High Accuracy Beam Energy Measurement for Linear and Circular Accelerators*

I. P. Karabekov, Yerevan Physics Institute and†
Continuous Electron Beam Accelerator Facility‡

R. Rossmanith, Continuous Electron Beam Accelerator Facility‡

Abstract

A new method of measuring the absolute beam energy is presented. The method uses the nonlinear variation of the intensity of a narrow line, filtered from the synchrotron radiation (SR), as a function of beam energy. The resolution of the method is mainly limited by the absolute accuracy of the magnetic field measurement using NMR, the absolute knowledge of the wavelength of the photons, and the quantum fluctuations of the SR at low beam currents. In order to achieve an energy resolution of 2×10^{-5} at a current of 10μA, the magnetic field has to be measured with an accuracy of 10^{-5}, and the wavelength, with 2.5×10^{-5}.

The method is applicable both in linear and circular machines.

INTRODUCTION

The measurement of the beam energy of accelerated or accumulated beams with an accuracy better than 10^{-4} is required both in high energy and nuclear physics [1, 2].

As known to the authors, two techniques were used so far to measure the absolute beam energy with an accuracy of the order of 10^{-5}. The first measures the energy by measuring the depolarization frequency of a stored beam [3]. In the second method the spectral and angular distributions of the synchrotron radiation (SR) are measured [4].

The second method has the advantage that it needs no polarization and it is in general faster [5]. It only requires synchrotron light and therefore it can be used in a greater variety of accelerators like linacs, linear colliders, recirculators and extracted beams. As an example, the SR method could permit measurement of the beam energy at CEBAF with an accuracy up to 2.5×10^{-5} at 300μA [4].

In this paper a new possibility of high accuracy energy measurement using synchrotron radiation is presented. In this method the nonlinear dependence of the synchrotron radiation intensity as a function of the beam energy and the magnetic field strength at a given wavelength is used. This technique promises a higher resolution and is free of several limitations under which the previous method described in [4] suffers. Applying this method at CEBAF, a resolution of 2.0×10^{-5} at $10\mu A$ can be achieved.

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† Yerevan Physics Institute, Alikhanian Brothers 2, Yerevan 375036, Republic of Armenia

‡ Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA USA

THE METHOD

The concept of the method is the following. A beam of charged particles passes through two magnets with different field strengths B_1 and B_2. The critical wavelengths λ_{c_1} and λ_{c_2} of the synchrotron radiation created in these magnets is

$$\lambda_{c_i} = \frac{7.12 \cdot 10^3}{\gamma^2 B_i}, \tag{1}$$

where γ is the mean Lorentz-factor of the beam, $i = 1, 2$, λ_c is in cm and B in Gauss.

The synchrotron radiation within a solid angle of 1 mrad horizontal Θ and 1 mrad vertical ψ has the following spectral distribution [6]:

$$N_k = A\gamma^2 (\lambda_c/\lambda)^2 K^2_{2/3}(\lambda_c/2\lambda), \tag{2}$$

where $K_{2/3}(\lambda_c/2\lambda)$ is the modified Bessel function, λ is the wavelength and A is a proportionality coefficient to obtain the result in units of photons per second.

$$A = 3.461 \cdot 10^6 \cdot I(\mathrm{mA}) \cdot \delta\lambda/\lambda \cdot \Theta(\mathrm{mrad}) \cdot \psi(\mathrm{mrad}) \tag{3}$$

The two photon distributions normalized by dividing through $A\gamma^2$ are shown in Figure 1. The extracted photon fluxes can be made equal when each of the photon beams passes a horizontal slit collimator. The vertical width of both collimators is the same and therefore the ratio of the horizontal slit widths ξ should be

$$\xi = M^2 K^2_{2/3}(\lambda_c/2\lambda_0)/K^2_{2/3}(\lambda_c/2M\lambda_0), \tag{4}$$

where $M = B_2/B_1$ and λ_c is defined in equation (1).

