CEBAF

0205 9100 003 274 1

CEBAF-TH-91-18

CEBAF

The Continuous Electron Beam Accelerator Facility
Theory Group Preprint Series

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150

Non-Local Condensates in the Nambu Model

David Kahana*[1] and Martin Lavelle†[2]

*Physics Div., CEBAF, 12000 Jefferson Ave.
Newport News, Virginia 23606
USA

and

†Institut für Theoretische Physik
Universität Regensburg
W-8400 Regensburg
F. R. Germany

ABSTRACT

We consider the condensate $\langle :\bar{\psi}(x)\psi(y): \rangle$ in the chirally symmetric Nambu - Jona-Lasinio model with two flavours. We Taylor expand the condensate and calculate the leading non-local correction in mean field approximation. The QCD generated up and down quark condensates are found to fall off with a scale of 1.6 fm. We also discuss non-local condensates in the context of composite models.

[1]From Sept. 1991 BITNET: KAHANA@KENTPHYS
[2]E-mail address: lavelle@vax1.rz.uni-regensburg.dbp.de

1. Introduction

The effects of non-local condensates on hadronic properties have been calculated both in QCD sum rules [1] and in various models [2]. Non-local condensates naturally occur in sum rules and are then approximated by the first (local) term in their Taylor expansion. Retaining finite widths for the condensates in sum rules seems to influence hadronic structure primarily, and to be less significant for spectroscopy. It has for example been claimed [1] that non-locality leads to drastically different answers for the sum rule for the pion distribution amplitude than other approaches [3].

The values of the local condensates are phenomenologically obtained from some QCD sum rules for use in all others. However, their values are still the objects of vigorous debate. Estimates of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ for example vary widely [4]. For non-local condensates the situation is much less clear. The functional dependence of the condensates on the separation of the fields is not calculable, and the widths of the ansatz functions are rough estimates.

In this context it is interesting to consider non-local condensates in a model where condensates supply the dominant physical contributions, namely the well known Nambu model in the mean field approximation. This model has recently received much attention as a possible low energy effective Lagrangian for hadronic physics [5]. In Sect. 2 we briefly discuss the non-local quark condensate in QCD. In Sect. 3 we evaluate the leading correction to the local quark condensate in the Nambu model. Finally, in Sect. 4 we present some conclusions and a rough estimate of the non-local structure of condensates in composite models.

2. $\langle \bar\psi(x)\psi(0) \rangle$ in QCD

In Ref(1), the following ansatz has been used to incorporate the non-local condensate $\langle \bar\psi(x)\psi(0) \rangle$ into QCD sum rules

$$\langle \bar\psi(x)\psi(0) \rangle = \langle \bar\psi\psi \rangle e^{-r^2/r_0^2}\,, \tag{1}$$

where an (implicit) string ensures gauge invariance of the condensate and we denote the separation between the quarks by r. The width r_0 of this ansatz is initially an unknown.

Similar ansätze were employed to describe other condensates. Specific to $\langle \bar\psi(x)\psi(0) \rangle$ however, is that its width can be estimated as we now describe. Expanding (1) in a Taylor expansion, the first non-trivial relation is

$$\frac{1}{r_0^2} = -\frac{\langle \bar\psi\sigma \cdot G\psi \rangle}{16\langle \bar\psi(x)\psi(0) \rangle} \tag{2}$$

where the equation of motion for light quarks has been used. The value of the condensate $\langle \bar\psi\sigma \cdot G\psi \rangle$ is known [4] from the sum rules, thus one finds $r_0 = 0.89$ fm.

For the higher terms in the Taylor expansion, the equation of motion provides no simplification. Therefore one cannot easily obtain the functional form of the non-local condensate. Also, for condensates other than $\langle \bar\psi(x)\psi(0) \rangle$ there does not generally exist a relationship like Eq(2). Further phenomenological constraints on the values of the non-local condensates may however, be provided by the analysis of systems with one heavy and one light quark [6].

