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The Axial Anomaly and the Dynamical Breaking of Chiral Symmetry in the $\gamma^*\pi^0 \to \gamma$ reaction

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Abstract

Using the quark triangle diagram for the Adler-Bell-Jackiw axial anomaly, we calculate the form factor for the $\gamma^*\pi^0 \to \gamma$ transition. This form factor depends on the quark mass, and we predict the right behavior with $m_q \simeq 250 MeV$, the same quark mass generated by the dynamical breaking of chiral symmetry through a Nambu-Jona-Lasinio mechanism.

The size of the axial anomaly[1] which occurs in the two-photon decay amplitude of a neutral pion ($\pi^0 \to \gamma\gamma$) can be fairly well explained in model independent approaches.[1-6] If we introduce an additional variable, such as the square of the four-momentum of one of the (virtual) photons, Q^2, or the temperature and density of hadronic matter, we may study the dynamics of the axial anomaly. In this letter we report on a study of the Q^2 dependence of the $\gamma^*\pi^0 \to \gamma$ transition form factor, $F_{\gamma\pi}$. We find that this dependence is very sensitive to the value of the dynamical quark mass used to calculate it, and that a value determined both by a fit to a number of pion observables, and from a consideration of the dynamical breaking of chiral symmetry, gives an excellent account of the recent data.

Recently, the form factor for the $\gamma^*\pi^0 \to \gamma$ transition was measured for space-like momentum ($Q^2 > 0$) of the virtual photon (γ^*).[7] Motivated by the Vector Meson Dominance (VMD) hypothesis, Behrend, *et. al.* parameterized the observed form factor as a monopole. At $Q^2 = 0$, this form factor coincides with the two-photon decay amplitude of the neutral pion, which was well explained a long time ago by an explicit calculation of the fermion triangle diagrams[2] shown in Fig. 1. The dependence of these diagrams on the fermion (quark) mass at $Q^2 = 0$ can be eliminated from the expression,[8] and the remaining dependence on the quark degrees of freedom is trivial; the amplitude is multiplied by the number of colors (n_c) and a factor depending in the quark charges ($e_u^2 - e_d^2 = \frac{1}{3}e^2$).

The two-photon decay of a neutral pion is also very successfully described by the chiral Lagrangians developed by Wess, Zumino[5] and Witten[6]. Beyond the tree level, however, the loop corrections are found to diverge if one of the two photons is virtual ($Q^2 \neq 0$), while these corrections cancel if the two photons are real.[9] Thus, the use of chiral perturbation theory to calculate the form factor requires a renormalization procedure.[10] Two other independent approaches, one[3] based on the noninvariance of the path-integral measure under chiral transformations, and one[4] based on the operator product expansion,[11] also do not determine the mass scale of the "off-shell" axial anomaly.

In this letter we assume that the constituent quark mass is generated through the spontaneous breaking of chiral symmetry in QCD, and that the Nambu-Goldstone boson

generated by the same mechanism is the pion. Using these relations, we are able to determine the size of the dynamical quark mass from a study of pion properties, and then use the resulting information to predict the form factor $F_{\gamma\pi}$.

To illustrate the sensitive dependence of the $\gamma^*\pi^0 \to \gamma$ form factor on the quark mass, we will first calculate it from the simple linear σ-model.[8] Then, using a covariant generalization[12] of the original Nambu-Jona-Lasinio model, we introduce a mechanism for generating the dynamical breaking of chiral symmetry. This model fixes the quark mass and removes the model dependence inherent in the simple linear σ model, and also gives a description of the internal $q\bar{q}$ momentum distribution of the pion.

The $\pi\gamma\gamma$ amplitude ($M_{\pi\gamma\gamma}$) is calculated from the triangle diagrams shown in Fig.1. In the simple linear σ model, these diagrams can be expressed in terms of the Feynman parameterized integral[8]

$$M_{\pi\gamma\gamma} = \epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma \frac{e^2 gm}{2\pi^2}\int_0^1 dx\int_0^{1-x} dy \frac{1}{m^2 - 2k_1\cdot k_2 xy + k_1^2(x^2 - x) + k_2^2(y^2 - y)}, \quad (1)$$

where $k_1(k_2)$ and $\epsilon_1(\epsilon_2)$ are the four momentum and polarization of photon 1(2). We assume that the fermion has an effective charge $e^2 = n_c(e_u^2 - e_d^2)$ and mass m. Here, p is the four momentum of the pion $[p^2 = (k_1 + k_2)^2 = m_\pi^2 = (138MeV)^2]$, and g is the coupling constant of the fermion field with the π and σ mesons: $\mathcal{L}_{int} = g\bar{\Psi}(\sigma + i\gamma^5\pi)\Psi$.

