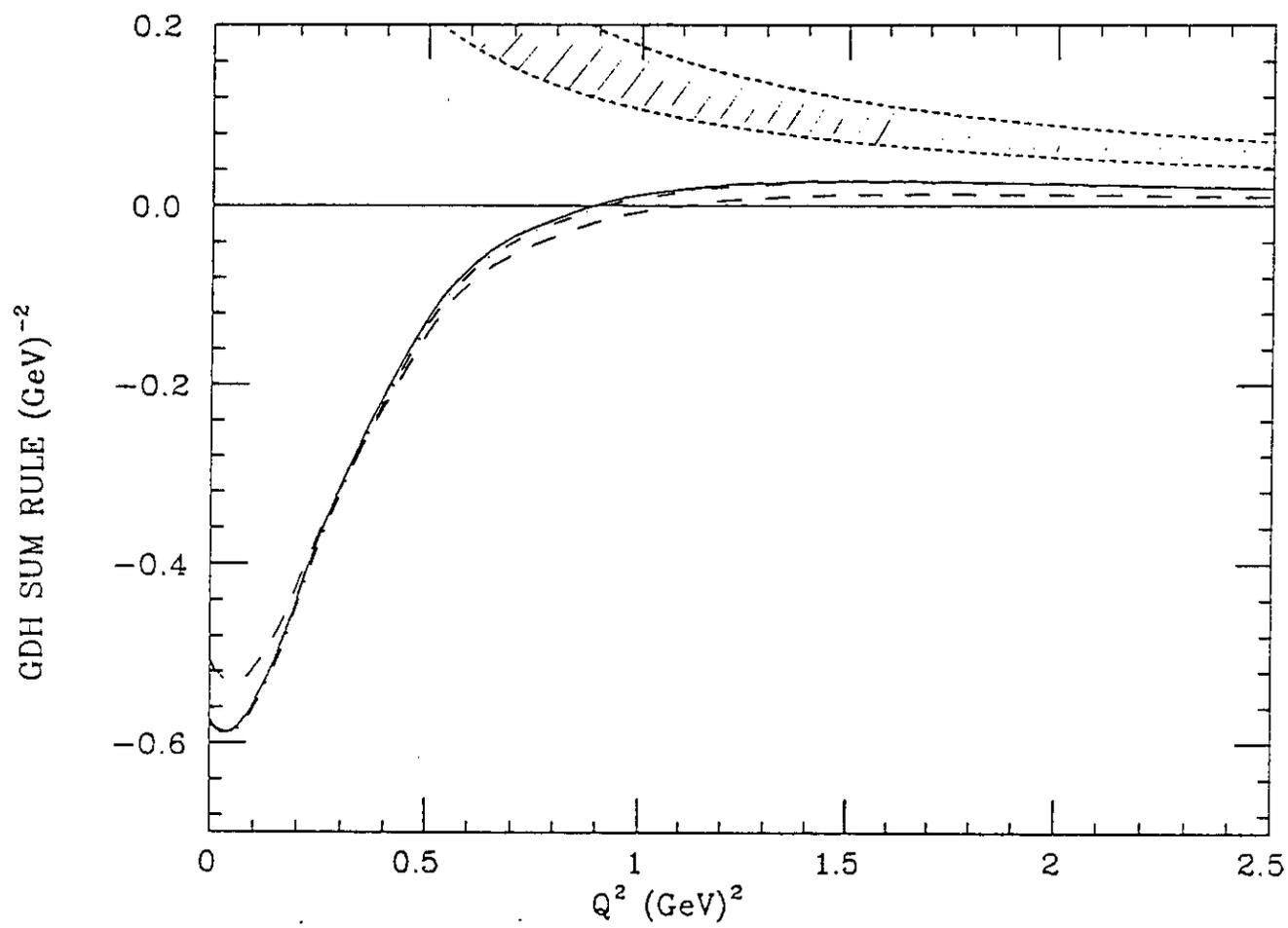


Figure 1

