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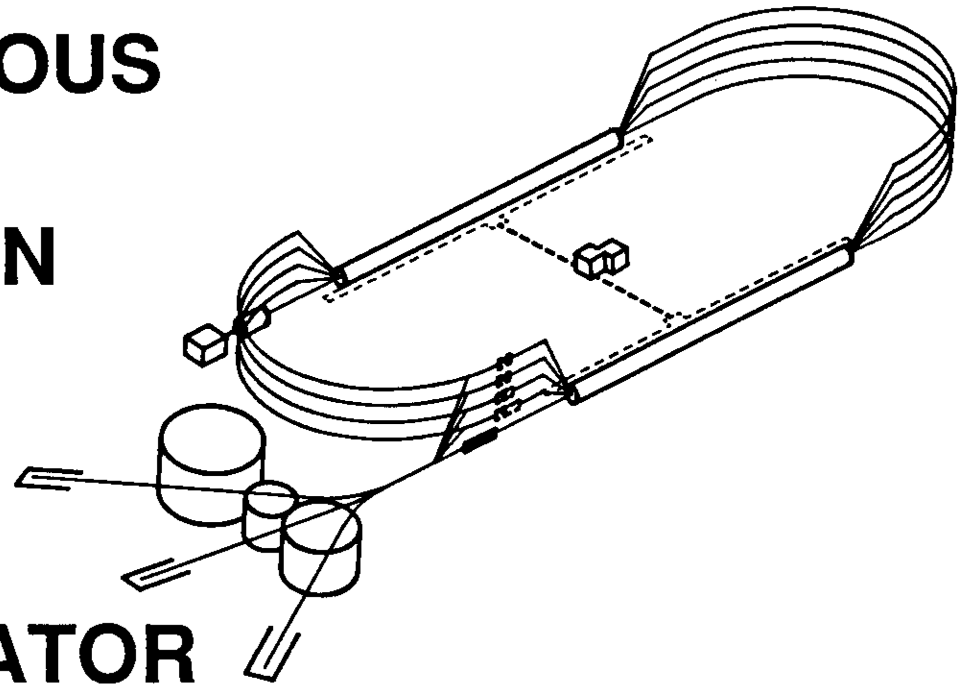

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ABSTRACT

The CEBAF accelerator beam transport system contains 104 sextupoles to correct chromatic aberrations. We describe the layout of these elements and discuss schemes for suppressing chromatic errors. Analytic results for the required sextupole strengths are given and computations of chromatic aberrations are documented. Numerical results using two correction methods are provided.

ACCELERATOR OVERVIEW

The CEBAF transport system is detailed elsewhere[1]. Here, we need only note that the five-pass accelerator comprises a pair of linacs joined by 9 recirculation transport channels. Each channel is 400 m in length and carries beam at a specified energy from one linac to the next. Each consists of 4 modules: a "spreader" (that separates beams at various energies and geometrically and optically matches each to the proper channel), an "extraction region" (used to generate external beams), a semi-circular "arc" (that connects the linac axes, which are 161.2 m apart), and a "recombiner" (that geometrically and optically matches each beam to a linac for reinjection). A switchyard carries beam from the end of the accelerator to experimental halls. The maximum beam path length is 6.5 km.

Of interest here are the arcs. Each consists of 4 dispersion suppressed, isochronous superperiods that are betatron matched to the linacs. Beam propagates in each superperiod via the "matched" betatron functions, with design horizontal and vertical phase advances of 5/4 and 3/4 oscillation per period, respectively. Each arc superperiod is equipped with a pair of sextupoles; these are located at points of nonzero dispersion and unequal horizontal and vertical envelope functions.

The system design is modular; errors and aberrations in individual channels are corrected locally. This motivates the choice of phase advance and superperiodicity; sextupoles powered in pairs separated by half wavelengths suppress chromatic errors without generating geometric aberrations[2]. A total of 72 sextupoles are available within the accelerator for chromatic correction. Chromatic correction is implemented in the beam switchyard using 32 sextupoles and is conceptually identical to that in the accelerator. It will not be separately addressed. The correction algorithm is based on a scheme of T. Collins. We will now outline the method and apply it to the CEBAF transport system.

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EFFECT OF A FOCUSSING PERTURBATION

Consider the effect of a point focussing perturbation at a location k in a beam line. Assume it has quadrupole symmetry and acts with focal length f_k; its effect is then described by a 4×4 thin lens matrix P that focusses in one transverse plane and defocusses in the other.