The amplitude for two-photon decay can be obtained from Eq. (1) using the real photon conditions $k_1^2 = k_2^2 = 0$,

$$\begin{aligned} M_{\pi^0\to\gamma\gamma} &= \epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma \frac{e^2}{4\pi^2}\frac{g}{m}\left[1 + \mathcal{O}(\frac{p^2}{12m^2})\right], \\ &= \epsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu k_1^\rho k_2^\sigma \frac{\alpha}{\pi f_\pi}, \end{aligned} \quad (2)$$

where $\alpha = \frac{e^2}{4\pi}$. The relation $m = gf_\pi$ (the Goldberger-Treiman relation to lowest order[8] in the linear σ model), and the soft pion limit, $\mathcal{O}(\frac{p^2}{12m^2}) \to 0$, are used to obtain the second equality. This gives the decay width $\Gamma_{\pi^0\to\gamma\gamma} = \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2} = 7.8eV$, which is independent of the fermion mass m. (The next order correction is $\sim \mathcal{O}(\frac{p^2}{12m^2})$. This would be larger than 1 if the current quark masses were used for the fermion, but with constituent masses it is small.) The $\gamma^*\pi^0 \to \gamma$ form factor, $F_{\gamma\pi}(q^2)$, is obtained by inserting $k_2^2 = (-q)^2 = -Q^2$ (an incoming virtual photon in initial state) and $k_1^2 = (p+q)^2 = 0$ (a real photon in the final state) into Eq. (1) and omitting ϵ_2,

$$M^{\mu}_{\gamma^*\pi^0\to\gamma} = \epsilon^{\mu\nu\rho\sigma} p_\nu \epsilon_{1\rho} q_\sigma \frac{e^2 g m}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy \frac{1}{m^2 + (q^2 - p^2)xy + q^2(y^2 - y)}$$

$$= \epsilon^{\mu\nu\rho\sigma} p_\nu \epsilon_{1\rho} q_\sigma F_{\gamma\pi}(Q^2). \qquad (3)$$

The numerical result is shown in Fig. 2a for $m = 100, 200$ and $300 MeV$, where $g = m/f_\pi$ has been used again. Note that the result with $m = 200\ MeV$ agrees with the experimental data, and this mass scale is consistent with the soft pion approximation, $\mathcal{O}(\frac{p^2}{12m^2}) \ll 1$, used in the calculation of two photon decay width. However, this simple linear σ model form factor is quite sensitive to the size of the fermion mass, and a more fundamental calculation requires that we find some other way to fix this mass.

To accomplish this, we introduce a mechanism for generating the quark mass, and relating it to the momentum distribution of the pion. The model, a generalization of the Nambu-Jona-Lasinio (NJL) model,[13] employs a covariant, separable form for the quark-antiquark ($q\bar{q}$) interaction, with a single mass scale (Λ).[12] Such a separable form is frequently used to treat the pairing problem in the BCS theory.[14] The dynamical quark mass which emerges from this model, and the $q\bar{q}$ momentum distribution in the pion wave function, are both related to the same parameters.

The Nambu-Jona-Lasinio (NJL) interaction is a contact interaction of the form $V^{NJL} = G[II - (\gamma^5\tau)(\gamma^5\tau)]$ with explicit chiral symmetry for the u- and d quarks. A momentum cutoff is introduced with $\Lambda_{NJL} = 0.7 \sim 1.0 GeV$[15] to avoid the divergence of loop integrals caused by the contact interaction. We modify this NJL interaction by using a covariant

separable interaction, still keeping the essential aspects of the original model. Our model interaction is

$$V_{\alpha\beta;\delta\gamma}(k',k) = g f(k'^2) f(k^2) [I_{\alpha\beta} I_{\delta\gamma} - (\gamma^5\tau)_{\alpha\beta}(\gamma^5\tau)_{\delta\gamma}], \qquad (4)$$

where a function $f(k^2)$ $[f(k'^2)]$ depends on the relative momentum, k $[k']$, of the $q\bar{q}$ pairs in the initial [final] state, and we use a monopole function $f(k^2) = 1/(k^2 - \Lambda^2)$ with a scale parameter Λ. Using this interaction, the Bethe-Salpeter (BS) equation for the bound state vertex function is

