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Form factors of semileptonic B and D meson decays in the quark confinement model

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Abstract

The extension of the quark confinement model (QCM) is suggested to study weak decays of heavy mesons containing a single heavy quark. It is assumed that the confinement forces define the interactions between light quarks only and does not influence the behavior of heavy quarks which are described as ordinary Fermi particles with large masses. The leptonic decay constants and weak form factors of heavy mesons are calculated which are needed for determining the Cabibbo-Kobayashi-Maskawa matrix elements. The comparison with the predictions of new symmetry discovered by Isgur and Wise for such decays and nonrelativistic quark model (NQM) is performed.

1. Introduction.

The study of weak decays of heavy mesons plays a crucial role in determining the Cabibbo-Kobayashi-Maskawa matrix elements. Of a special interest are heavy mesons containing a single heavy quark along with light degrees of freedom. Nowadays, it is established that heavy quarks interact with other color degrees of freedom as spinless point sources of color. This gives rise to the new symmetries first discovered by Isgur and Wise [1] for weak decays of such mesons. These symmetries allow one to reduce the number of independent form factors characterising the semileptonic decays of B and D mesons. The remained independent form factors should be obtained from a dynamical model.

Traditionally, the nonrelativistic quark model (NQM) is used to describe these form factors [2, 3]. There is a lot of calculations of these form factors in other approaches: relativistic quark potential model [4], phenomenological analysis involving vector dominance and current algebra [5], QCD sum rules [6]-[9], lattice simulations [10].

In the papers [11, 12] we have developed the relativistic quark model with taking into account confinement of light quarks. In this model the hadron interactions are described by quark diagrams averaged over vacuum gluon backgrounds. The confinement hypothesis means that this averaging provides an abcence of singularities in quark diagrams corresponding to quark production. All physical matrix elements are defined by the universal confinement function describing the behavior of quarks at large distances. The calculations of numerous low-energy effects of the meson-meson and meson-baryon interactions performed in the QCM [12] have shown that the model allows one to describe with quite a good accuracy both the static hadron characteristics, such as decay widths, magnetic moments etc., and more sophisticated ones as form factors, phase shifts, etc.

In the paper [13] we gave the extension of the QCM for applying to study the weak decays of heavy mesons containing a single heavy quark along with light degrees of freedom. It was assumed that the confinement forces defines the interactions between the light quarks only and does not influence the behavior of heavy quarks which are described as usual Fermi particles with large masses.

The main aim of this work is to describe the behavior of form factors characterizing semileptonic decays $P \to P'l\nu$ ($P = D, B; P' = \pi, K$) and $B \to Dl\nu$. We use these form factors to calculate the electron spectrum in these decays. The comparison of our results with the predictions of new symmetry developed by Isgur and Wise for such decays and nonrelativistic quark model (NQM) is performed.

2. QCM: light and heavy quarks.

In the QCM, hadron interactions are described by the quark diagrams, e.g., the quark loops in the case of light mesons, see, Fig.1. The hadron-quark verteces are given by the interaction Lagrangian

$$L_H(x) = g_H H(x) J_H(x) \tag{1}$$

where H is a hadron field and J_H is the corresponding quark current. The coupling constant g_H is defined by the so-called compositeness condition [11] which means that the renormalization constant of hadron wave function is equal to zero

$$Z_H = 1 + \frac{3g_H^2}{(2\pi)^2}\tilde{\Pi}'_H(m_H^2) = 0 \tag{2}$$

where $\tilde{\Pi}'_H$ is the derivative of the mass operator. Physically it means that the probability of finding the hadron H in the bare state is equal to zero. In other words, a hadron H is a bound state of quarks.

The following assumption is about the confinement of light quarks. It has been proposed that there exist vacuum gluon configurations that ensure confinement of colored objects. We do not use any concrete gluon backgrounds, only suggest that the averaging over vacuum background fields B_{vac} of the quark diagrams leads to the smearing of quark masses so that no quarks can be found in the observable hadron spectrum. Technically, we change the averaging over B_{vac} with measure $d\sigma_{vac}$ to the one-multiple integral

$$\begin{aligned} &\int d\sigma_{vac} tr[M(x_1)S(x_1x_2|B_{vac})...M(x_n)S(x_nx_1|B_{vac})] \rightarrow \\ \rightarrow &\int d\sigma_v tr[\Gamma_1 S_v(x_1 - x_2)...\Gamma_n S_v(x_n - x_1)] \end{aligned} \tag{3}$$

where

$$S_v(x_1 - x_2) = \int \frac{d^4p}{(2\pi)^4 i} e^{-ip(x_1 - x_2)} \frac{1}{v\Lambda_q - \not{p}}.$$

The measure $d\sigma_v$ is defined as

$$\int \frac{d\sigma_v}{v - z} = G(z) = a(-z^2) + zb(-z^2) \tag{4}$$

The parameter Λ_q characterizes the confinement range. The function $G(z)$ called the confinement function is an entire analytical function which decreases faster than any

degree of z in an Euclidean direction $z^2 \to -\infty$. This requirement provides the absence of singularities corresponding to quark productions in the physical matrix elements and makes all diagrams to be finite. $G(z)$ is a universal function, i.e., is independent of color and flavor. The choice of $G(z)$ is one of the model assumptions. However, as calculations have shown, only integral characteristics of the function $G(z)$ are important for the description of low-energy physics [12] and its shape is chosen from a convenience under calculations. Some restrictions on the shapes $a(u)$ and $b(u)$ can be obtained from the condition of coincidence of the QCM-predictions with the well-established low-energy approaches as chiral theory, vector dominance model etc. for small external momenta.

To demonstrate this coincidence let us consider the main low-energy physical values. The interaction Lagrangian is chosen as

$$L_H = \frac{g_\pi}{\sqrt{2}}\bar{q}\hat{\pi}i\gamma^5 q + \frac{g_\rho}{\sqrt{2}}\bar{q}\hat{\rho}_\mu\gamma^\mu q + \frac{g_\omega}{\sqrt{2}}\bar{q}\hat{\omega}_\mu\gamma^\mu q \tag{5}$$

where $\hat{\pi} = \pi^i\tau^i$, $\hat{\rho}_\mu = \rho^i_\mu\tau^i$. Electroweak interactions are introduced in a standard manner. The quark diagrams describing physical processes are calculated according to confinement ansatz (3) (for details see [11, 12]). The relations between the physical values obtained in the limit of zero hadron masses are shown in Table 1. The notation is $B_0 = \int_0^\infty du b(u)$, $A_0 = \int_0^\infty du a(u)$. One can see, if the following conditions are valid

$$\begin{aligned} &a(0) = b(0) = 1; \qquad -a'(0) = 1; \qquad a''(0) = 2; \\ &A_0 = B_0 = 2; \qquad \Lambda_q = 236 MeV. \end{aligned} \tag{6}$$

the well-known low-energy relations are reproduced. It is to be remarked that the numerical value of the parameter $\Lambda_q = 236$ MeV turns out to coincide with the constituent mass of a light quark. Further, we will use the simplest shape of the functions $a(u)$ and $b(u)$ satisfying the conditions (6):

$$\begin{aligned} a(u) &= (1 - a_1 u + a_2 u^2) exp\{-[(a_2 - 1) + \frac{(1 - a_1^2)}{2}]u^2 - (1 - a_1)u\}; \\ b(u) &= exp\{-u^2 + b_1 u\}. \end{aligned} \tag{7}$$

Here, $b_1 = 1.18$ and the parameters a_1 and a_2 are connected with each other by the condition $A_0 = 2$. The best agreement with experimental data is achieved at $a_1 = 1.5$ and $a_2 = 2.675$. The numerical results obtained with taking into account the physical hadron masses are shown in Table 1.

The calculations of numerous low-energy effects of the meson-meson and meson-baryon interactions performed in the QCM [12] have shown that the model allows one to describe with quite a good accuracy both the static hadron characteristics, such as decay widths, magnetic moments etc., and more sophisticated ones as form factors, phase shifts, etc.

It is clear that additional physical ideas are needed for applying the QCM to heavy quark physics. It have known that heavy quarks weakly interact with vacuum gluon fields, e.g. instantons [14], and they can be considered as spinless point sources of color [1, 2]. Therefore, we can adopt the following picture for describing the processes with heavy and light quarks. The interaction of light quarks is completely defined by the confinement mechanism whereas a heavy quarks is considered as an ordinary Dirac particle with a large mass. According to this notion, we propose the following ansatz for averaging quark diagrams containing a single heavy quark (see, Fig.2):

$$\begin{aligned} & \int d\sigma_{vac} tr[\Gamma_1 S(x_1 x_2|B_{vac})...\Gamma_n S^{heavy}(x_n x_1|B_{vac})] \rightarrow \\ \rightarrow & \int d\sigma_v tr[\Gamma_1 S_v(x_1 - x_2)...\Gamma_n S^{heavy}(x_n - x_1)]. \end{aligned} \tag{8}$$

where

$$S^{heavy}(x) = \int \frac{d^4p}{(2\pi)^4 i} e^{-ipx} \frac{1}{M_Q - \hat{p}}$$

Here, M_Q is a constituent mass of the heavy quark. In other words, it is suggested that the heavy quark is described by an ordinary free propagator with mass M_Q.

