CEBAF-TH-91-06
WM-91-104

How the OZI Rule Evades Large Loop Corrections

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ABSTRACT

Arguments based on unitarity indicate that hadronic loop diagrams should produce large violations of the OZI (Okubo-Zweig-Iizuka) rule. The mechanism by which these corrections are evaded has long been a mystery. We have found that there is an exact cancellation of all such loops in a particular limit and that, at least for the $\rho-\omega-\phi$ system which we have studied in detail, the cancellation is maintained in a realistic calculation which takes into account departures from this limit.

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Since its articulation some 25 years ago, experimental support for the OZI rule [1,2,3] has steadily accumulated, but a firm theoretical underpinning has remained elusive. The rule declares that processes involving "hairpin-turn" quark lines are suppressed (see Fig. 1), but, as has been emphasized by Lipkin [4], it is difficult to understand how such a rule can be respected: OZI-violation can always proceed by a strong *two step* process involving the amplitudes for virtual decay channels (see Fig. 2, corresponding to Fig. 1(b)). Though such amplitudes vanish in certain limits of QCD, notably the large N_c limit, in our $N_c = 3$ world they produce hadronic decay widths of order Λ_{QCD}, so that one would naively expect the scale of OZI violation (as measured, for example, by the $\omega - \rho$ mass difference) to satisfy $\Lambda_{OZI} \sim \Lambda_{QCD}$.

In this Letter we report the results of our study [5] of a mechanism by which virtual-decay contributions to OZI violation may naturally be reduced from this naive "unitarity" prediction to the observed result $\Lambda_{OZI} \sim m_\omega - m_\rho \sim 10$ MeV. The explanation we propose is a simple one. Refer to Fig. 2 and consider, for definiteness, $u\bar{u} \leftrightarrow d\bar{d}$ mixing in the meson propagator. We see that pair creation produces a virtual decay from the initial meson to an essentially arbitrary intermediate state and thence to the final meson, resulting in an OZI-violating amplitude A apparently of order a typical hadronic width, Γ:

$$A(E) \equiv \sum_n \frac{\langle d\bar{d}|H_{pc}^{u\bar{u}}|n\rangle\langle n|H_{pc}^{d\bar{d}}|u\bar{u}\rangle}{(E - E_n)} \sim \Gamma \quad , \tag{1}$$

where $H_{pc}^{f\bar{f}}$ is the quark pair creation operator for the flavor f and the set $\{|n\rangle\}$ is a complete set of two-meson intermediate states. However, in a "closure approximation", in which the variation of the energy denominators associated with this sum is neglected, A

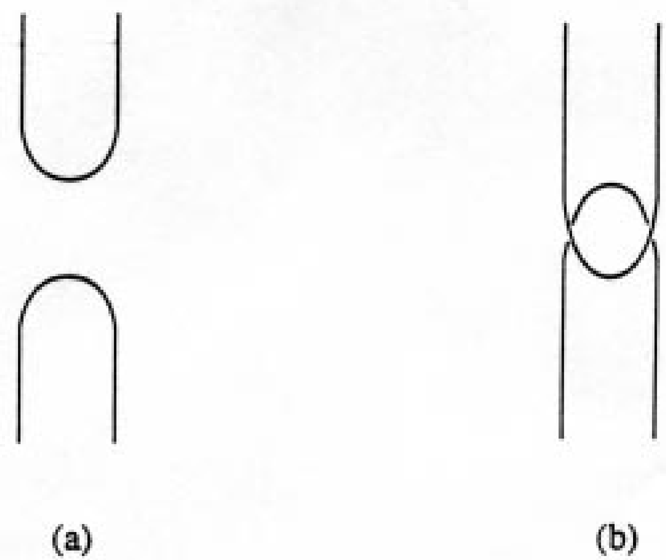

Figure 1: A diagram associated with OZI rule violation shown in two time orderings.