$$\Gamma_{\alpha\beta}(k';p) = i\int \frac{d^4k}{(2\pi)^4} V_{\alpha\beta;\delta\gamma}(k',k) S_{\gamma\gamma'}(k+\frac{p}{2}) \Gamma_{\gamma'\delta'}(k;p) S_{\delta'\delta}(k-\frac{p}{2}). \qquad (5)$$

where the quark propagator is $S(p) = [\not{p} - m_0 - \Sigma(p^2) + i\epsilon]^{-1}$, with m_0 the current quark mass, and $\Sigma(p^2)$ the quark self energy. The solution for the pion (at the pion mass $p^2 = m_\pi^2$) has the form $\Gamma(k;p) = \mathcal{N}\gamma^5 \mathcal{F}(k^2)\chi_f\chi_c$, where $\mathcal{N}$ is the wave function normalization determined by charge normalization, and the normalized flavor and color wave functions are given by χ_f and χ_c, respectively. For a zero mass pion, $p^2 = 0$, Eq. (5) becomes

$$\mathcal{F}(k'^2) = 4in_f g f(k'^2) \int \frac{d^4k}{(2\pi)^4} f(k^2) \frac{\mathcal{F}(k^2)}{k^2 - [m_0 + \Sigma(k^2)]^2}, \qquad (6)$$

where $n_f = 2$ is the number of quark flavors.

The quark self-energy, $\Sigma(p^2)$, is given by the Schwinger-Dyson (SD) equation

$$\Sigma(k'^2) = i\int \frac{d^4k}{(2\pi)^4} V_{\alpha\beta,\delta\gamma}(k',k) S_{\gamma\delta}(k),$$

$$= 4in_f f(k'^2) g \int \frac{d^4k}{(2\pi)^4} f(k^2) \frac{m_0 + \Sigma(k^2)}{k^2 - [m_0 + \Sigma(k^2)]^2}. \qquad (7)$$

Note that in the chiral limit, i.e. $m_0 = 0$, the equations for the quark self-energy (7) and for the momentum distribution of the zero mass pion (6) are identical; if a nontrivial solution of the quark mass Eq. (7) exists, so that $\Sigma(p^2) \neq 0$ and a non-zero quark mass is generated, then a zero mass pion must also exist with the $q\bar{q}$ momentum distribution $\mathcal{F}(k^2)$. For such solutions to exist, the coupling strength $g \geq g_c$, where the critical value is $g_c = 2\pi^2\Lambda^2$.

We now make the assumption that the dynamical quark mass in the quark propagators can be approximated by a constant, mean value, $< \Sigma > \cong M = m_q - m_0$. Furthermore, since the pion bound state equation and the dynamical quark mass equation are identical in the chiral limit, we can draw conclusions accurate to order $\frac{m_0}{m_q}$ from only one of them (which we choose to be the pion equation), insuring that all subsequent approximations are consistent. To determine the parameters Λ and m_q we calculated the weak decay constant (f_π) and the two-photon decay width ($\Gamma_{\pi^0 \to \gamma\gamma}$) for several choices of these parameters. Some results are (Λ [MeV], m_q [MeV]: f_π [MeV], $\Gamma_{\pi^0 \to \gamma\gamma}$ [eV]) = (700, 200: 90.0, 11.0), (700, 250: 105.0, 7.2), (700, 300: 123.2, 5.8), (400, 200: 75.1, 12.0), (400, 250: 89.3, 8.0), and (400, 300: 100., 5.6). Note that these observables are quite stable under small variations of the parameters. The best fit is given by $\Lambda = 450 MeV$, $m_q = 248 MeV$, $f_\pi = 93.0 MeV$, and $\Gamma_{\pi^0 \to \gamma\gamma} = 7.74 eV$. [See Ref. 12 for more details.]

The eigenvalue equation for the pion mass which follows from Eq. (5) is

$$1 = -i n_f g \int \frac{d^4k}{(2\pi)^4} f^2(k^2) \frac{4m_q^2 + p^2 - 4k^2}{[(k - \frac{p}{2})^2 - m_q^2][(k + \frac{p}{2})^2 - m_q^2]}. \quad (8)$$

In the chiral limit, where $p^2 = 0$ and $m_0 = 0$, this equation reduces to

$$1 = 4 i n_f g \int \frac{d^4k}{(2\pi)^4} f^2(k^2) \frac{1}{k^2 - M^2}. \quad (9)$$

With the best choices, $\Lambda = 450 MeV$ and $m_q = 248 MeV$, and using $m_\pi = 138.0 MeV$, the coupling constant obtained from (8) is $g = 2.795\ \pi^2\Lambda^2$. Using this same coupling constant in (9) gives the mean value $< \Sigma > = M = 234\ MeV$. This gives a reasonable estimate for the current quark mass, $m_0 \cong 14\ MeV$.