It should be emphasized that the ansatz (8) provides confinement of both the heavy quark and a light one, i.e., absence of imaginary parts corresponding to quark productions.

3. Week coupling constants of B and D mesons.

The Lagrangian describing the strong interactions of heavy mesons ($H = B, D, B^*, D^*$) with quarks is written as

$$\begin{aligned} L_I &= g_B[B^0(\bar{b}i\gamma^5 d) + B^-(\bar{b}i\gamma^5 u)] + g_D[D^0(\bar{u}i\gamma^5 c) + D^-(\bar{d}i\gamma^5 c)] + \\ &+ g_B^*[B_\mu^{*0}(\bar{b}\gamma^\mu d) + B_\mu^{*-}(\bar{b}\gamma^\mu u)] + g_D^*[D_\mu^{*0}(\bar{u}\gamma^\mu c) + D_\mu^{*-}(\bar{d}\gamma^\mu c)] + h.c. \end{aligned} \tag{9}$$

The coupling constants $g_{H(H^*)}$ are determined from the compositeness condition (2) where the mass operator $\tilde{\Pi}(p^2)$ is defined by the diagram, Fig.3a, and is written in the following form:

$$\Pi_{HH}(p^2) = \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v tr[i\gamma^5 \frac{1}{v\Lambda_q - \hat{k}} i\gamma^5 \frac{1}{M_Q - (\hat{k} + \hat{p})}] = \tag{10}$$
$$= -\frac{1}{\Lambda_q} \int \frac{d^4k}{4\pi^2 i} tr[G(-\frac{\hat{k}}{\Lambda_q}) \cdot \frac{1}{M_Q - (\hat{k} + \hat{p})}].$$

Here and further, we will use the dimensionless variables:

$$k = \Lambda_q K, \qquad p = \Lambda_q P, \qquad M_Q = \Lambda_q \mu_Q, \qquad m_H = \Lambda_q \mu_H. \tag{11}$$

Then the expression (10) can be written as

$$\Pi_{HH}(p^2) = -\Lambda_q^2 \int \frac{d^4k}{4\pi^2 i} tr[G(-\hat{K}) \cdot \frac{1}{\mu_Q - (\hat{K} + \hat{P})}]. \tag{12}$$

This integral is calculated in the following way:

(1) the trace is calculated;

(2) the transition to Euclidean region is performed both the internal momentum $K_0 \to iK_4$, $K^2 = -K_E^2$ and external one $P_0 \to iP_4$, $P^2 = -P_E^2$;

(3) integration over sphere angles is carried out;

(4) analytical continuation to physical region over external momentum is fulfilled.

The typical integral is

$$\int \frac{d^4K}{\pi^2 i} \frac{f(-K^2)}{\mu_Q^2 - (K+P)^2} = \int_0^\infty du f(u) C(u, P^2)$$

where $u = K_E^2$ and

$$C(u, P^2) = \frac{[\sqrt{(u + \mu_Q^2 - P^2)^2 + 4uP^2} - (u + \mu_Q^2 - P^2)]}{2P^2}. \tag{13}$$

Finally, we have

$$\Pi_{HH}(p^2) = -\Lambda_q^2 I_{HH}(P^2), \tag{14}$$

$$I_{HH}(P^2) = \frac{1}{2} \int_0^\infty du u b(u) + \int_0^\infty du C(u, P^2) \{\mu_Q a(u) + \frac{1}{2}(P^2 - \mu_Q^2 + u) b(u)\}.$$

Substituting (14) into (2) we have the following expression for the coupling constant g_H:

$$g_H = \frac{2\pi}{\sqrt{3}} \frac{1}{\sqrt{I'_{HH}(\mu_H^2)}} \tag{15}$$

where

$$I'_{HH}(\mu_H^2) = \int_0^\infty du\{\mu_Q a(u) C'_{\mu_H^2}(u,\mu_H^2) + \frac{1}{2} b(u)[C(u,\mu_H^2) + (u - \mu_Q^2 + \mu_H^2) C'_{\mu_H^2}(u,\mu_H^2)]\}.$$

The invariant matrix element defining the leptonic weak decay $H \to l\nu$ (see, Fig.2b) is written as

$$M(H \to l\nu) = G_F V_{qQ} f_H \bar{l}\hat{p}(1-\gamma^5)\nu \tag{16}$$

where V_{qQ} is the CKM-matrix element and

$$f_H = \Lambda_q \frac{3g_H}{(2\pi)^2} I_H(\mu_H^2), \tag{17}$$

$$I_H(\mu_H^2) = \int_0^\infty du\{\mu_Q b(u) \frac{[u - (u + \mu_Q^2 - \mu_H^2) C(u,\mu_H^2)]}{2\mu_H^2} - a(u) \frac{[u - (u + \mu_Q^2 + \mu_H^2) C(u,\mu_H^2)]}{2\mu_H^2}\}.$$

The dependence of weak couplings $f_H(H = d, b)$ on the parameter $\Delta = M_H - M_Q$ is shown on Fig.2. It should be remarked that this parameter is not directly connected with the binding energy because the light quark is supposed to be confined. One can see, there is a strong dependence on the parameter Δ. The magnitude of f_D is greater than f_B for the same Δ. If we take $\Delta = 500$ Mev that corresponds to the choice of QCD sum rules (see, e.g.[17]), one can obtain

$$f_D = 150 Mev \qquad f_B = 110 Mev.$$

It is interesting to consider the limit $M_Q = m_H \to \infty$. We have

$$g_H \Rightarrow \frac{2\pi}{\sqrt{3}} \sqrt{\frac{2\mu_Q}{A_0 + B_{\frac{1}{2}}}}, \tag{18}$$

$$I_H \Rightarrow \frac{1}{\mu_Q} \cdot [A_{\frac{1}{2}} + \frac{1}{2} B_1],$$

$$f_H \Rightarrow \frac{\Lambda_q}{\sqrt{\mu_Q}} \cdot \frac{1}{\pi}\sqrt{\frac{3}{2}} \cdot \frac{[A_{\frac{1}{2}} + \frac{1}{2}B_1]}{\sqrt{A_0 + B_{\frac{1}{2}}}}. \tag{19}$$

Here,

$$A_0 = \int_0^\infty du a(u) = 2, \qquad B_0 = \int_0^\infty du b(u) = 2,$$

$$A_1 = \int_0^\infty du u a(u) = 2.04, \qquad B_1 = \int_0^\infty du u b(u) = 1.68,$$

$$A_{\frac{1}{2}} = \int_0^\infty du \sqrt{u} a(u) = 1.91, \qquad B_{\frac{1}{2}} = \int_0^\infty du \sqrt{u} b(u) = 1.72.$$

Such behavior of the weak coupling f_H for large masses is in an accordance with the nonrelativistic potential model (see, e.g.[15]).

For comparison, the numerical results of other approaches are shown in Table 2.

4. The form factors of decays $D^0 \to \pi^+(K^+)e\nu$ and $B^0 \to \pi^+ e\nu$.

The semileptonic weak decays $H \to Pe\nu$ ($P = K, \pi$) are defined by the triangle quark diagram and resonant one (see, Fig.5). It should be emphasized that we use the full propagator for intermediate vector particle obtained as result of summation of the one-loop self-energy insertions. After cumbersome calculations we have the following expression for the matrix element:

$$M(H^0 \to P^+ e\nu) = \frac{G_F}{\sqrt{2}} V_{qQ}(\bar{l}O^\mu \nu_l)[(p+p')^\mu f_+(t) + q^\mu f_-(t)] \tag{20}$$

where $q = p - p'$, $t = q^2$. The weak form factors $f_\pm(t)$ are written as

$$\begin{aligned} f_+(t) &= \frac{3 g_H g_P}{(2\pi)^2} I^+_{PPV}(p^2, p'^2, t) \frac{I^{(1)}_{VV}(m^2_V)}{[I^{(1)}_{VV}(m^2_V) - I^{(1)}_{VV}(t)]}, \\ f_-(t) &= f_+(t) \frac{1}{[I^{(1)}_{VV}(m^2_V) - I^{(1)}_{VV}(t) - t I^{(2)}_{VV}(t)]} \\ &\cdot \{(p^2 - p'^2) + [I^{(1)}_{VV}(m^2_V) - I^{(1)}_{VV}(t)] \frac{I^-_{PPV}(p^2, p'^2, t)}{I^+_{PPV}(p^2, p'^2, t)}\}. \end{aligned} \tag{21}$$

The structure integrals $I^\pm_{PPV}$ and $I^{1,2}_{VV}$ are shown in Appendix.

