QCD SUM RULE CALCULATION OF THE ISGUR-WISE FORM FACTOR [1]

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Abstract

Within the QCD sum rule approach, we develop a formalism that enables one to calculate the form factors of the heavy-light mesons in the $m_Q \to \infty$ limit. It is shown that the behaviour of the universal Isgur-Wise form factor is determined by the quark propagation function in imaginary time.

1. Introduction. Recently, Isgur and Wise discovered that form factors of the $B \to De\nu$ type decays, in the limit when heavy quark masses tend to infinity, can be described by a universal function $\xi(w)$ that depends only on the velocities v, v' of the heavy mesons $w = (vv')$ [1]. This function accumulates information about the long-distance dynamics and cannot be calculated within the perturbative QCD framework. Our goal in the present paper is to outline an approach to this problem based on the QCD sum rules [2]. First, we reformulate the QCD sum rule analysis of the mesons containing one infinitely massive quark, developed originally by Shuryak [3], to cast it into a relativistically covariant form. Then, we generalize this analysis to apply it to the form factor problems. Finally, we construct the QCD sum rule for the Isgur-Wise function and compare our result with two existing quark model estimates [4, 5].

2. QCD sum rules for mesons containing one infinitely heavy quark. The basic idea of the QCD sum rule approach [2] is to calculate a correlator of two local currents $j(x)$

$$\Pi(P) = i \int e^{iPx} \langle 0|T\,(j(x)\,j(0)\,)|\,0\rangle\, d^4x \tag{1}$$

for momenta P belonging to a region where one can incorporate the asymptotic freedom property of QCD *via* the operator product expansion, and then construct the hadronic spectrum reproducing, as closely as possible, the behaviour of $\Pi(P)$ obtained in this way.

In the case of our interest, $j(x) = \bar{q}(x)\Gamma Q(x)$ with $Q(x)$ being the heavy quark field and $q(x)$ that of the light one. The matrix $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$ specifies the type of the state: scalar, pseudoscalar, vector or axial.

The heavy quark limit of the two-point function can be studied using some of the conventional ways to analize the asymptotic behaviour of Feynman diagrams, say, the α-representation analysis (see, *e.g.*, [6]). It is easy to establish that when $P^2 \sim m_Q^2 \to \infty$, the T-product of the two currents reduces to the product of two factors

$$\langle 0|T\,(j(x)\,j(0)\,)|\,0\rangle \;= \langle 0|\mathrm{Tr}\{\Gamma S_Q^c(-x)\Gamma S_q^c(x,A)\}|0\rangle, \tag{2}$$

where the first is the perturbative heavy quark propagator (with the radiative corrections included) and the second one is the total "gauge invariant" light quark propagator

$$S_q^c(x,A) = \langle 0|T(q(x)P\exp(ig\int_0^x A_\mu(z)dz^\mu)\bar{q}(0)|0\rangle \tag{3}$$

containing both the perturbative part and the nonperturbative one (the nonlocal condensate, see, *e.g.*, [7]).

To make the dependence on the heavy quark mass m_Q explicit, we write the probing momentum P as the sum of $O(m_Q)$ term and the finite part: $P_\mu = (m_Q + E)v_\mu$, with v_μ being the 4-velocity of the heavy quark. In the momentum representation, the heavy quark propagator (to lowest order in α_s) looks then like

$$\frac{\not{P} - \not{k} + m_Q}{(P-k)^2 - m_Q^2} = \frac{m_Q(1+\not{v}) - \not{k}}{2m_Q(E-(kv)) + E^2 - 2E(kv) + k^2} =$$
$$= \left(\frac{1+\not{v}}{2}\right)\frac{1}{E-(kv)} + O(1/m_Q), \tag{4}$$

where k is the momentum associated with the light quark. Note, that dependence on the heavy quark mass m_Q has disappeared from the leading term. Another observation is that

$$\Gamma(\frac{1+\not{v}}{2})\Gamma \sim \frac{1+p\not{v}}{2},$$

where $p = \pm 1$ is the parity of the relevant current. As emphasized by Shuryak [3], the sum rule for the vector state is the same as for the pseudoscalar one. These states are degenerate in this limit: the energy levels do not depend on how spin of the (infinitely) heavy quark is oriented. Another degenerate set is formed by scalar and axial states. However, there is no degeneracy in parity: the states with opposite parity have completely different properties.