As a test of our method, including our choice of parameters, the pion charge form factor obtained using our best set of parameters is shown in Fig. 3. The agreement with

experimental data is very good. The charge radius, r_π, is found to be 0.74fm. Additional effects from two-body interaction currents associated with the rank 1 separable interaction are not included in Fig. 3, although it is found [12] that the interaction current reduces the form factor about 10(40)% at $Q^2 = 2(10) GeV^2/c^2$.

The calculation of the $\pi\gamma\gamma$ amplitude predicted by the generalized NJL model presented above differs from the simple sigma model result, Eq. (1), primarily by the appearance of the BS vertex functions $\Gamma(k;p)$ at the $\pi q\bar{q}$ vertices in the Feynman triangle diagrams shown in Fig. 1. These vertex functions contain the $q\bar{q}$ momentum distribution associated with the pion structure. The amplitude can be written

$$M_{\pi\gamma\gamma} = T(k_1, \epsilon_1 : k_2, \epsilon_2) + T(1 \leftrightarrow 2),$$

where

$$T(k_1, \epsilon_1 : k_2, \epsilon_2) = -i\mathcal{N} Tr\{Q_q Q_q \lambda_f\}\sqrt{n_c}$$

$$\times \int \frac{d^4k}{(2\pi)^4} \mathcal{F}(k^2) Tr\left\{ \not\epsilon_2 S(k - \frac{k_1 - k_2}{2}) \not\epsilon_1 S(k + \frac{p}{2}) \gamma^5 S(k - \frac{p}{2}) \right\}. \quad (10)$$

This prediction for the $\gamma^*\pi^0 \to \gamma$ form factor is shown in Fig.2b. Excellent agreement with the experimental data is obtained. For comparison, Fig. 2b also shows the monopole fit used in the experimental analysis.

Our predictions are in remarkable agreement with the experimental data for both the charge and $F_{\gamma\pi}$ form factors. The VMD model also works well for these two form factors, even though the vector mesons are far away from their mass shell at these space-like momenta. VMD effects are not explicitly included in our calculations, but duality[17,18] suggests that the $q\bar{q}$ contribution to the quark loop in Fig. 1 should give the same result as the propagation of vector mesons when we are far from the meson pole.

In conclusion, we emphasize that while the $\pi^0 \to \gamma\gamma$ amplitude can be calculated in a model independent way in which any dependence on the quark mass can be removed, the same is not true for the form factor which determines the $\gamma^*\pi^0 \rightarrow \gamma$ reaction. Here it is essential to use a quark mass $m_q = 200 \sim 250 MeV$ in order to predict the right behavior of these form factors, and this quark mass is determined by the low energy properties of the pion, consistent with the Schwinger-Dyson equation, and in agreement with the value used by Sakurai, Schilcher and Tran in their discussion of quark-meson duality.[17]

The first author (H. I.) thanks Prof. G. E. Brown for very useful and valuable advice. The work of H. I. is supported in part by Department of Energy Grant Number DE-FG05-86-ER40270. The work of W.B. was partially supported by the National Science Foundation, Grant RII-8704038. The work of F.G. was partially supported by the Department of Energy Grant Number DE-FG05-88ER40435.

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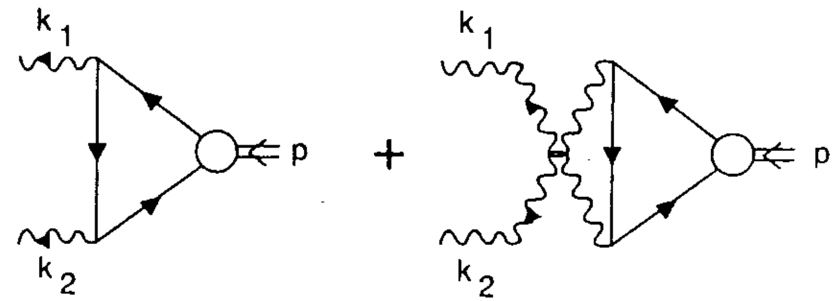

Fig. 1

Triangle diagrams for the $\pi\gamma\gamma$ amplitude, where the solid line, wavy line and double line are the fermion, photon and pion respectively.