The differential decay rate for $H^0 \to P^+ e\nu$ has the form [24]

$$\frac{d^2\Gamma}{dxdy} = |V_{qQ}|^2 \frac{G_F^2 m_H^5}{16\pi^3} |f_+(ym_H^2)|^2 (1-2x)[y_{max}(x) - y], \tag{22}$$

where

$$y = \frac{t}{m_H^2}, \qquad x = \frac{E_e}{m_H}.$$

For fixed electron energy, y varies over the region

$$0 \le y \le \frac{4x(x_m - x)}{(1-2x)},$$

where $x_m = (m_H^2 - m_P^2)/(2m_H^2)$ is the maximum value of x.

The decay width can be obtained by integration over x and y. The result is

$$\Gamma(H^0 \to P^+ e\nu) = m_H \frac{(G_F m_H^2)^2}{192\pi^3} |V_{qQ}|^2 \int_0^{y_-} |f_+(ym_H^2)|^2 [(y-y_+)(y-y_-)]^{\frac{3}{2}} \tag{23}$$

where $y_\pm = (m_H \pm m_P)^2/m_P^2$.

It turned out that the form factors depend very slowly on the parameter $\Delta = M_H - M_Q$. Therefore, we will choose $\Delta = 0.5$ in accordance to QCD sum rules (see, [17]). The masses of heavy mesons are taken from Review of Particle Properties [27] ($M_D = 1.87$ GeV, $M_{D^*} = 2.01$ GeV, $M_B = 5.28$ GeV) except for the mass of the B^*-resonance which is taken from the NQM predictions [25] as $m_{B^*} = \sqrt{m_B^2 + 1} = 5.37$ GeV.

The behavior of form factors $f_+(t)$ in the kinematical region is shown on Fig.6. The separate contribution of the triangle diagrams is present. One can see that the resonance contributions play a role only near $t_m = (m_H - m_P)^2$, in a complete accordance with the conclusion of the paper [26].

Most of the models allow one to calculate only the $f_+(0)$ because it is the overlap integral of wave functions. In the Table 3. we give the results for $f_+(0)$ obtained in other approches. But it is more interesting to have the predictions for $f_+(t_m)$ because these values mainly define the electron spectrum.

Here, we give our results for $f_+(t_m)$ taking into account triangle diagrams only and NQM predictions without pole contribution [25]:

	QCM (*triangle diagram*)	NQM (*without pole*)
$D\pi$	1.22	1.34
DK	1.41	1.38
$B\pi$	1.85	1.98

Now let us construct electron spectra for the semileptonic decays $H^0 \to P^+ e\nu$ using obtained form factors. We will use the shapes of an electron spectrum given by formula (19) and normalized on decay width which is determined by (20). For comparison we show the NQM predictions [3, 24]. The shapes of electron spectra are shown on Fig.7 (a,b,c). One can see that our results are in complete accordance with the NQM ones. But the decay widths differ very strongly one from another. It can be easily explained by that the NQM form factors fall very rapidly $\sim exp[-(t_m - t)]$, when t becomes less than t_m.

Of course, it is needed to take into consideration other final states, e.g. vector mesons, for complete analisis of electron spectra. We are planning to do this in our next work.

5. The form factors of decay $B \to De\nu$

Of the special interest in our approach is a consideration of the decay $B \to De\nu$ (see, Fig.8) because two heavy quarks come to the quark loop. It this case, the threshold singularities corresponding to production of b and c quarks may appear at $t \geq (M_b + M_c)^2$. But due to the kinematics of the decay this threshold cannot be reached because $t_{max} = (m_B - m_D)^2 < (M_b + M_c)^2$. Therefore, we will use the free propagators for both heavy quarks.

The important consequence of the Isgur-Wise symmetry for semileptonic decay of heavy meson containing two quarks with large masses is that amplitudes are independent of masses and are determined by a single universal function depending on the dot product of the four-velocities [1, 26].

To check this property in our approach, we calculate the matrix element corresponding to diagram Fig.8

$$M^\mu(p,p') = \frac{3}{(2\pi)^2} g_H g_{H'} \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v tr[\gamma^\mu \frac{1}{M - \hat{k} - \hat{p}} \gamma^5 \frac{1}{\Lambda_q v - \hat{k}} \gamma^5 \frac{1}{M' - \hat{k} - \hat{p}'}] = \tag{24}$$

$$= \frac{3}{(2\pi)^2} g_H g_{H'} \frac{1}{\Lambda_q} \int \frac{d^4k}{4\pi^2 i} tr[\gamma^\mu \frac{1}{M - \hat{k} - \hat{p}} \gamma^5 G(\frac{\hat{k}}{\Lambda_q}) \gamma^5 \frac{1}{M' - \hat{k} - \hat{p}'}].$$

Performing the transition to the dimensionless variables according to (11) we have

$$M^\mu(p, p') = \frac{3}{(2\pi)^2} g_H g_{H'} \Lambda_q \int \frac{d^4K}{4\pi^2 i} tr[\gamma^\mu \frac{1}{\mu - \hat{K} - \hat{P}} G(-\hat{K}) \frac{1}{\mu' - \hat{K} - \hat{P}'}]. \tag{25}$$

When $p^2 = m^2 = M^2 \to \infty$, and $(p')^2 = (m')^2 = (M')^2 \to \infty$, one can easily get the following representations for typical integrals:

$$\int \frac{d^4K}{\pi^2 i} \frac{F(-K^2)}{[\mu^2 - (K+P)^2]} \Rightarrow \frac{1}{\mu} \int_0^\infty du \sqrt{u} F(u), \tag{26}$$

$$\int \frac{d^4K}{\pi^2 i} \frac{F(-K^2)}{[\mu^2 - (K+P)^2][(\mu')^2 - (K+P')^2]} \Rightarrow \frac{1}{2\mu\mu'} \Phi(w) \int_0^\infty du F(u), \tag{27}$$

for any function $F(u)$ decreasing faster than $1/u^n$ for any n.

The function $\Phi(w)$ has the following form

$$\Phi(w) = \frac{1}{2\sqrt{w^2 - 1}} ln \frac{w + \sqrt{w^2 - 1}}{w - \sqrt{w^2 - 1}}, \tag{28}$$

where the variable w is the dot product of the four-velocities of initial and final particles:

$$w = \frac{2pp'}{2mm'} = \frac{m^2 + (m')^2 - q^2}{2mm'}.$$

The value w varies over the region

$$1 \leq w \leq w_0 = \frac{m^2 + (m')^2}{2mm'}.$$

Taking into account the behavior of the coupling constants g_H for large masses (see, (18)), and making use of the expressions for typical integrals, we obtain the final result

$$M^\mu(p, p') = f_+(q^2)(p + p')^\mu + f_-(q^2) q^\mu, \tag{29}$$

where

$$f_\pm(q^2) = \pm \frac{M \pm M'}{\sqrt{4MM'}} \xi(w). \tag{30}$$

The obtained representation is in a complete accordance with the Isgur-Wise symmetry prediction [1, 26]. The function $\xi(w)$ is equal to

$$\xi(w) = \frac{\Phi(w)A_0 + \frac{2}{(1+w)}B_{\frac{1}{2}}}{A_0 + B_{\frac{1}{2}}}. \tag{31}$$

The values A_0 and $B_{\frac{1}{2}}$ are defined above.

The behavior of $\xi(w)$ up to $w_0 = 2$ (B-D-transition) is shown on Fig.9. For comparison the NQM curve is drawn.

The "charge radius" ρ of ξ defined by the expansion

$$\xi(w) = 1 - \rho^2(w-1) \tag{32}$$

is equal to

$$\rho^2_{QCM} = \frac{2A_0 + 3B_{\frac{1}{2}}}{6(A_0 + B_{\frac{1}{2}})} = 0.41.$$

The numerical result for $f_+(0)$ is shown in Table 3.