The next standard step is to perform the Borel transformation $\Pi^{\pm}(v,E) \to M^{\pm}(v,\epsilon)$

$$M^{\pm}(\epsilon) = \int \frac{d^4k}{(2\pi)^4} e^{-(vk)/\epsilon} \mathrm{Tr}\Big\{\Big(\frac{1\mp\not{v}}{2}\Big)\tilde{S}_q(k)\Big\} = \mathrm{Tr}\Big\{\Big(\frac{1\mp\not{v}}{2}\Big)S_q\left(\frac{iv}{\epsilon}\right)\Big\}. \tag{5}$$

Thus, all the dynamical information is accumulated in the function $S_q(\frac{iv}{\epsilon})$. Note, that in the rest frame of the heavy quark one has $v_\mu = (1, \mathbf{0})$ and, hence, this function describes the propagation of the light quark in the imaginary time $\tau = 1/\epsilon$. As mentioned above, it has two parts: the perturbative propagator

$$S^{pert}(x) = -\frac{\not{x}}{2\pi^2(x^2)^2} + O(\alpha_s) + \ldots$$

and the nonperturbative part given by the sum of two nonlocal quark condensates

$$\langle 0|\bar{q}(0)q(x)|0\rangle \equiv \langle\bar{q}q\rangle\Phi(x^2), \tag{6}$$

and

$$\frac{\gamma^\mu}{4}\langle 0|\bar{q}(0)\gamma_\mu q(x)|0\rangle = -\frac{i\not{x}}{162}\pi\alpha_s\langle\bar{q}q\rangle^2\Psi(x^2), \tag{7}$$

all taken at the point (or, better, moment) $x = iv/\epsilon$.

For the sake of brevity, we omitted the P-exponential, *i.e.*, the temporal gauge $v^\mu A_\mu = 0$ is implied, in which it reduces to unity. Numerically, the contribution due to the vector nonlocal condensate is much smaller than that due to the simplest scalar combination and, in what follows, we neglect it.

For the scalar nonlocal condensate, we use the Gaussian model [7]

$$\Phi(x^2) = \exp(x^2\lambda^2/8),$$

with the width parameter λ determined by the standard value [8] of the ratio

$$\langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle \equiv \lambda^2 = \frac{1}{2}\langle\bar{q}(\sigma G)q\rangle/\langle\bar{q}q\rangle = 0.4 \pm 0.1\, GeV^2. \tag{8}$$

Taking the usual ansatz "first resonance plus continuum" for the hadronic spectral function gives the following sum rules

$$f^2 e^{-E_R/\epsilon} = \int_0^{E_c} \frac{s^2}{2} e^{-s/\epsilon} ds - p\kappa^3 \exp\left(-\frac{\lambda^2}{8\epsilon^2}\right), \tag{9}$$

where E_R is the lowest energy level (with negative or, respectively, positive parity) of the light quark in the static color field and κ is the basic energy scale for this system settled by the quark condensate value:

$$\kappa = (-\frac{\pi^2}{6}\langle\bar{q}q\rangle)^{1/3} \approx 260\, MeV. \tag{10}$$

Applying the requirement of the best agreement between the two sides of the sum rule we obtain, for the negative parity states: $E_R = 360\, MeV$ for the energy of the first resonance, $E_c = 680\, MeV$ for the continuum threshold parameter and $f^2 = 3.9$ for the coupling constant f^2. In the positive parity case, we get $E_R = 1000\, MeV$ for the energy of the lowest resonance, $E_c = 1270\, MeV$ for the continuum threshold parameter and $f^2 = 18.6$ for the coupling constant f^2. These results agree with those obtained by Shuryak [3].