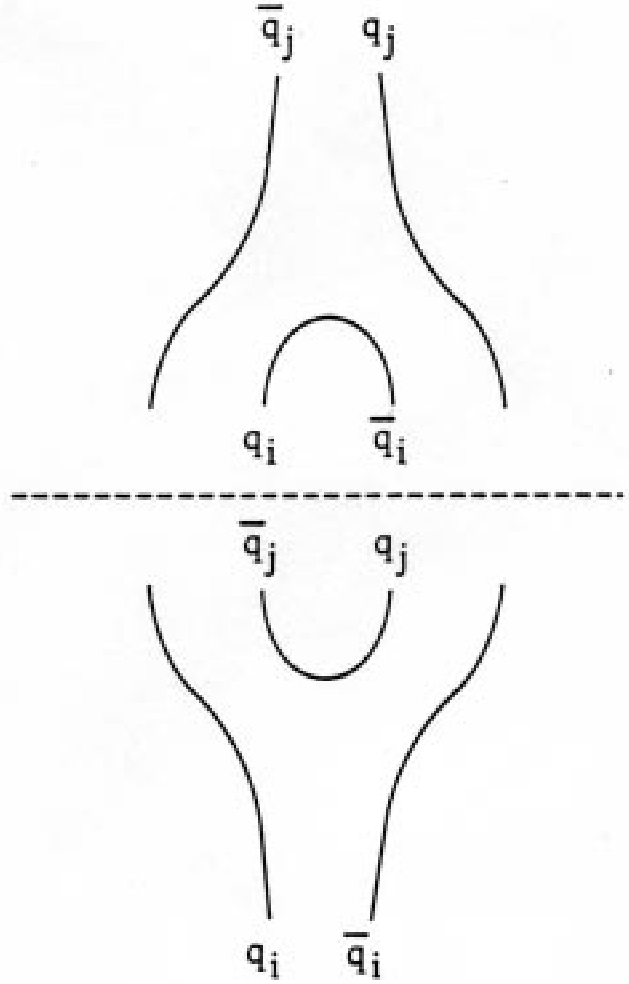

Figure 2: OZI violation via two OZI-allowed amplitudes.

is proportional to

$$B \equiv \sum_n \langle d\bar{d}|H_{pc}^{u\bar{u}}|n\rangle\langle n|H_{pc}^{d\bar{d}}|u\bar{u}\rangle = \langle d\bar{d}|H_{pc}^{u\bar{u}}H_{pc}^{d\bar{d}}|u\bar{u}\rangle \quad , \tag{2}$$

hence in this approximation Figure 2 will be suppressed relative to Γ whenever the created (destroyed) pair has only a small overlap with the final (initial) meson and the pair creation (destruction) operator treats the original (final) quark pair as spectators. As we will see below, pair-creation in the familiar flux-tube breaking model possesses just these features: the spectator approximation applies and the $q\bar{q}$ pairs are created and destroyed with $^3P_0$ quantum numbers so that, except in the scalar meson sector, they are orthogonal to the final and initial states. Hence *this source of OZI violation in the established meson nonets vanishes identically in the closure approximation.*

This picture of the OZI rule was foreshadowed by the earlier work of Lipkin and others[4]. In particular, Lipkin stressed the importance to the OZI rule of cancellations between different intermediate states and argued that its validity must ultimately involve cancellations not only between states of a given flavor or flavor-spin multiplet, but also between states of different generalized G-parity. He also explicitly recognized that the closure and spectator approximations could be important to understanding these cancellations. The reader will see below that our solution to the OZI puzzle has all of the characteristics which Lipkin anticipated. The role of cancellations in the OZI rule (as well as in the analogous suppression of exotic exchanges) was also noted by Schmid, Webber, and Sorensen and by Berger and Sorensen [4]. They pointed out within the context of Regge theory that the cancellations between exchange degenerate trajectories of opposite

G-parity occurred naturally and could potentially be arranged to preserve the OZI rule.