In conclusion, we calculate the decay width according to the formula (23) for different values of parameter $\Delta = M_H - M_Q \in (0.1 - 0.8)$. Our result is

$$\Gamma(B \to De\nu) = (7.3 - 10.5) \cdot 10^{-12}|V_{cb}|^2 GeV.$$

Using the available experimental data [26]

$$\Gamma(B \to De\nu) = (4.9 \pm 1.6) \cdot 10^{-14} GeV,$$

we find that

$$|V_{cb}| = 0.027 - 0.043.$$

These values does not contradict the modern treatment of experimental data [30]

$$|V_{cb}| = 0.040 \pm 0.010.$$

6. Summary and outlook.

The relativistic scheme is developed to study the weak decays of heavy mesons containing a single heavy quark along with light degrees of freedom. It was assumed that the confinement forces define the interactions between the light quarks only and does not influence the behavior of heavy quarks which are described as ordinary Fermi particles

with large masses. This assumption coincides with the representation about heavy quark as spinless point source of color. The form factors of the semileptonic decays $P \to P'l\nu$ ($P = D, B; P' = \pi, K$) and $B \to Dl\nu$ are calculated in the framework of this approach. It was shown that the main properties of these form factors are in agreement with a new symmetry discovered by Isgur and Wise. The comparison with the nonrelativistic quark model and other approaches are performed. In the next work we are planning to calculate the form factors of decays $H \to Ve\nu$.

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Appendix.

The structure integrals defining the $H \to Pl\nu$-decay are

$$I^{\mu}(p,p') = \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v tr[\gamma^{\mu} \frac{1}{M_Q - \hat{k} - \hat{p}} \gamma^5 \frac{1}{\Lambda_q v - \hat{k}} \gamma^5 \frac{1}{\Lambda_q v - \hat{k} - \hat{p}'}] =$$

$$= (p+p')^{\mu} I^{+}_{PPV}(\frac{p^2}{\Lambda_q^2}, \frac{p'^2}{\Lambda_q^2}, \frac{q^2}{\Lambda_q^2}) + q^{\mu} I^{-}_{PPV}(\frac{p^2}{\Lambda_q^2}, \frac{p'^2}{\Lambda_q^2}, \frac{q^2}{\Lambda_q^2})$$

$$I^{+}_{PPV}(p^2, p'^2, q^2) = \frac{1}{2} \int_0^1 d\alpha \{ z I_S^{a_1}(W_\alpha, P_\alpha^2) + I_0^b(W_\alpha) -$$

$$- \quad z^2 I_S^{b_1}(W_\alpha, P_\alpha^2) - [(1-\alpha)p^2 + \alpha q^2] I_V^{b_1}(W_\alpha, P_\alpha^2) \},$$

$$I^{-}_{PPV}(p^2, p'^2, q^2) = -I_V^b(q^2) - \frac{1}{2} \int_0^1 d\alpha \{ z I_S^{a_1}(W_\alpha, P_\alpha^2) + I_0^b(W_\alpha) -$$

$$- \quad z^2 I_S^{b_1}(W_\alpha, P_\alpha^2) - [(1-\alpha)p^2 + \alpha q^2 - 2\alpha p'^2] I_V^{b_1}(W_\alpha, P_\alpha^2) \}.$$

Here, $W_\alpha = \alpha(1-\alpha)p'^2$; $P_\alpha^2 = (1-\alpha)p^2 - \alpha(1-\alpha)p'^2 + \alpha q^2$, $z = \mu_Q$.

$$I_0^b(y) = B_0 + \frac{y}{4} \int_0^1 du b(-u\frac{y}{4}) \sqrt{1-u};$$

$$I_V^b(x) = \int_0^\infty du b(u) \frac{[u - (u + z^2 + x) C(u,x)]}{2x};$$

$$I_S^{a_1}(y,x) = \int_0^\infty du a(u-y) C'_u(u,x);$$

$$I_S^{b_1}(y,x) = \int_0^\infty du b(u-y) C'_u(u,x);$$

$$I_V^{b_1}(y,x) = \int_0^\infty du b(u-y) \frac{[1 - C(u,x) - (u + x + z^2) C''_u(u,x)]}{2x}.$$

The structure integrals defining the two-point vector-vector loop are

$$I^{\mu\nu}_{VV}(p) = \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v tr[\gamma^{\mu} \frac{1}{M_Q - \hat{k} + \hat{p}} \gamma^{\nu} \frac{1}{\Lambda_q v - \hat{k}}] =$$

$$= g^{\mu\nu} I_{VV}^{(1)}(\frac{p^2}{\Lambda_q^2}) + \frac{p^\mu}{\Lambda_q}\frac{p^\nu}{\Lambda_q} I_{VV}^{(2)}(\frac{p^2}{\Lambda_q^2});$$

$$I_{VV}^{(1)}(x) = B_1 + zI_S^a(x) - z^2 I_S^b(x) - xI_V^b(x) + 2I_{T1}^b(x);$$

$$I_{VV}^{(2)}(x) = I_V^b(x) + I_{T2}^b(x).$$

$$I_S^f(x) = \int_0^\infty du f(u) C(u,x);$$

$$I_V^f(x) = \int_0^\infty du f(u) \frac{[u - (u + z^2 + x)C(u,x)]}{2x};$$

$$I_{T1}^f(x) = \int_0^\infty du f(u) \frac{\{u(u + z^2 - x) - [(u + z^2 - x)^2 + 4ux]C(u,x)\}}{12x};$$

$$I_{T2}^f(x) = \int_0^\infty du f(u) \frac{\{-u(u + z^2 + 2x) + [(u + z^2 + x)^2 - xz^2]C(u,x)\}}{3x^2}.$$

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List of Figures

1. Light quark loop.
2. Light and heavy quark loop.
3. Diagrams describing (a) mass operator and (b) leptonic decay of heavy mesons.
4. Dependence of weak couplings on the masses of heavy quarks.
5. Diagrams describing semileptonic decay of B and D mesons.
6. HP-form factors.
7. $H \to Pe\nu$ electron spectra.
8. Diagrams describing decay $B \to Dl\nu$
9. Form factor $\xi(w)$ of the $B \to De\nu$ decay.

List of Tables

1. Main fit.
2. Leptonic decay constants $f_H(H = D, B)$.
3. Weak form factors at zero momentum.

Table 1.

Process	$m_H = 0$	Physical masses	Expt.
$P \to P$	$g_P = 2\pi\sqrt{\frac{2}{3B_0}}$		
$V \to V$	$g_V = 2\pi\sqrt{\frac{1}{B_0}}$		
$\pi \to \mu\nu$	$f_\pi \equiv \sqrt{2}F_\pi = \frac{2\Lambda_q}{g_\pi}\frac{A_0}{B_0}$	$130MeV$	$132MeV$
$\rho \to \gamma$	$g_{\rho\gamma} = \frac{1}{2\pi}\sqrt{\frac{B_0}{2}}$	0.18	0.20
$\pi^0 \to \gamma\gamma$	$g_{\pi\gamma\gamma} = \frac{a(0)}{4\pi^2 F_\pi}$	$0.276\ GeV^{-1}$	$0.276\ GeV^{-1}$
$\omega \to \pi\gamma$	$g_{\omega\pi\gamma} = 3\pi g_{\pi\gamma\gamma}\sqrt{\frac{2}{B_0}}$	$2.12\ GeV^{-1}$	$2.54\ GeV^{-1}$
$\rho \to \pi\pi$	$g_{\rho\pi\pi} = \frac{1}{g_{\rho\gamma}} = 2\pi\sqrt{\frac{2}{B_0}}$	6.5	6.1
$\gamma \to 3\pi$	$g_{\gamma 3\pi} = \frac{-a'(0)}{4\pi^2 F_\pi^3}$		
5π-vertex	$g_{5\pi} = \frac{1}{80\pi^2 F_\pi^5}\frac{a''(0)}{2}$		
Anomaly Ward identity	$p_\alpha T_5^{\alpha\mu\nu} = 2\Lambda_q T_5^{\mu\nu}\frac{b(0)}{[-a'(0)]} - \frac{i}{2\pi^2}\epsilon^{\mu\nu q_1 q_2}\frac{b(0)a(0)}{[-a'(0)]}$		

Table 2.

References	f_D, MeV	f_B, MeV
M.Suzuki [15]	117 ± 14	75 ± 9
N.Isgur [2]	243	174
P.Cea et al. [16]	182	231
E.Shuryak [17]	220	140
V.Chernyak [18]	160	90
V.Aliev [19]	170	130
C.A.Dominguez [20]	224 ± 26	145-211
S.Narison [21]	173 ± 16	182 ± 18
C.Bernard [22]	$174 \pm 26 \pm 46$	$105 \pm 17 \pm 30$
M.Gavela [23]	194 ± 15	120
QCM ($M_c = 1.3 - 1.8, M_b = 4.7 - 5.2$)	$83 - 165$	$33 - 127$

Table 3.