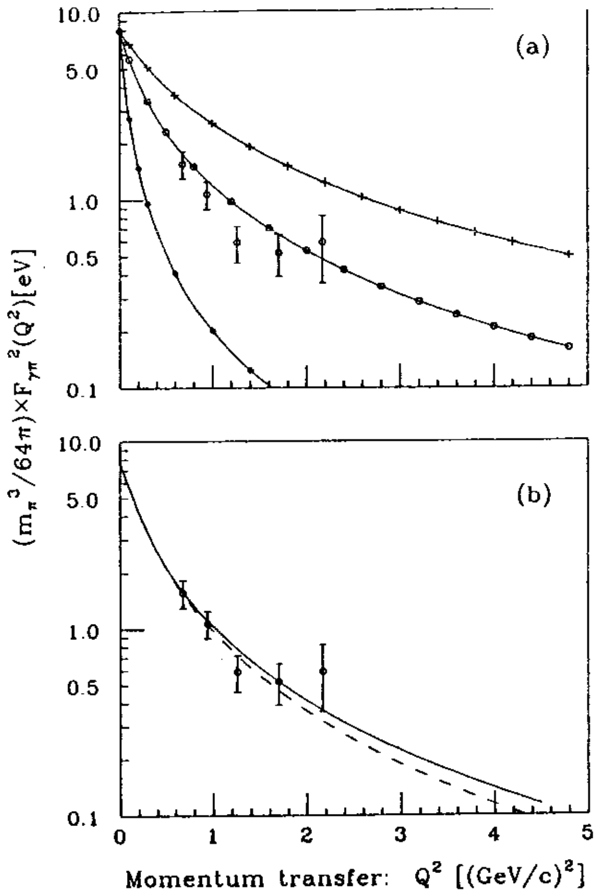

Fig. 2

The theoretical predictions and the experimental data[7] for the $\gamma^*\pi^0 \to \gamma$ form factor, $F_{\gamma\pi}(Q^2)$, where $Q^2 = -q^2 > 0$. (a) The calculation with the linear σ model, Eq. (3), with $m = 300MeV$ (crosses), $m = 200MeV$ (circles), $m = 100MeV$ (diamonds). (b) The result of this letter with the generalized Nambu-Jona-Lasinio model, Eq. (10), with $\Lambda = 450MeV$ and $m_q = 248MeV$. The dash line is the dipole fit[7] to the experimental data, $F_{\gamma\pi}(Q^2) \sim 1/[Q^2 + \lambda^2]$ with $\lambda = 748 \pm 30MeV$, where the charge radius is $0.65 \pm 0.03fm$.

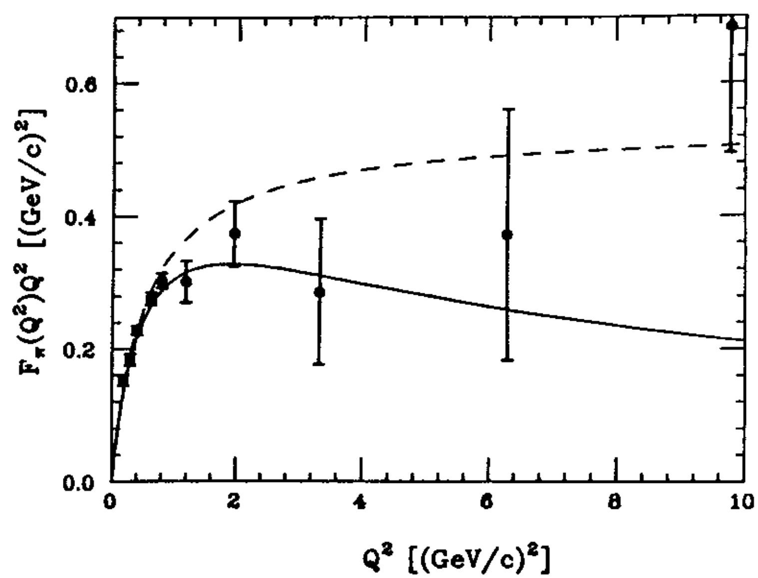

Fig. 3

The charge form factor of the pion, $F_\pi(Q^2)$. The solid line is the theoretical result obtained with the present model ($\Lambda = 450MeV$ and $m_q = 248MeV$), and the experimental data are from Ref.16. The dashed line is given by the Vector Meson Dominance model with the pole mass of $\lambda = 748MeV$.