With the closure and spectator approximations in mind, we have examined OZI violation in the $\rho - \omega - \phi$ system by calculating hadronic loop corrections to the ρ and ω masses. Recall that a meson mass matrix in the $\{u\bar{u}, d\bar{d}, s\bar{s}\}$ basis may be written as

$$\begin{bmatrix} m+A & A & A \\ A & m+A & A \\ A & A & m+\Delta m+A \end{bmatrix} ,$$

where A is the $q_i\bar{q}_i \leftrightarrow q_j\bar{q}_j$ mixing amplitude, assumed to be $SU(3)_f$ symmetric. Transforming to the ideally mixed basis $\{(u\bar{u}-d\bar{d})/\sqrt{2},\ (u\bar{u}+d\bar{d})/\sqrt{2},\ s\bar{s}\}$, the mass matrix becomes

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m+2A & \sqrt{2}A \\ 0 & \sqrt{2}A & m+\Delta m+A \end{bmatrix} ,$$

and it is immediately apparent that in the almost ideally-mixed nonets (*i.e.*, all known nonets except the pseudoscalars), the strength of the OZI-violating amplitude A is measured by the mass difference between the mostly nonstrange isospin-zero meson and its isospin-one partner. A survey of the data gives [6] $A_\omega = +7 \pm 1$ MeV, $A_{f_2} = -22 \pm 3$ MeV, $A_{f_1} = +11 \pm 15$ MeV, $A_{h_1} = -32 \pm 12$ MeV, and $A_{\omega_3} = -12 \pm 4$ MeV, from the $\omega - \rho$, $f_2 - a_2$, $f_1 - a_1$, $h_1 - b_1$, and $\omega_3 - \rho_3$ mass differences, and we see that these amplitudes are indeed typically an order of magnitude smaller than meson widths.

As we have indicated, our calculation of the "virtual decay" piece of A_ω, *i.e.*, the piece arising from the time-ordering of Fig. 1(b), was done in the context of a flux-tube breaking model of quark pair creation. This model, as well as its considerable phenomenological success in predicting strong meson decay amplitudes, is described in Refs. 7, 8 and 9. In this model, as in the old dual string model, the leading effects of gluonic interactions are string-like confinement (leading to the linear Regge trajectories) and $q\bar{q}$ pair creation (and annihilation) by string breaking (and healing). Here we simply outline its essential features. The decay vertices of Fig. 2 arise in the model from the breaking of the chromoelectric flux-tube that joins the quark and antiquark in the initial meson. (Thus in leading order the diagrams of Figs. 1 and 2 are defined by their string world sheets and do not require dressing by further gluon exchanges.) It is plausible in such a model (and strongly indicated by decay data) that the new $q\bar{q}$ pair is created in a $^3P_0$ state [7], and that the decay then proceeds by rearrangement. (In the naive $^3P_0$ model the pair is created with the same amplitude over all space. In the flux tube model of Ref. 7, the pair is created in a cigar-shaped region defined by the overlap of the initial and final flux tube wavefunctions. Here for simplicity it is assumed that the pair is created in a spherical region about the meson's center of mass. This approximation leads to the spectator model for the angular momentum of the original $q\bar{q}$ pair being exact; we will comment on the effect of this approximation below.) This simple picture leads to the $A \to BC$ meson decay amplitude:

$$M(A \to BC) = \frac{2}{(2\pi)^{3/2}} \gamma_0 \phi \vec{\Sigma} \cdot \int d^3k\, d^3p\, d^3p'\, \Psi(\vec{p}, \vec{p}\,')\, \Phi_B^*(\vec{k} + \frac{\vec{p}\,'}{2}) \Phi_C^*(\vec{k} - \frac{\vec{p}\,'}{2})$$

$$\times (\vec{k} + \frac{\vec{q}}{2}) exp[-\frac{2r_q^2}{3}(\vec{k} + \frac{\vec{q}}{2})^2]\, \Phi_A(\vec{k} - \frac{\vec{q}}{2} - \vec{p}). \qquad (3)$$

Here the Φ's are momentum space meson wavefunctions, $\Psi(\vec{p}, \vec{p}\,')$ represents the overlap of the initial and final flux-tube wavefunctions described above, the term $exp[-\frac{2r_q^2}{3}(\vec{k} + \frac{\vec{q}}{2})^2]$ is a form factor for the pair creation vertex which takes into account the finite size, r_q, of the created constituent quarks, $\vec{q}$ is the momentum of meson B, ϕ is a flavour overlap, $\vec{\Sigma}$ is a spin overlap, and γ_0 is a parameter of the model, the intrinsic pair creation strength.