References	$D\pi$	DK	$B\pi$	BD
J.M.Cline [5]	0.77 ± 0.04	0.77 ± 0.04		
M.Wirbel [4]	0.69-0.78	0.76-0.82	0.33-0.39	0.7
T.M.Aliev [6]		0.6 ± 0.1		
C.Dominguez [7]	0.75 ± 0.05	0.75 ± 0.05	0.4 ± 0.1	
M.Voloshin [8]	0.72	0.72	0.3-0.4	
A.Ovchinnikov [9]				$1. \pm 0.2$
Lattice UCLA [10]	0.63 ± 0.15	0.75 ± 0.20		
Lattice ELC [10]	0.70 ± 0.20	0.74 ± 0.17		
QCM	0.84-0.92	$0.95 - 1.08$	$0.6 - 0.62$	$0.8 - 1.25$

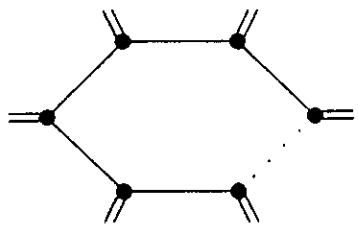


Fig.1

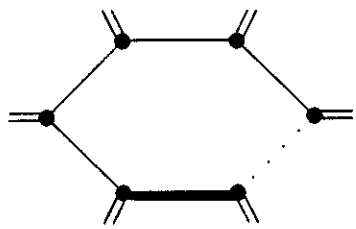
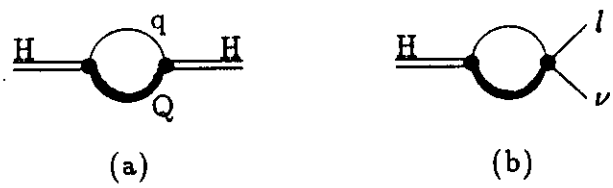


Fig.2



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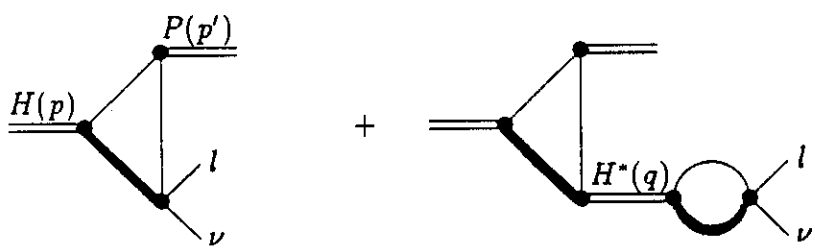


Fig.5

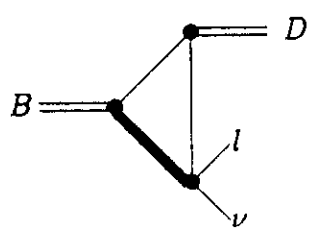
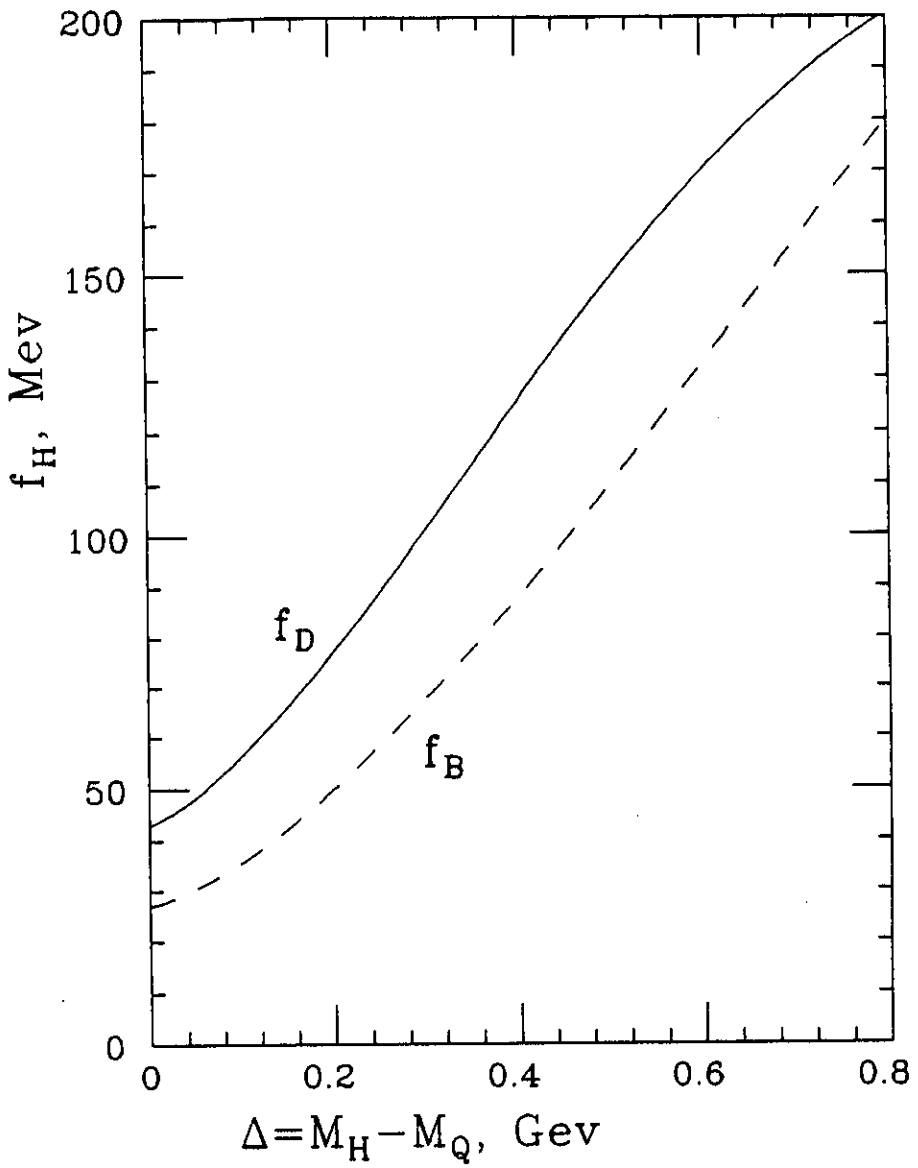


Fig.8

Weak couplings f_H (H=B,D)