According to Figure 1 the curves are crossing at λ_0. In this case $\xi = 1$. The general case ($\xi \neq 1$) is shown in Figure 2. The two curves have different slopes in the vicinity of the crossing point. Therefore the photon flux changes nonlinearly with beam energy and the magnetic field strength.

$$\begin{aligned}\Delta N_k(\Delta\gamma) = A \cdot P^2(\gamma_0 + \Delta\gamma)^{-2}\lambda_0^{-2}B_1^{-2}\Big[K^2_{2/3}\big(\frac{P}{2}\lambda^{-1}(\gamma_0+\Delta\gamma)^{-2}B^{-1}\big) - \\ - M^{-2}K^2_{2/3}\big(\frac{P}{2}\lambda^{-1}(\gamma_0+\Delta\gamma)^{-2}M^{-1}B^{-1}\big)\Big]\end{aligned} \tag{5}$$

where P $= 7.12 \cdot 10^3$ and A is defined in (3).

The analytical expression for $\Delta N_k(\Delta\gamma) = \partial N_k/\partial\gamma \cdot \Delta\gamma$ permits us to find the value for M for which the sensitivity of the method is maximum for a given wavelength λ_0.

$$\begin{aligned}\Delta N_k = \Delta\gamma \cdot A \cdot \frac{3a}{2\gamma^3}\Big[K^2_{2/3}(z) - M^{-2}K^2_{2/3}(z/M)\Big] + \\ + \frac{2a^3}{\gamma^5}\Big[K_{1/3}(z)K_{2/3}(z) - M^{-3}K_{1/3}(z/M)K_{2/3}(z/M)\Big]\end{aligned} \tag{6}$$

where $a = 7.12 \cdot 10^3/B\lambda$ and $z = a/2\gamma^2$.

For the maximum sensitivity $\partial \Delta N_k / \partial M = 0$. This partial derivative is

$$\frac{\partial N_k}{\partial M} = 2zM^{-4}K_{1/3}(z)K_{2/3}(z) - 2/3M^{-3}K^2_{2/3}(z/M), \tag{7}$$

and accordingly

$$M = 3z\frac{K_{1/3}(z/M)}{K_{2/3}(z/M)}. \tag{8}$$

RESULTS

In order to obtain optimum resolution the different parameters have to be optimized. First, the optimum wavelength for λ_0 has to be chosen. This wavelength should be in the hard x-ray region ($\lambda = 0.5 \div 2.0\mathring{A}$), since the difference of the photon fluxes can be measured with an ionization chamber [7] sensitive to X-rays.

The next parameter which has to be chosen is the magnetic field strength B_1. This field strength has to be chosen in such a way that at the lowest beam energy and the lowest current for the selected wavelength λ_0 the photon flux is high enough. The field strength has to be measured by NMR.

For further optimization the maximum field B_1 was chosen to $1.0 \times 10^4 \div 2.0 \times 10^4$ Gauss and λ_0 to be within $1.0\mathring{A} \div 1.7\mathring{A}$.

For CEBAF a maximum field strength of $B_1 = 2 \times 10^4$ and $\lambda_0 = 1.7\mathring{A}$ was chosen for beam energies equal to 2 GeV and higher. For 1 GeV a λ_0 of $2.5\mathring{A}$ was chosen. It was assumed that the beam current is 10^{-5}A and the maximum horizontal angle of collimation for the lowest energy is $\Theta = 10^{-2}$ rad.

The results of the calculations are summarized in Table 1. According to these calculations the absolute accuracy of the energy measurement for CEBAF for an electron beam of $10\mu A$ is better than 2×10^{-5}.

As can be seen from the table the main contribution to the systematic error comes from the error in measuring the wavelength of the photons. For a crystal monochromator the uncertainty $\delta\lambda$ in the determination of λ_0 and the uncertainty $\delta\theta$ in the measurement of the angle between the SR beam axis and the crystal planes (θ_B is the Bragg angle) are related to each other by

$$\delta\lambda/\lambda_0 = \delta\theta ctg\theta_B. \tag{9}$$

For a short λ_0 the Bragg angle θ_B is small and for the realization of demanding $\delta\lambda/\lambda_0$ the value of θ will be measured $ctg\theta_B$ times more accurate.

In principle, in order to achieve high resolution one will try to use the longest possible wavelength and highest magnetic field strength. But for soft X-Rays and VUV-radiation the construction of high resolution monochromators and photon flux difference detectors causes problems.