The calculation of A_ω proceeds by inserting (3) into (1), with a Φ_A and $\vec{\Sigma}$ appropriate to the vector meson initial and final states. However, before describing this calculation it is instructive to examine the terms in the closure sum (2); understanding how this sum vanishes identically is the first step in understanding how A_ω can be small in the full calculation that includes energy denominators. In the general term in the closure sum, n stands for the set $\{n_B, \ell_B, m_{\ell_B}, s_B, m_{s_B};\ n_C, \ell_C, m_{\ell_C}, s_C, m_{s_C};\ q, \ell,$ and $m\}$, *i.e.* the radial, orbital, and spin quantum numbers of the intermediate mesons B and C, as well as the (magnitude of the) momentum and the angular momentum of their relative coordinate. The sums over $m_{\ell_B}, m_{\ell_C}, m$, and the quark spins may be done analytically, leaving terms that are functions of q, labelled by n_B, n_C, ℓ_B, ℓ_C, and ℓ. We display these functions in Fig. 3, ordered by $L \equiv \ell_B + \ell_C + \ell$ and $N \equiv n_B + n_C - 2$ (our convention is that the ground state of every ℓ has $n = 1$, so the sums start with $N = 0$). The graphs were obtained by inserting harmonic oscillator wavefunctions into Eq. (2). (The oscillator parameter β, defined by $\Phi(\vec{k})$ ~(polynomial)$exp[-\frac{k^2}{2\beta^2}]$, was taken to be 0.4 GeV, r_q was taken to be 0.15 fm, and the string overlap function was $\Psi(\vec{p}, \vec{p}\,') = \delta^3(\vec{p})(\frac{2\pi}{b})^{3/2} exp[-\frac{p'^2}{2b}]$, where $b = 0.18$ GeV^2 is the string tension. These choices are based on fits to meson spectral and decay data (without reference to OZI violation), as described in Ref. 5. As we explain below, cur results are not very sensitive to them; moreover the closure sum must be zero independent of the values of these parameters.)

The closure sum converges rapidly towards zero: the sequence of partial sums corre-