$\Delta = 0.$

$M_d = M_D = 1.87$ Gev

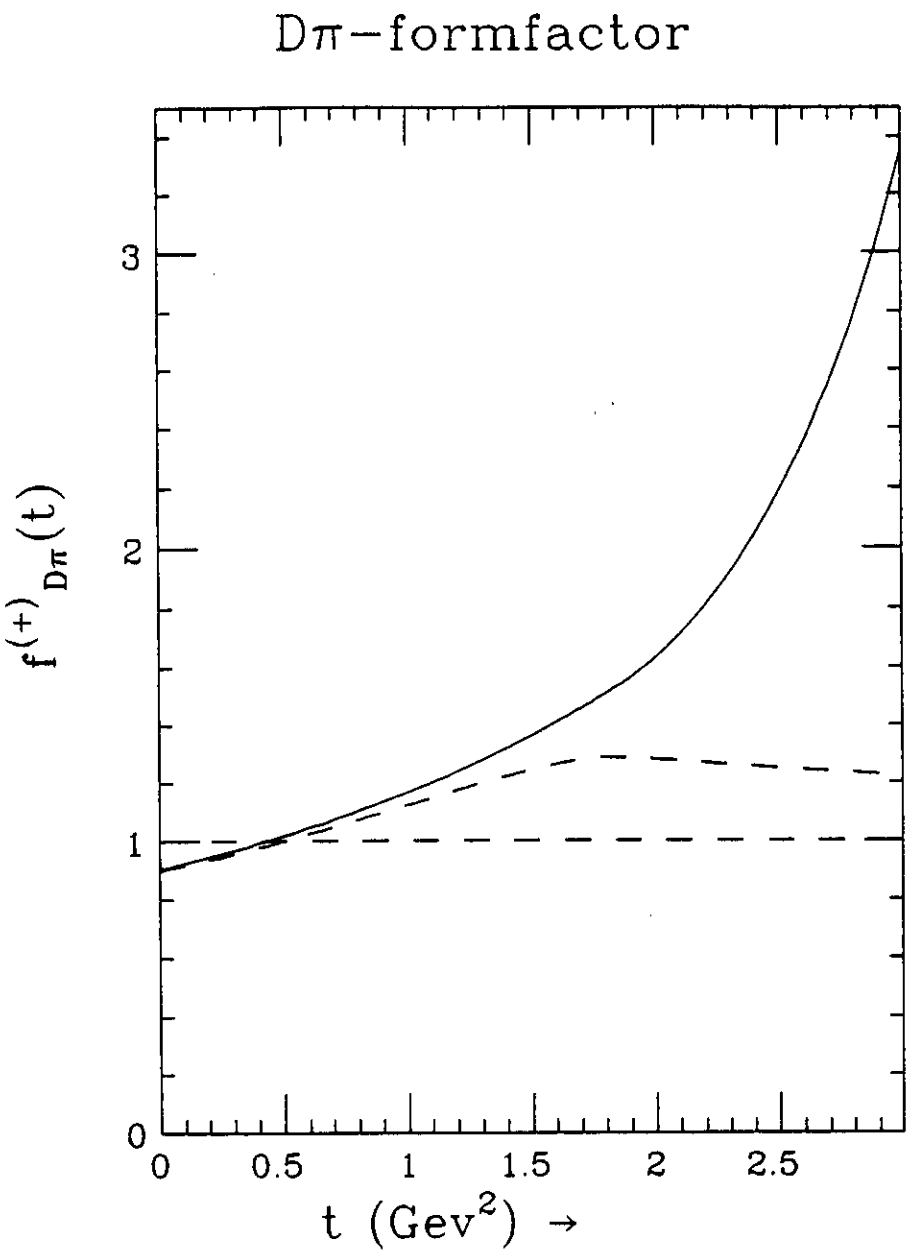
$M_b = M_B = 5.28$ Gev

$\Delta = 0.8$

$M_d = 1.07$ Gev

$M_b = 4.48$ Gev

Fig.4

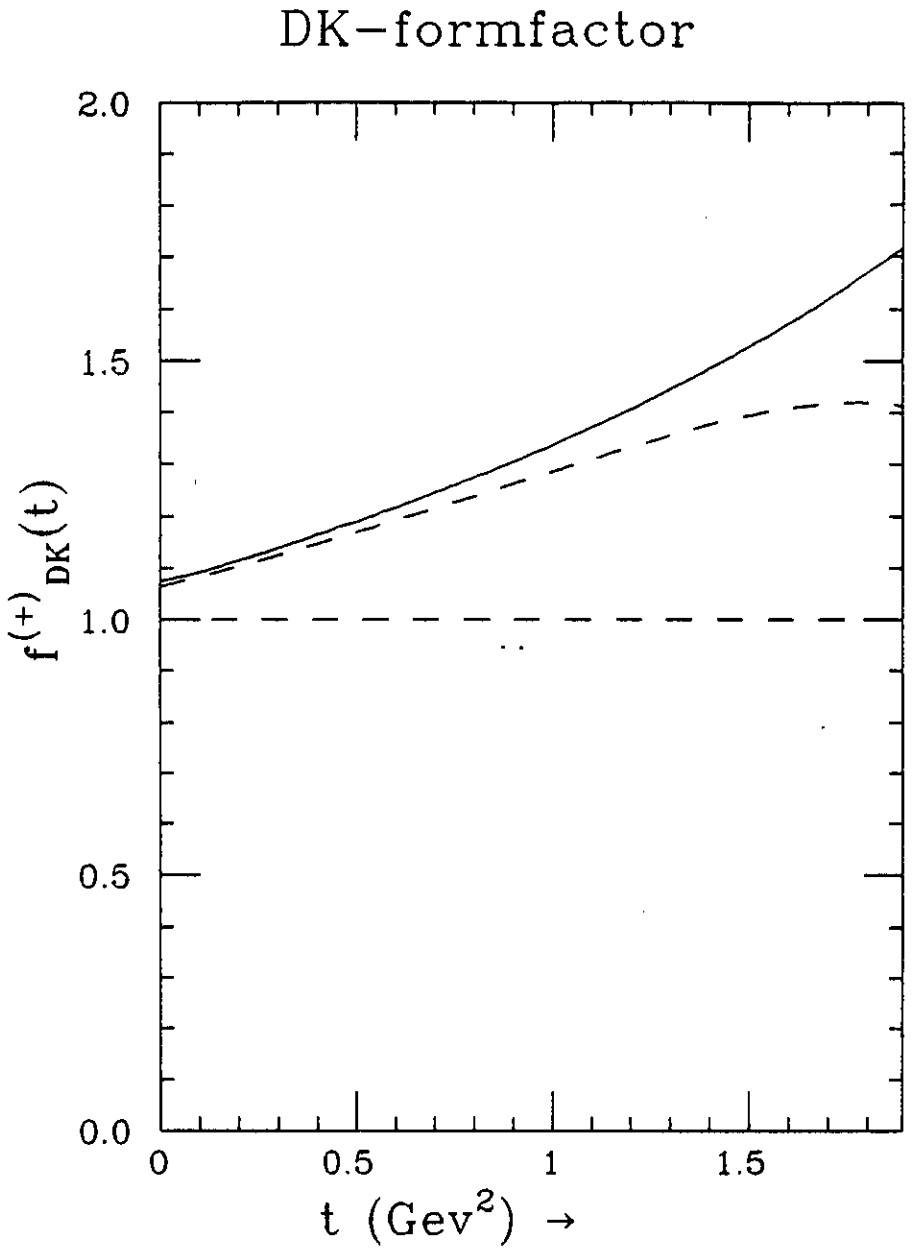


$f^{(+)}{}_{D\pi}(0.)=0.91$ $f^{(+)}{}_{D\pi}(t_{max})=3.36$

——— total

- - - - - triangle diagram

Fig.6a



$f^{(+)}_{DK}(0.)=1.08$ $f^{(+)}_{DK}(t_{max})=1.72$

——— total