3. $\langle \bar\psi(x)\psi(0) \rangle$ in the Nambu Model

The Nambu model we consider is the chirally symmetric one with two quark flavours. The Lagrangian is:

$$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi + \frac{g^2}{2}\left\{(\bar\psi\psi)^2 - (\bar\psi\gamma_5\tau\psi)^2\right\} + m\bar\psi\psi\,, \tag{3}$$

where the τ_i are the usual Pauli matrices. The dynamically generated mass term has as usual been added and subtracted in Eq(3), so that it does not appear in the equation of motion. This last is:

$$i\partial\!\!\!/\psi = g^2\left\{\psi(\bar\psi\psi) - \gamma_5\tau_i\psi(\bar\psi\gamma_5\tau_i\psi)\right\}\,. \tag{4}$$

Consider now the Taylor expansion of the non-local quark condensate in this model

$$\langle \bar\psi(x)\psi(0) \rangle = \langle \bar\psi\psi \rangle + r_\mu\langle \bar\psi\partial_\mu\psi \rangle + \frac{1}{2}r_\mu r_\nu\langle \bar\psi\partial_\mu\partial_\nu\psi \rangle + \cdots\,. \tag{5}$$

Clearly the expectation value of the first correction vanishes. Lorentz averaging yields for the second term

$$\frac{1}{2}r_\mu r_\nu\langle \bar\psi\partial_\mu\partial_\nu\psi \rangle = \frac{1}{8}r^2\langle \bar\psi\partial^2\psi \rangle = -\frac{1}{8}r^2\langle \bar\psi i\partial\!\!\!/ i\partial\!\!\!/\psi \rangle\,. \tag{6}$$

We now want to use the equation of motion to rewrite this term as a local condensate. After some algebra we obtain ($\gamma_5^2 = 1\,, \tau_i\tau_j = i\epsilon_{ijk}\tau_k + \delta_{ij}$)

$$\langle\bar\psi\partial^2\psi\rangle = -g^4\langle(\bar\psi\psi)^3 - i\epsilon_{ijk}(\bar\psi\tau_i\gamma_5\psi)(\bar\psi\tau_j\gamma_5\psi)(\bar\psi\tau_k\psi) - (\bar\psi\psi)(\bar\psi\tau_i\gamma_5\psi)(\bar\psi\tau_i\gamma_5\psi)\rangle\,. \quad (7)$$

To obtain this result we have used the following trick

$$\begin{aligned}\langle\bar\psi\gamma_\mu\psi\partial_\mu\bar\psi\psi\rangle =&\langle\partial_\mu((\bar\psi\gamma_\mu\psi)(\bar\psi\psi))\rangle - \langle(\partial_\mu J_\mu)\bar\psi\psi\rangle\\ =&\partial_\mu\langle(\bar\psi\gamma_\mu\psi)(\bar\psi\psi)\rangle = 0\,,\end{aligned} \quad (8)$$

where we employed conservation of vector current. Note that since we have used chiral quarks and there is no anomaly in the model under consideration, axial current is also conserved.

The result (7) is, as promised, local and we can now use the mean field approximation to calculate it. To illustrate this calculation we sketch the procedure for the condensate $\langle\bar\psi\psi\rangle$. This is defined by $\langle\bar\psi\psi\rangle = \langle\Omega|N_0(\bar\psi_0\psi_0)|\Omega\rangle$, where $|\Omega\rangle$ is the physical vacuum, N_0 is the normal ordering operator for the bare quark fields, ψ_0 in the bare (perturbative) vacuum $|0\rangle$. Since for some x the bare and full fields, ψ_f, are identical, we may initially consider the following identity at that point

$$\langle\Omega|T(\bar\psi_0(x)\psi_0(x))|\Omega\rangle = \langle\Omega|T(\bar\psi_f(x)\psi_f(x)|\Omega\rangle\,. \quad (9)$$

From Wick's theorem both sides can be written as

$$\langle\Omega|T(\bar\psi_0(x)\psi_0(x))|\Omega\rangle = \langle\Omega|N_0(\bar\psi_0(x)\psi_0(x)|\Omega\rangle - S_0(x,x)\,, \quad (10)$$

and

$$\langle\Omega|T(\bar\psi_f(x)\psi_f(x))|\Omega\rangle = \langle\Omega|N_f(\bar\psi_f(x)\psi_f(x)|\Omega\rangle - S_f(x,x)\,, \quad (11)$$

where N_f is the normal ordering operator for the full fields in the full vacuum and S_0 and S_f are the bare and dressed quark propagators respectively. Hence the normal ordered term in (11) vanishes. Since we use chiral bare quarks the singular integral

$$S_f = \int\frac{d^4k}{(2\pi)^4}\frac{i}{\not k} = 0\,, \quad (12)$$

can be consistently set to zero, as a careful analysis shows. In this way we obtain

$$\langle\bar\psi\psi\rangle = -\mathrm{tr}\int^\Lambda\frac{d^4k}{(2\pi)^4}\frac{i}{\not k - M}\,, \quad (13)$$

where Λ is the cutoff and M is the mass of the pseudoparticle. The terms in (7) may be found through a straightforward extension of the above. We obtain