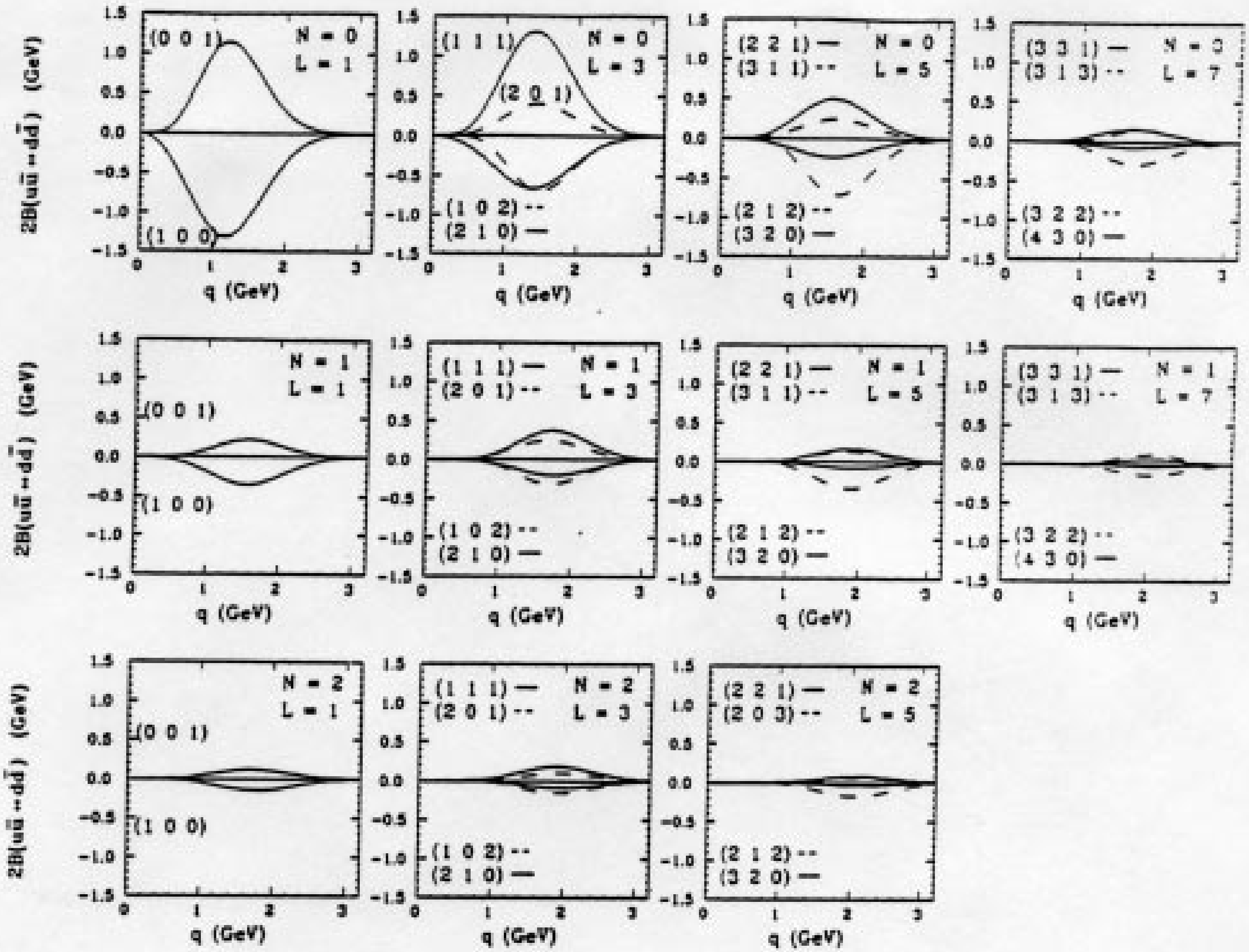

Figure 3: The terms in the closure sum for B in Eq. 2 ordered by $L \equiv \ell_B + \ell_C + \ell$ and $N \equiv n_B + n_C - 2$, calculated using our canonical parameters $\beta = 0.4$ GeV, $b = 0.18$ GeV^2, and $r_q = 0.15$ fm. The curves are labelled by (ℓ_B, ℓ_C, ℓ); when $\ell_B \neq \ell_C$, (ℓ_B, ℓ_C, ℓ) is an abbreviation for $(\ell_B, \ell_C, \ell)+(\ell_C, \ell_B, \ell)$. To avoid overcrowding on the graphs with $L \geq 5$, we show only the "leading" terms, *i.e.*, the $(\frac{L-1}{2}, \frac{L-1}{2}, 1)$ and $(\frac{L+1}{2}, \frac{L-1}{2}, 0)$ ones, and the two largest remaining terms.