- - - - - triangle diagram

Fig.6b

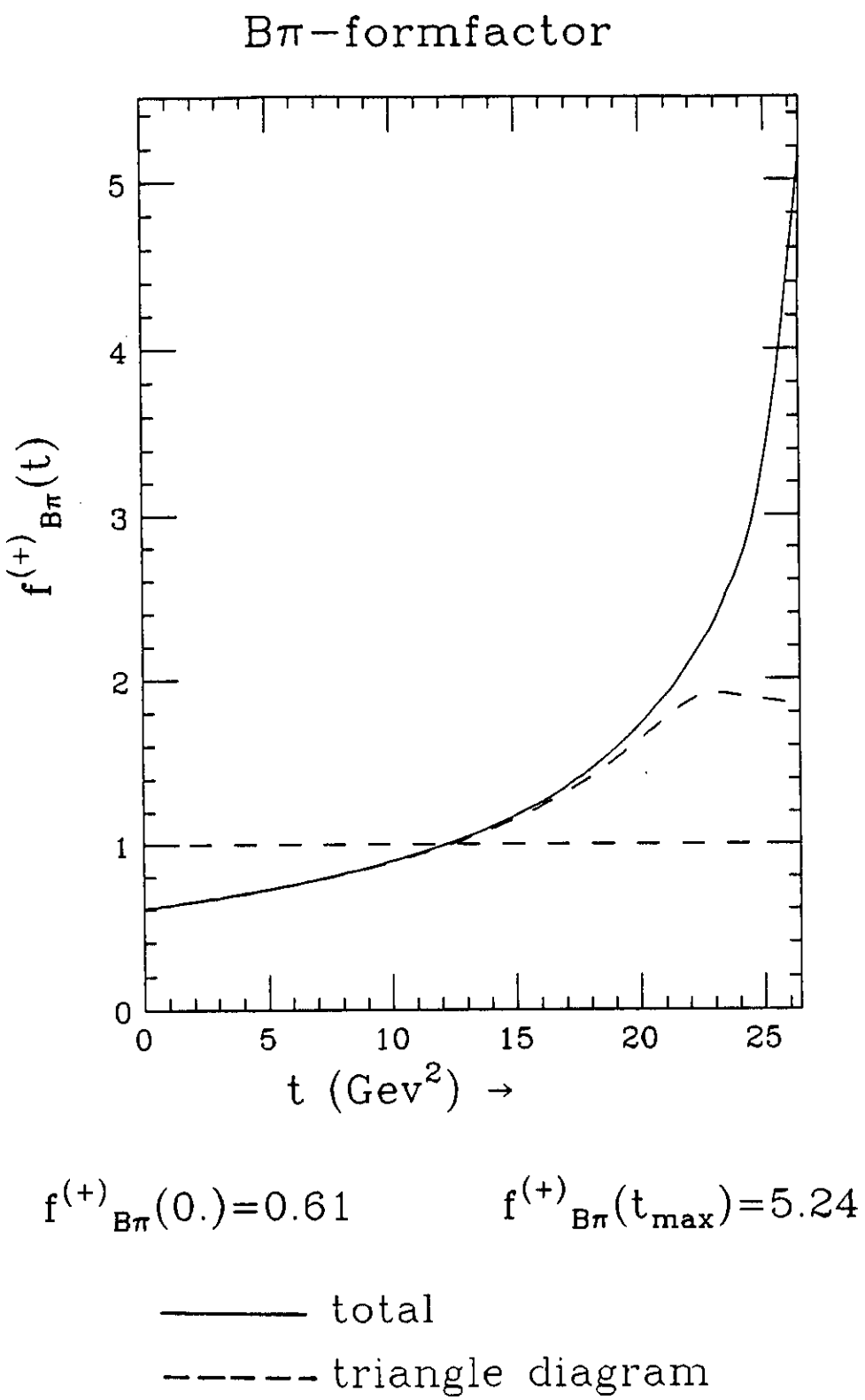
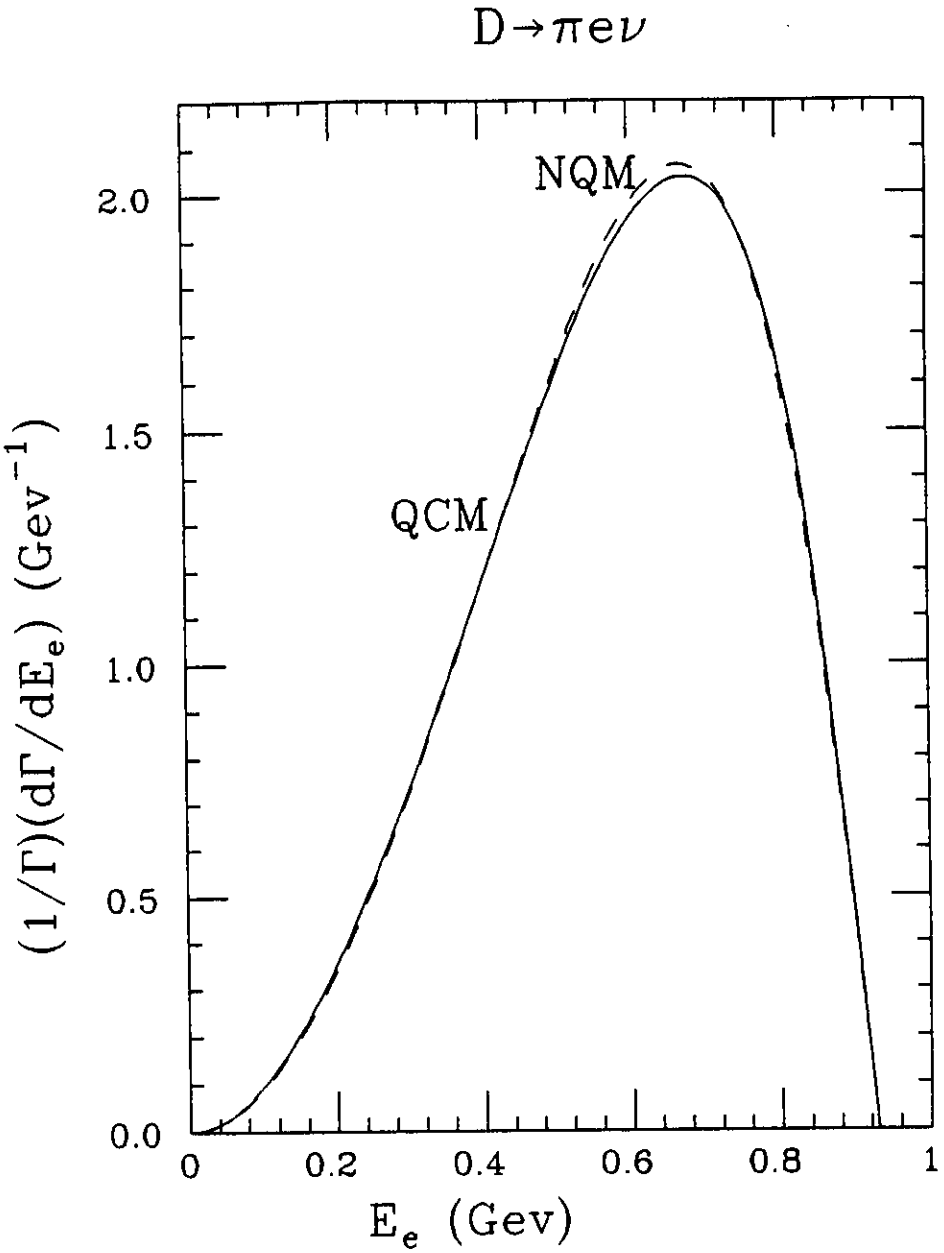

Bπ−formfactor
$f^{(+)}_{B\pi}(t)$
0
1
2
3
4
5
0
5
10
15
20
25
t (Gev^2) →
$f^{(+)}_{B\pi}(0.)=0.61$
$f^{(+)}_{B\pi}(t_{max})=5.24$
total
triangle diagram

Fig.6c



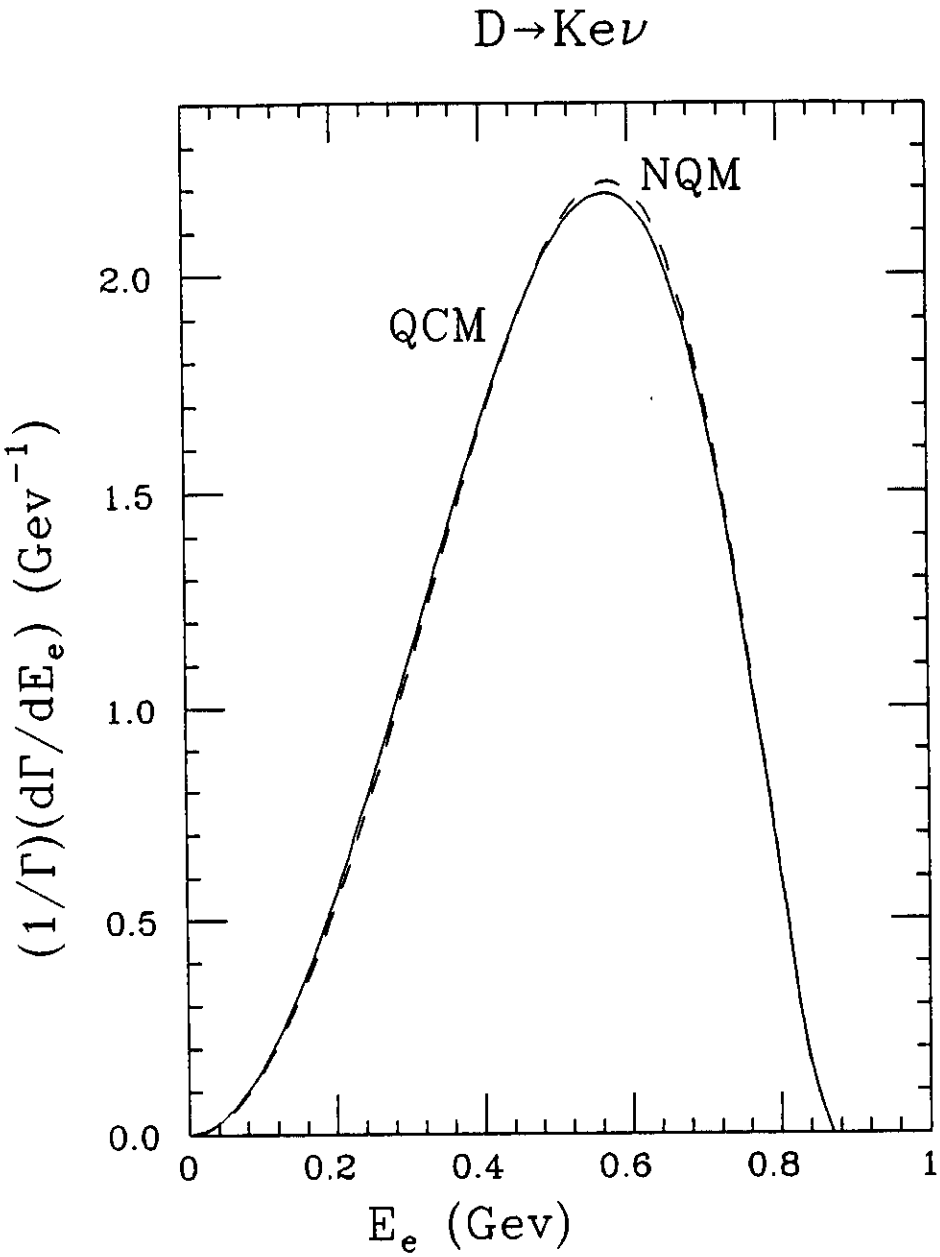
QCM: $\Gamma(D\to\pi e\nu)=15\cdot 10^{-14}\ |V_{dc}|^2$ Gev=