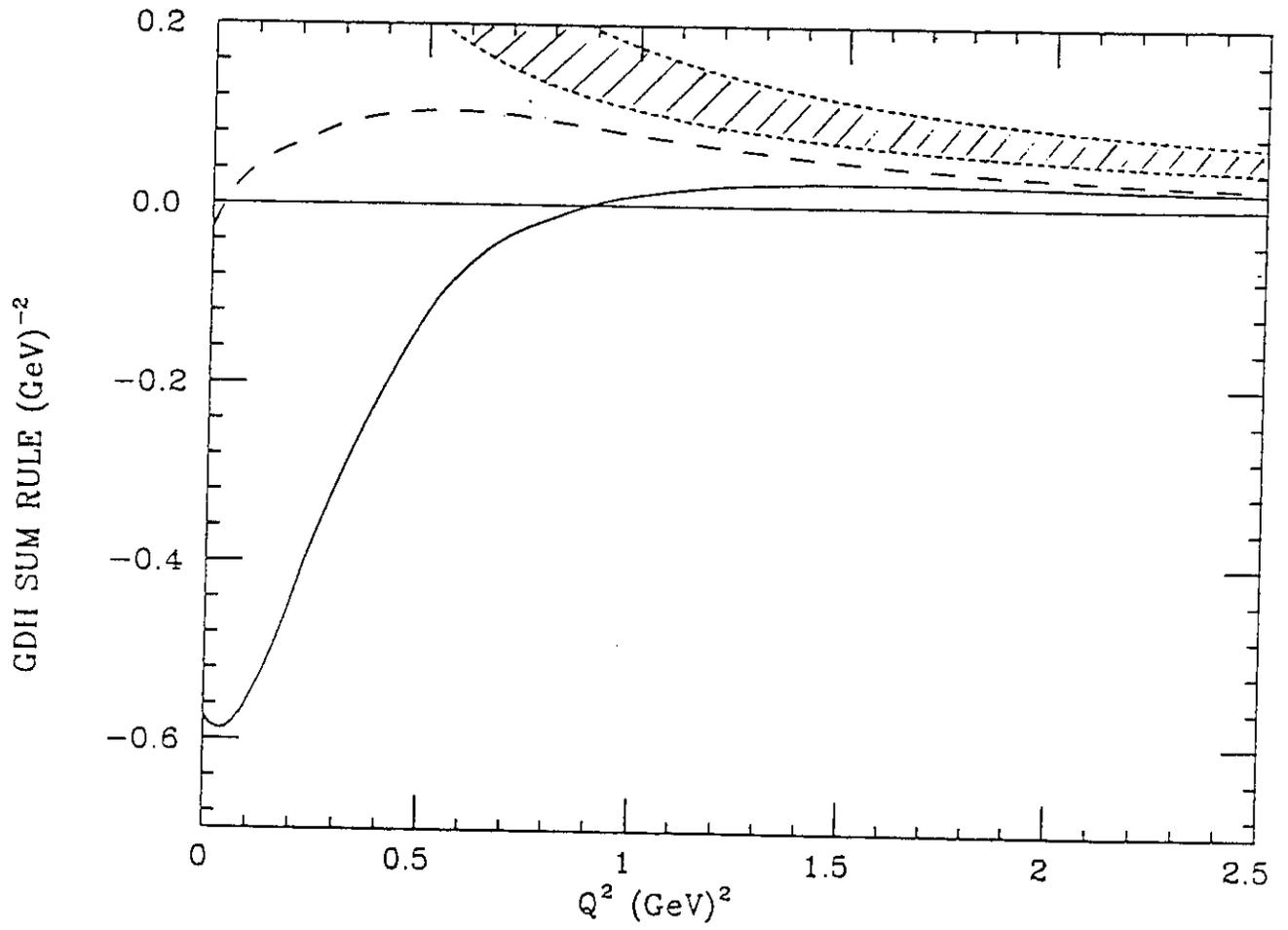


Figure 2