sponding to the graphs of Fig. 3 with ($N=0$, $L=1$), ($N \leq 1$, $L \leq 3$), ($N \leq 2$, $L \leq 5$), ... is -184.3, -13.6, -30.2, -0.1, -2.8, 0.0, ... in units of $\gamma_0^2\beta^2/6\pi^2$. Fig. 3 also indicates that the required cancellation of the large OZI-violating amplitudes in (1) cannot occur if only S-wave intermediate-state mesons are summed over; we will soon see that at the very least, states with one S-wave and one P-wave are required. This surprising result runs counter to the naive expectation that any cancellations which occur ought to take place within some flavor or flavor-spin symmetry group (*e.g.*, SU(6)) and corresponds to the cancellation between states of opposite generalized G-parity anticipated in Refs. 4. This and other important gross features of the terms in the sum, such as their tendency to cancel in pairs, can be understood in terms of a peculiar "magic" limit. When $\frac{\lambda}{1-\lambda} \equiv (\frac{\beta^2}{b} - \frac{\beta^2 r_q^2}{3} - \frac{1}{4})$ and $\gamma \equiv (\frac{\beta^2 r_q^2}{3} - \frac{1}{4})$ are both zero (corresponding to $r_q = \frac{\sqrt{3}}{2\beta}$ and $b = 2\beta^2$), the integrals in the closure sum simplify so that only the radial ground state terms with $(\ell_B, \ell_C, \ell) =$ (1, 0, 0), (0, 1, 0) and (0, 0, 1) are nonvanishing: the entire closure cancellation therefore occurs in the analog of the $N = 0$, $L = 1$ graph of Fig. 3! Further structural features of the graphs in Fig. 3 follow from expansions (in λ and γ) about the "magic" limit [5]. For example, these expansions explain why the terms in the closure sum tend to cancel "locally" (*i.e.*, cancellations occur between terms with neighbouring values of N and L). This is important because such terms will have similar energy denominators in the full calculation of A_ω, so their cancellations will largely be maintained. The "magic" limit thus has the virtue of allowing one to see analytically the features of the closure sum apparent numerically in Fig. 3. It is, however, somewhat more than an analytic toy since the ratios of the phenomenological parameters $b^{\frac{1}{2}}, \beta$, and r_q^{-1} are reasonably close to the "magic" values. As a result, the physical closure sum zeroes in a very rapid and orderly fashion.

Our full calculation of $\Delta m_\omega - \Delta m_\rho = 2A_\omega$ converges rapidly. With our canonical parameters the sequence of partial sums with $(N=0,\ L=1)$, $(N \leq 1,\ L \leq 3)$, $(N \leq 2,\ L \leq 5)$, ... in MeV is $-106, -17, -7, +7, +11, \ldots$ converging to $+13$ MeV, but this agreement with the measured value is accidental: for reasonable variations in our model parameters the calculated mass difference varies by tens of MeV's (for $r_q = 0.30$ fm, $\omega - \rho = -33$ MeV; for $\beta = 0.3$ GeV, $\omega - \rho = +31$ MeV; and for $b = 0.12$ GeV^2, $\omega - \rho = +32$ MeV). (We also studied the delicacy of our calculation to a number of other factors like the pion mass and other details of the assumed hadronic spectrum, but never found any significant sensitivity.) The value of this calculation is not to be measured in such terms, but rather in its showing how the scale of Λ_{OZI} is naturally reduced from Λ_{QCD} to a mass of order 10 MeV. (Note, in this regard, that a sum over just S-wave intermediate-state mesons gives $\Delta m_\omega - \Delta m_\rho = 141$ MeV.) We should also emphasize that even were this calculation accurate, it should not give the experimentally observed $\omega - \rho$ splitting since in addition to the source considered here, this splitting will receive contributions from other sources, for example the "pure annihilation" time ordering shown in Fig. 1(a).

There are a number of reasons why the full calculation leaves nearly intact the closure-approximation result that $\Delta m_\omega = \Delta m_\rho$. We note that if $2N + L$ is large, the cancellation of the dominant $\ell = 0$ and 1 terms is maintained because the energy denominators of these terms are then approximately equal and independent of q. At smaller N and L, cancellations persist because differences in energy denominators are systematically compensated by differences in matrix elements[5]. For the low mass intermediate states where large spin splittings produce large and effectively random deviations from the closure limit, cancellations are aided by the simple fact that the integrals have no support at $q = 0$, where the energy denominators are most different.

To summarize, the closure mechanism we have described here offers a framework for understanding the remarkable cancellations that must occur among OZI-violating hadronic loops in order to make $\Lambda_{OZI} \ll \Lambda_{QCD}$, but much remains to be done to substantiate this picture. Among the remaining tasks are: studying model-dependence (including the effects of small transverse gluon exchanges on the dominant $^3P_0$ amplitude), explicitly examining $\omega - \phi$ mixing, and extending the calculations to other sectors (including the scalar mesons where the effects of virtual decay channels may be very different).

P.G. would like to thank the CEBAF Theory Group for its hospitality and financial support during the period when this work was completed. N.I. acknowledges the support of the College of William and Mary. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada and by the U.S. D.O.E. under Contract No. DE-AC05- 84ER40150.

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