$$\begin{aligned}\langle(\bar\psi\psi)^3\rangle =&\langle\bar\psi\psi\rangle^3\left\{1 - \frac{3}{8N_c} + \frac{2}{(8N_c)^2}\right\}\,,\\ -i\epsilon_{ijk}\langle(\bar\psi\tau_i\gamma_5\psi)(\bar\psi\tau_j\gamma_5\psi)(\bar\psi\tau_k\psi)\rangle =&0\,,\\ \langle(\bar\psi\psi)(\bar\psi\tau_i\gamma_5\psi)(\bar\psi\tau_i\gamma_5\psi)\rangle =&\langle\bar\psi\psi\rangle^3\left\{\frac{-3}{8N_c} + \frac{3}{(8N_c)^2}\right\}\,,\end{aligned} \quad (14)$$

Thus we obtain our main result

$$\langle\bar\psi(x)\psi(0)\rangle = \langle\bar\psi\psi\rangle - \frac{g^4}{8}r^2\langle\bar\psi\psi\rangle^3\left\{1 - \frac{1}{(8N_c)^2}\right\} + \mathcal{O}(r^4)\,. \quad (15)$$

Note that using the gap equation we find $g^4\langle\bar\psi\psi\rangle^2 = M^2$. Therefore the fall-off scale is simply related to the constituent mass. If we assume, both for simplicity and to facilitate comparison with Ref. 1, a Gaussian fall-off of the form of (1) we obtain $r_0 = 1.6$ fm. (We use $M = 330$MeV.) Although our model was for two flavours, naïvely inserting the strange quark constituent mass of ~ 500 MeV, yields a scale $r_0 \sim 1$ fm.

4. Conclusions.

We have found the scale of the fall-off of the quark condensate in the Nambu model in mean field approximation. That we could calculate this scale so directly can be understood as a consequence of the fact that in this approximation the vacuum factorisation hypothesis is satisfied by the higher condensates. We obtained the scale $r_0 = 1.6$ fm, which is rather larger than that found in QCD with the standard condensate values of QCD sum rules. We note however, that our scale is very similar to that found from a solution of the free Dirac equation with constituent masses [7] where a complicated functional fall-off in terms of Bessel functions was obtained. For strange quarks a scale of roughly 1 fm was found.

Our scale is rather larger than (say) the nucleon radius, and this implies that the mean field approximation, from the point of view of its using constant (separation independent) condensates, may be expected to be a reasonable approximation for many applications (e.g. most hadronic static properties). However, we believe that if scales comparable to r_0 occur [8], then results obtained using the mean field approximation should be treated with care. We stress that although the mean field vacuum has a non-local structure, this has no effect on any hadronic properties calculated in this approximation! Introducing non-locality may be a simple way of going beyond the mean field.

There is at present much interest in a Nambu-like mechanism for generating electroweak symmetry breaking [9]. In this context it is amusing to estimate the scale r_0 appropriate to a $\bar{t}t$ condensate. Using $M_t = 160$ GeV, we find $r_0 = 0.003$ fm.

Acknowledgements.

DK thanks CEBAF and the DOE for financial support. ML thanks CEBAF for hospitality, Prof. E. Werner for a discussion on non-local condensates and the BMFT for financial support. We thank Prof. A. Radyushkin for useful discussions.

References

[1] S. V. Mikhailov & A. V. Radyushkin, Soviet J. Nucl. Phys. 49 (1989) 494.

[2] D. Gromes, Phys. Lett. B115 (1982) 482.
P. V. Landshoff & O. Nachtmann, Z. Phys. C31 (1987) 405.

[3] V. L. Chernyak & A. R. Zhitnitsky, Nucl. Phys. B201 (1982) 492, B214 (1983) 547 E.
M. J. Lavelle, Z. Phys. C29 (1985) 203.

[4] S. Narison, *QCD Spectral Sum Rules*, World Scientific Lectures in Physics v.26, (1990).

[5] Y. Nambu & G. Jona-Lasinio Phys. Rev. 122, (1961) 345.
A. Kočic, Phys. Rev. D33, (1986) 1785.
A. H. Blin, B. Hiller & M. Schaden, Z. Phys. A 331 (1988) 75.

[6] N. Isgur, private communication.

[7] V. Elias, T. G. Steele & M. D. Scadron, Phys. Rev. D38 (1988) 1584.

[8] See, e. g., U. Meissner, Phys. Lett. B220 (1989) 1.

[9] D. E. Kahana & S. H. Kahana, Phys. Rev. D43 (1991) 2361, and references therein.