The parameter f^2 is related to the $Q \to \mu\nu$ decay constant f_Q by

$$f^2 = \frac{\pi^2}{6} f_Q^2 M_Q,$$

so that $f_Q \sim 1/\sqrt{M_Q}$ [3]. For the D-meson this gives $f_D = 150\, MeV$ and for the B-meson $f_B = 90\, MeV$. To get the experimental masses of the D and B mesons ($M_D = 1.86\, GeV$, $M_B = 5.28\, GeV$) one should take $m_c = 1.5\, GeV$ and $m_b = 4.9\, GeV$, which are reasonable estimates for the c- and b- quark masses normalized at a near-mass-shell point. However, any comparison with experimental data would be meaningful only after an accurate calculation of the $O(1/m_Q)$- and $O(\alpha_s)$-corrections.

3. How many poles has the quark propagator? Since the infinitely heavy quark can be treated as just a source of the color field, a heavy-light meson, in the $M_Q \to \infty$ limit, can be considered as a system composed by a single light quark. In other words, this is the simplest possible color neutral system in QCD, providing the best laboratory to study the properties of an isolated quark in the color field compensating its color charge. To this end it is instructive to return to eq.(refeq:borel) rewriting it in terms of the imaginary time τ

$$\tilde{M}^{\pm}(\tau) = \mathrm{Tr}\Big\{\Big(\frac{1 \mp \not{v}}{2}\Big) S_q(iv\tau)\Big\}. \tag{11}$$

Its most wonderful property is that it states that a Green-like function describing mesons is simply proportional to (the projection of) the quark propagator. Note now, that the mesonic function $\tilde{M}^{\pm}(\tau)$, in the narrow-resonance approximation, should be given by the sum of exponentials

$$\tilde{M}^{\pm}(\tau) = \sum_{i=1} f_i^2 e^{-E_i\tau} \tag{12}$$

with E_i being the energy of the ith state. Inverting then eq.(11) one concludes that, at large τ, the quark propagator behaves like $\exp(-E_1\tau)$. This means that the quark propagator has a pole at $k^2 = E_1^2$, and, hence, $E_1 \equiv E_R$ can be interpreted as the constituent (dynamic or whatever) quark mass! Fortunately enough, the value $E_R = 360\, MeV$ is just what one would expect from such a parameter. In fact, eq.(11) implies more than that. It states that the quark propagator has not a single, but many poles: the number of the poles is determined by the number of the states the quark has in the external color source.

The statements above might sound paradoxical. However, they become evident, if one incorporates a simple quantum-mechanical analogy. For the d-dimensional harmonic oscillator, *e.g.*, which is an exactly confining system, one has

$$\tilde{M}^{osc}(\tau) = \left(\frac{\omega}{\sinh(\omega\tau)}\right)^{d/2} = (2\omega)^{d/2}\sum_{k=0}^{\infty}\frac{(k+d/2-1)!}{(d/2-1)!k!}e^{-(d/2+2k)\omega\tau} \qquad (13)$$

for the Green function in the imaginary time (see, *e.g.* ref. [9]), *i.e.*, it is given just by the sum of poles (or, more precisely, δ-functions) when taken in the energy representation.

Of course, what we are discussing now, is the quark propagator in the presence of an external color source whose colour is opposite to that of our quark (taken together with accompanying gluonic field). We are not discussing how a single quark propagates in an empty space or in a color neutral background. Such an propagator, however, is of little practical importance in the real world with confined quarks.

4. QCD sum rules for the $B \to De\nu$ type decays. To calculate the decay form factor, one should add an the extra current $J^\mu(y)$ into the correlator and analize the three-point function $T(P, P')$, with $P = (m_Q + E)v$ and $P' = (m_{Q'} + E')v'$. In the $m_Q, m_{Q'} \to \infty$ limit, this function reduces to the product of the light quark propagator and two heavy quark propagators. The latter, as discussed above, acquire a very simple form. As a result, $T(P, P')$ depends on the initial and final momenta only through v, v', E, E'. The dynamic information again is accumulated in the light quark propagator. For the double Borel transform we get

$$M(\epsilon_1,\epsilon_2,v,v') = \int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big\{\Gamma'\Big(\frac{1+\not{v}'}{2}\Big)\Gamma^\mu\Big(\frac{1+\not{v}}{2}\Big)\Gamma S_q(k)\Big\}e^{-(kv)/\epsilon_1-(kv')/\epsilon_2} =$$
$$= \mathrm{Tr}\Big\{\Gamma'\Big(\frac{1+\not{v}'}{2}\Big)\Gamma^\mu\Big(\frac{1+\not{v}}{2}\Big)\Gamma S_q(\frac{iv}{\epsilon_1}+\frac{iv'}{\epsilon_2})\Big\}. \qquad (14)$$