Table 1. Results of the calculations

Beam energy E_0 in Gev	1.0	2.0	3.0	4.0
Magnetic fields ratio $M = B_2/B_1$	0.5	0.5	0.5	0.2156
Slits length ratio $\xi = \theta_{h_1}/\theta_{h_2}$	21.77	2.05	1.0298	1.0
Working wavelength λ_0 in Å	2.5	1.7	1.7	1.7
Number of photons in each beam N_k per sec.	$4.2 \cdot 10^8$	$7.0 \cdot 10^9$	$1.704 \cdot 10^{10}$	$2.56 \cdot 10^{10}$
Number of photons per sec. in flux difference ΔN_k created by $\Delta\gamma = 10^{-5}$	$3.1 \cdot 10^4$	$1.7 \cdot 10^5$	$2.19 \cdot 10^5$	$6.3 \cdot 10^5$
Number of photons caused by quantum fiuctuation	$2.95 \cdot 10^4$	$1.2 \cdot 10^5$	$1.84 \cdot 10^5$	$2.26 \cdot 10^5$
Number of photons caused by angle error $\delta\theta = 10^{-5}$rad.	$2.0 \cdot 10^4$	$2.44 \cdot 10^5$	$3.14 \cdot 10^5$	$1.0 \cdot 10^6$
Number of photons caused by magnet error $\Delta B/B_0 = 10^{-5}$	$2.0 \cdot 10^4$	$8.3 \cdot 10^4$	$1.06 \cdot 10^5$	$3.32 \cdot 10^5$
Error flax caused by wave band $\delta\lambda/\lambda = 2.10^{-3}$	$2.34 \cdot 10^3$	$2.73 \cdot 10^4$	$5.5 \cdot 10^4$	$1.44 \cdot 10^5$
Resolution in part of E_O	$1.32 \cdot 10^{-5}$	$1.75 \cdot 10^{-5}$	$1.74 \cdot 10^{-5}$	$1.72 \cdot 10^{-5}$

TECHNICAL REALIZATION

The equipment consists of two magnets with different field strengths B_1, B_2 for generating synchrotron radiation with the critical wavelengths λ_1, λ_2. For compensating the overall deflection this magnet system will be part of a three pole wiggler (Figure 3).

The detector consists of two ionization chambers IC_1, IC_2 with connected collecting plates. The high voltage plates are connected to equal voltage but opposite sign. This technique permits us to measure the difference of the photon flux with the accuracy better than 10^{-5} [7].

Two crystal monochromators will be used for selecting the required wavelength. For even higher accuracy in selecting and measuring the wavelength a special K-edge absorption spectrometer may be used.

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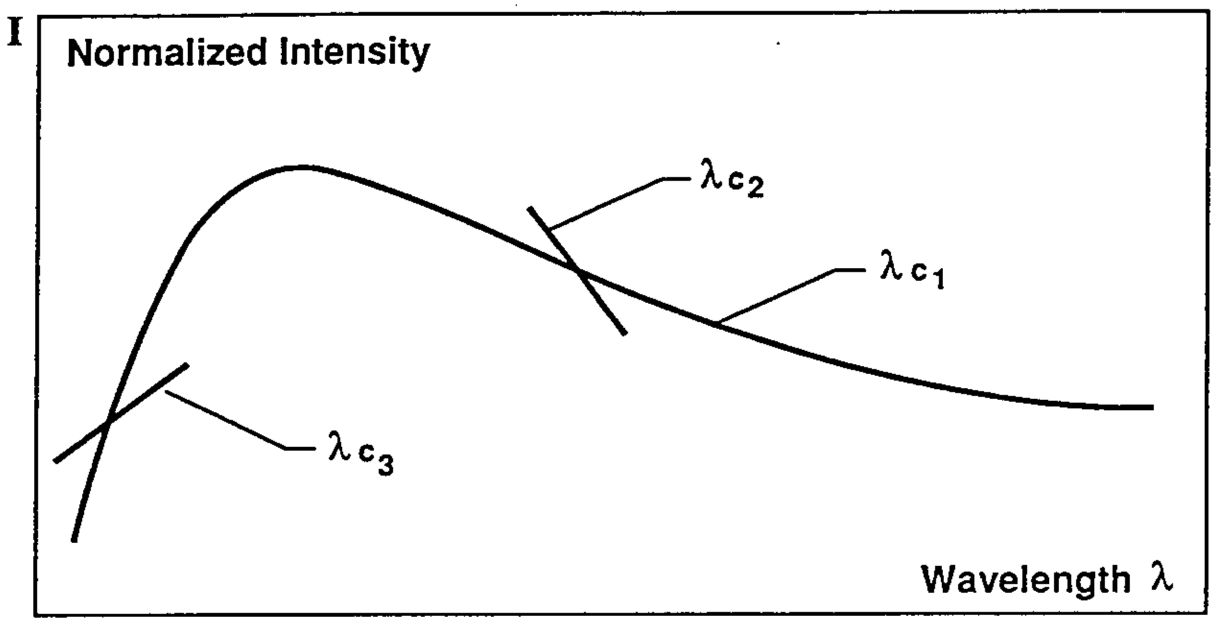