$=0.23\cdot 10^{12}\ |V_{dc}|^2\ \mathrm{sec}^{-1}$

NQM: $\Gamma(D\to\pi e\nu)=5.4\cdot 10^{-14}\ |V_{dc}|^2$ Gev=

$=0.08\cdot 10^{12}\ |V_{dc}|^2\ \mathrm{sec}^{-1}$

Fig.7a



QCM: $\Gamma(D \to Ke\nu) = 11 \cdot 10^{-14}\ |V_{sc}|^2$ Gev=

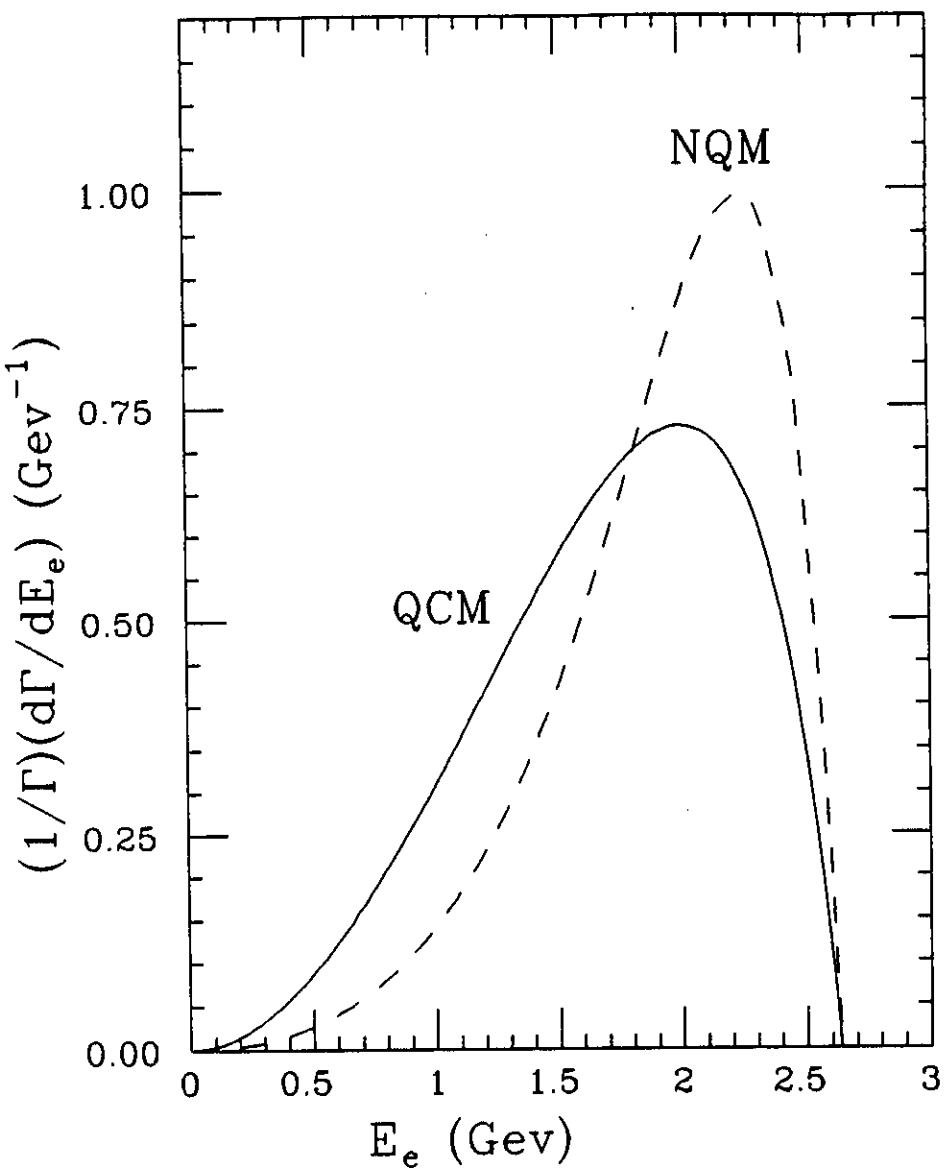
$= 0.17 \cdot 10^{12}\ |V_{sc}|^2\ \mathrm{sec}^{-1}$

NQM: $\Gamma(D \to Ke\nu) = 6.7 \cdot 10^{-14}\ |V_{sc}|^2$ Gev=

$= 0.10 \cdot 10^{12}\ |V_{sc}|^2\ \mathrm{sec}^{-1}$

Fig.7b

$B \to \pi e \nu$



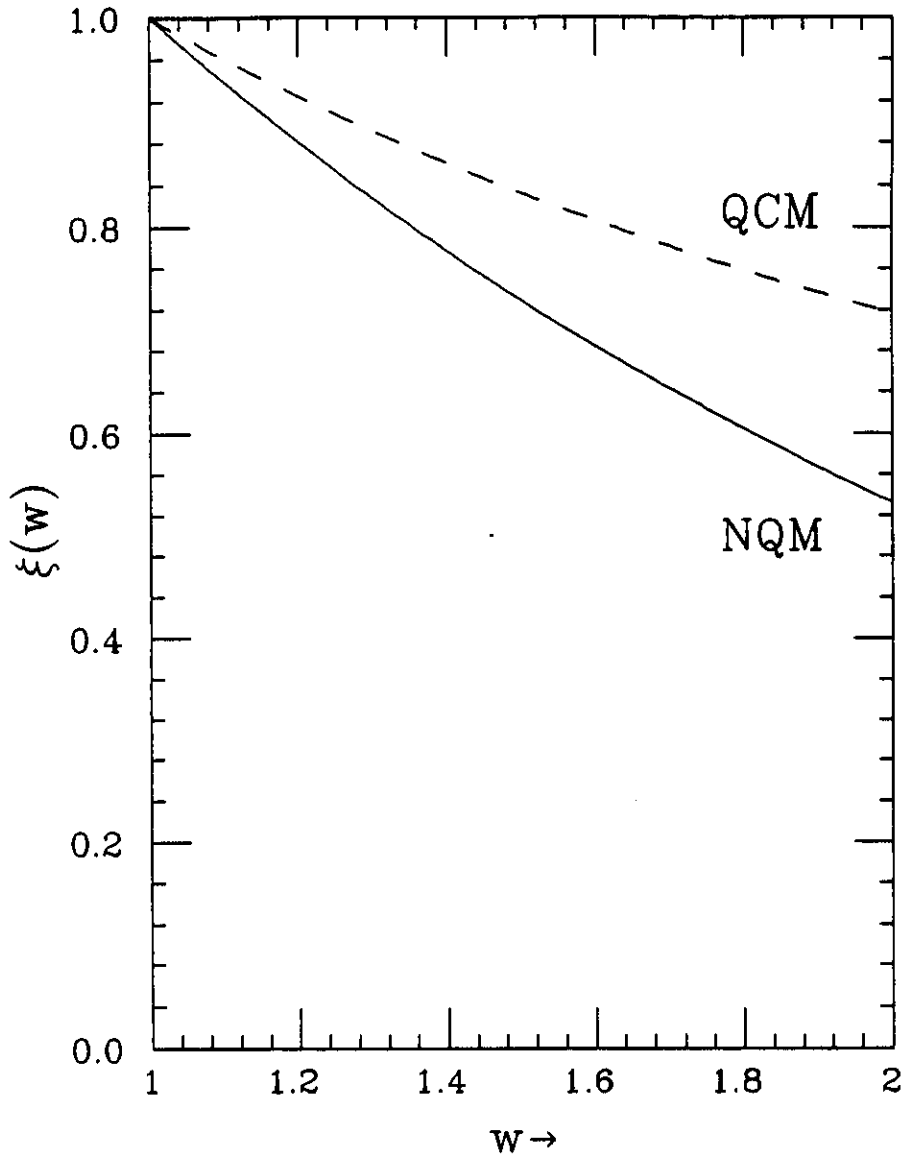
QCM: $\Gamma(B \to \pi e \nu) = 15 \cdot 10^{-12} \; |V_{ub}|^2 \; Gev =$

$= 0.22 \cdot 10^{14} \; |V_{ub}|^2 \; sec^{-1}$

NQM: $\Gamma(B \to \pi e \nu) = 15 \cdot 10^{-13} \; |V_{ub}|^2 \; Gev =$

$= 0.22 \cdot 10^{13} \; |V_{ub}|^2 \; sec^{-1}$

Fig.7c



$$\xi^{QCM}(2.)=0.72$$
$$\xi^{NQM}(2.)=0.53$$

Fig.9