Thus, one should use now the function describing the light quark propagation from the past point $-iv/\epsilon_1$ to the future point iv'/ϵ_2. Using the explicit form for $S^{pert}(x)$, we obtain the perturbative spectral density in the lowest α_s order,

$$\rho^{pert}(s_1,s_2,w) = \frac{(s_1+s_2)\,\theta(s_1e^{-\gamma} < s_2 < s_1e^{\gamma})}{4(1+w)\sqrt{w^2-1}}, \qquad (15)$$

where γ is the angle between v and v' : $w = \cosh(\gamma)$. The nonperturbative contribution is determined by the nonlocal quark condensate

$$\langle\bar{q}q\rangle\Phi\left(-\left[\frac{1}{\epsilon_1^2}+\frac{1}{\epsilon_2^2}+\frac{2w}{\epsilon_1\epsilon_2}\right]\right). \qquad (16)$$

Taking $\epsilon_1 = \epsilon_2 = 2\epsilon$ and assuming again the "first resonance plus continuum" ansatz we obtain the sum rule for the Isgur-Wise form factor $\xi(w)$:

$$f^2\xi(w)e^{-E_R/\epsilon} = \int_0^{E_c} ds_1 \int_{ys_1}^{s_1} ds_2 \frac{(s_1+s_2)e^{-(s_1+s_2)/2\epsilon}}{2(1+w)\sqrt{w^2-1}} + \kappa^3 \exp\left[-\frac{\lambda^2}{8\epsilon^2}\left(\frac{w+1}{2}\right)\right]. \qquad (17)$$

Note, that this sum rule, in the limit $w \to 1$, coincides with that for f^2, since

$$\rho^{pert}(s_1, s_2, w \to 1) = \delta(s_1 - s_2).$$

This means that $\xi(w)$ is normalized at zero recoil: $\xi(1) = 1$ (cf.ref.[1]). Taking $E_c = 680\,MeV$ for the continuum threshold and varying the Borel parameter ϵ within the stability region $\epsilon = 400 - 800 MeV$, we obtain the resulting curve for the $\xi(w)$ function. Within the accuracy limits of the method, it can be approximated by

$$\xi(w)|_{QCD\,SR} \approx \exp(-0.37\sqrt{w^2-1}). \qquad (18)$$

Numerically, the QCD SR prediction is rather close to the valence quark model result

$$\xi(w)|_{QM} = \exp(-0.63(w-1)) \qquad (19)$$

obtained by Isgur [4], especially in the region $w > 1.5$, where the QCD SR errors caused by the use of a rough model for the continuum are less significant. On the other hand, our curve goes lower than that corresponding to the formula

$$\xi(w)|_{CQM} = \frac{1}{1.86}\left(\frac{\gamma}{\sinh\gamma} + \frac{1.72}{1+\cosh\gamma}\right) \qquad (20)$$

obtained by M.Ivanov [5] in his confined quark model.

5. *Conclusions.* Physics of the hadrons containing one heavy quark, is essentially a light-quark problem. This means it requires a nonperturbative approach. As we observed, the QCD sum rules provide such an approach enabling one to systematically calculate both static and dynamic characteristics of these hadrons. From the other side, the heavy-light systems, in the $M_Q \to \infty$ limit, seem to be an ideal tool to study the nonperturbative properties of the light quark propagation functions, nonlocal condensates, *etc.*