Figure 2. Different slopes of spectral lines at crossing points

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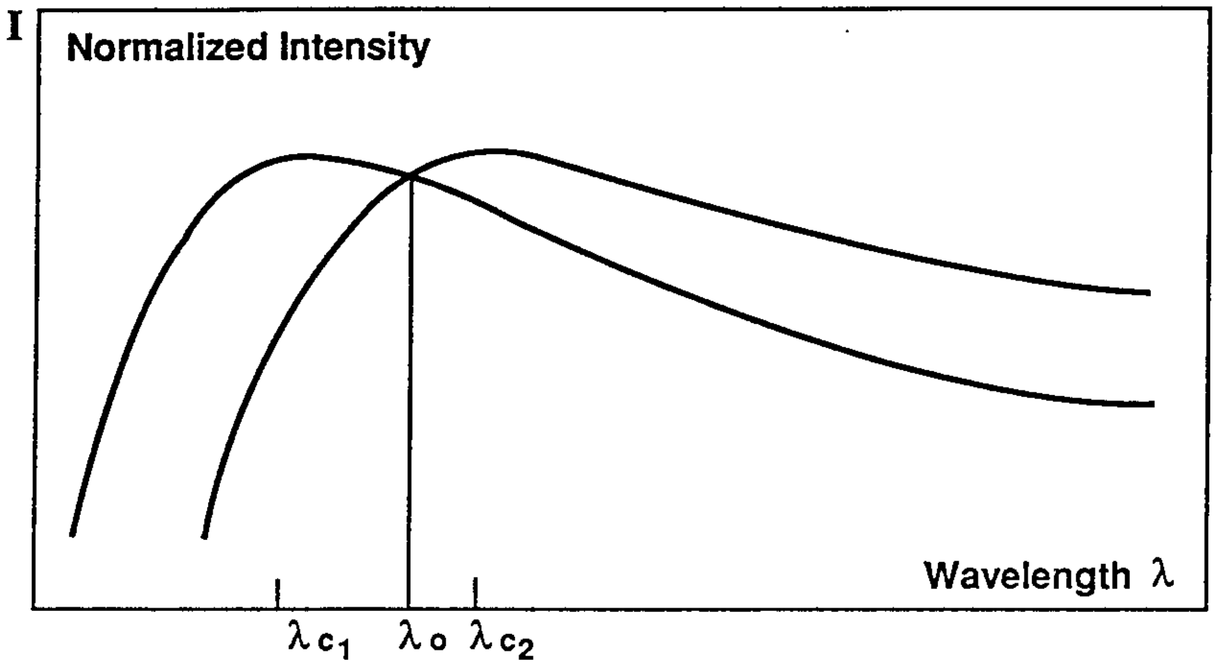