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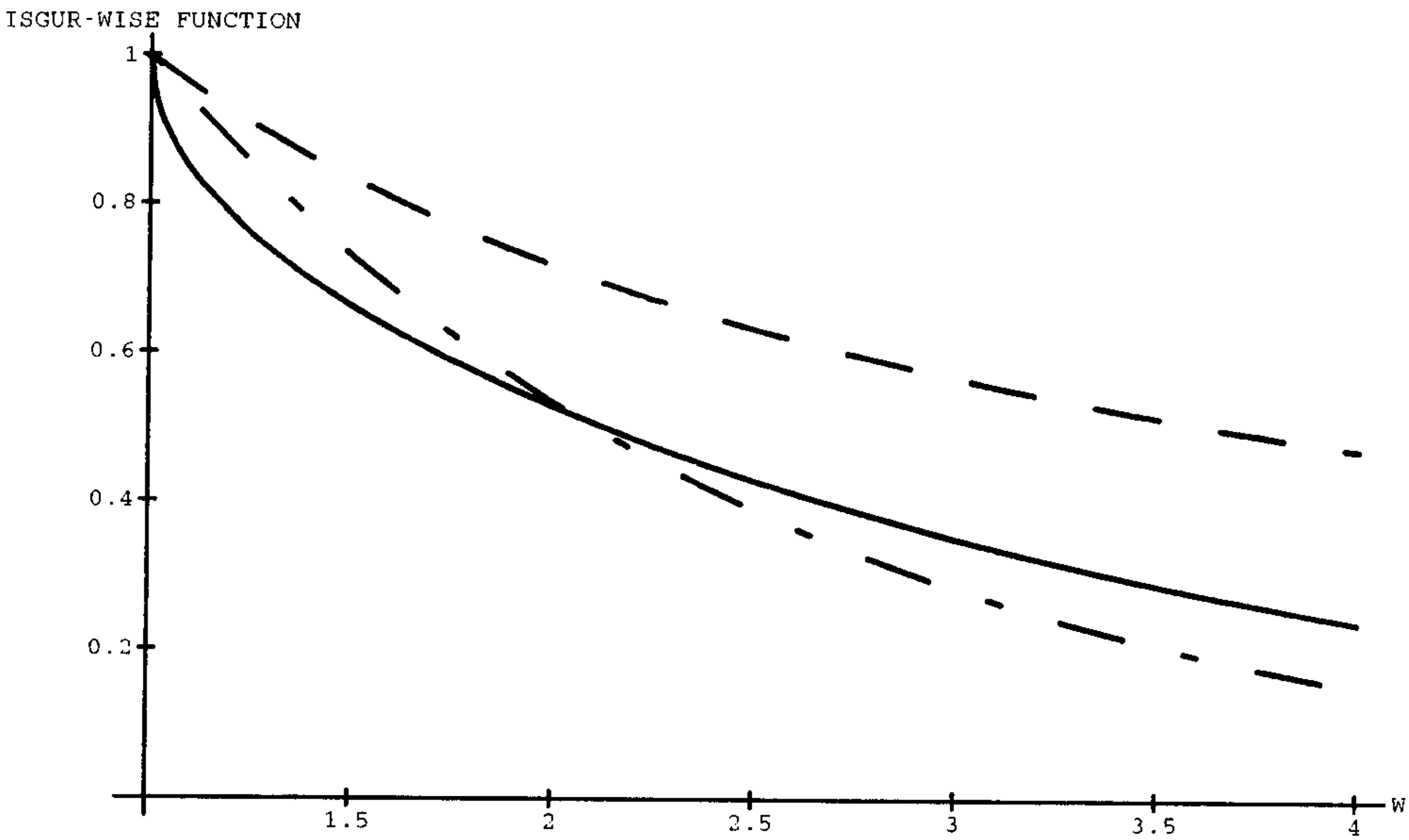
FIGURE CAPTION

Fig.1 Comparison of the QCD sum rule calculation of the Isgur-Wise form factor $\xi(w)$ (eq.(18), solid line) with the results obtained in the valence quark model (eq.(19), dash-dotted line) and in the confined quark model (eq.(20), dashed line).



ISGUR-WISE FUNCTION
1
0.8
0.6
0.4
0.2
1.5
2
2.5
3
3.5
4
W