Figure 1. Spectra for different critical wavelengths

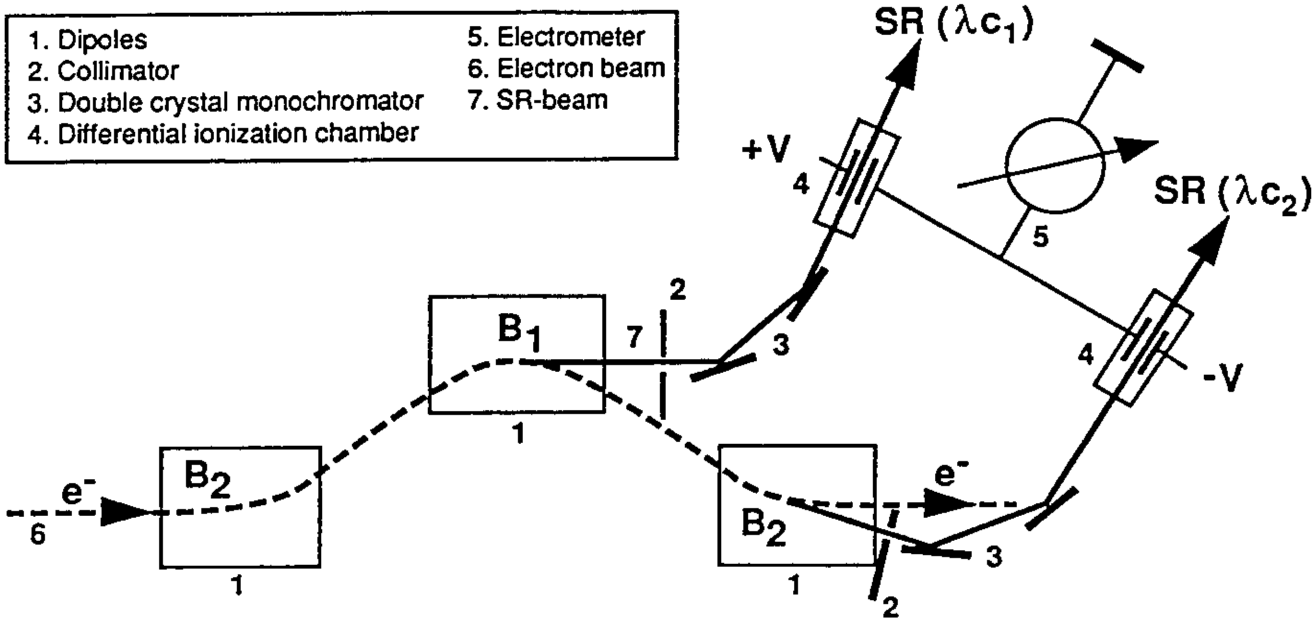
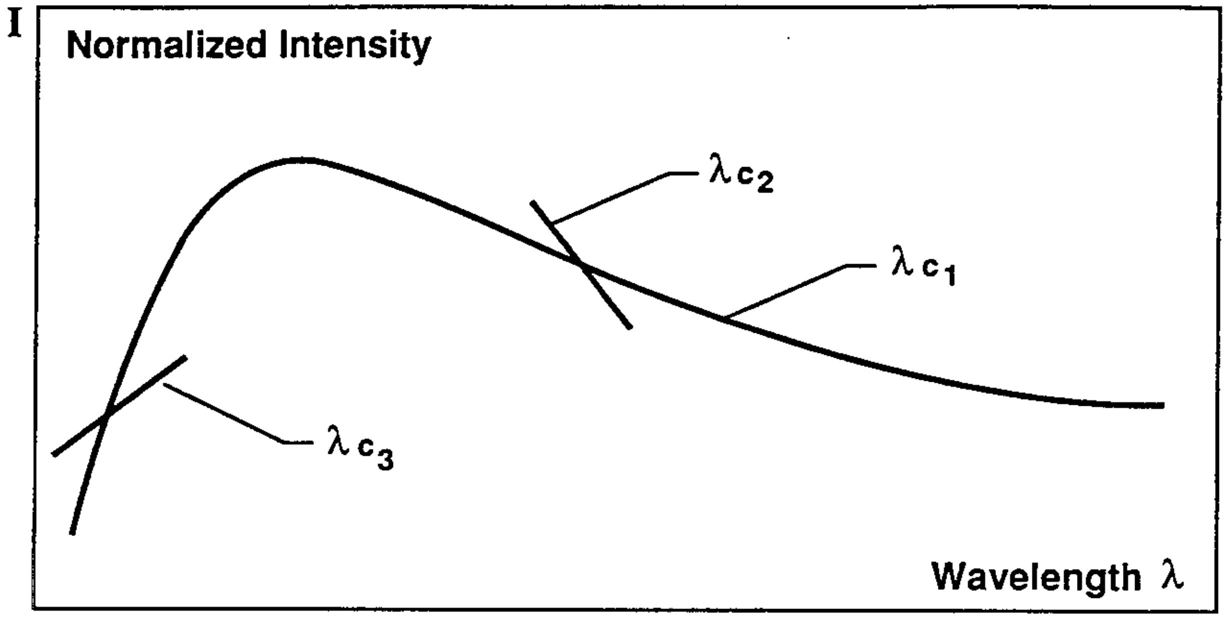


Figure 3. Principal layout of the energy measurement.



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