Collins[3] notes that $1/f_k$ is a small parameter in the computation of lattice functions β_o^x, α_o^x, β_o^y, α_o^y, ψ_o^x and ψ_o^y at a downstream observation point "o". The unperturbed lattice functions β_k^x, α_k^x, β_k^y, α_k^y, ψ_k^x and ψ_k^y at k transform to the perturbed lattice functions at o via the matrix MP, where M is the *unperturbed* transformation matrix from k to o. When lattice functions are propagated with MP in the usual manner[4], the result at o contains terms linear and quadratic in $1/f_k$. As $1/f_k$ is small, we retain only linear terms; differences between the perturbed and unperturbed functions at o are readily computed. Assume then that N such errors occur upstream of o, and that all terms in $1/f$ of nonlinear order are negligible. The resulting total deviations at o, which follow, are simply the sum of the effects of the separate perturbations.

$$\begin{aligned}
\frac{\Delta\beta_n}{\beta_o^n} &= \mp \sum_{k=1}^{N} P_k \beta_k^n \sin 2(\psi_o^n - \psi_k^n) \\
\Delta\alpha_n &= \pm \sum_{k=1}^{N} P_k \beta_k^n \left(\cos 2(\psi_o^n - \psi_k^n) - \alpha_o^n \sin 2(\psi_o^n - \psi_k^n)\right) \\
\Delta\psi_n &= \pm \frac{1}{2\pi} \sum_{k=1}^{N} P_k \beta_k^n \sin^2(\psi_o^n - \psi_k^n), \qquad n = x \text{ or } y
\end{aligned} \tag{1}$$

Here, the upper sign applies to x, and the lower to y; $P_k \equiv 1/f_k$; all β, α, and ψ values are the unperturbed linear lattice functions.

We emphasize that these functions and deviations are those *propagated* through the transport system; they are not constrained by periodicity. (Thus, *e.g.* ψ^x and ψ^y represent phase advances from one point to another, not tunes). The effect of a perturbation on downstream lattice functions therefore depends on the choice of input lattice functions, which we will assume to be fixed.

SCHEMES FOR CHROMATIC CORRECTION

Formalism

Correction of chromatic aberrations may be accomplished through the use of sextupoles at locations of nonzero dispersion. The focal length of a sextupole at location

k acting on a test particle with momentum offset δ is

$$\frac{1}{f_k} = 2k_{2,k} l \eta_k^x \delta \ , \qquad (2)$$

where η_k^x the orbit dispersion, l the length, and $k_{2,k}$ (defined by $B_y(x, y=0) \equiv B\rho k_2 x^2$) the strength. This perturbation may be treated by (1). Substitution of (2) into (1) yields results for the downstream linear chromatic aberrations induced by a set of sextupoles at N locations k. A method of chromatic correction is then clear: aberrations of concern are computed, and sextupoles are activated to generate compensatory deviations. Given lattice functions and values for the aberrations of interest, (1) may be inverted to specify the values $1/f_k$ required to provide the desired cancellation.

Certain restrictions apply. Foremost is any requirement that each sextupole be powered in series with a partner one half betatron-wavelength away. This has particular implications for lattices with quarter-integer phase advances. Consider a beam line with an embedded four-period structure, on which the following conditions are imposed: 1) optical matching into the structure (lattice functions propagate via the "matched" betatron functions), 2) quarter-integer per period phasing (each period has phase advance $(2m+1)/4$; m may differ for each transverse plane) 3) a single pair of sextupoles per period, and 4) sextupoles separated by half betatron-wavelengths are powered in series.

Under these conditions, evaluation of (1) reveals that only *four* independent terms are available for correcting *six* linear chromatic aberrations. Denoting by $k_{2,1}$ and $k_{2,2}$ the strengths of the sextupoles in the first (and third) period, and by $k_{2,3}$ and $k_{2,4}$ the strengths of the two in the second (and fourth), conditions 1) – 4) applied to (1) yields a system dependent on only $(k_{2,1} \pm k_{2,3})$, $(k_{2,2} \pm k_{2,4})$. The parameters $(k_{2,1} + k_{2,3})$ and $(k_{2,2} + k_{2,4})$ occur only in the phase equations; $(k_{2,1} - k_{2,3})$ and $(k_{2,2} - k_{2,4})$ occur only in the β and α equations. It is therefore possible to correct only one of the pairs $(\Delta\beta_x(\delta), \Delta\beta_y(\delta))$, $(\Delta\alpha_x(\delta), \Delta\alpha_y(\delta))$, together with the phase advances. In the following, we will focus attention on the compensation of β aberrations; compensation of α may be carried out in like fashion.

Four-Family Correction

We now apply equations (1) to the suppression of the linear variation in δ of $\beta_{x,y}$ and $\psi_{x,y}$. Imposing conditions 1) – 4) from above on (1) and (2) results in the following relation between sextupole excitation and induced linear chromatic aberrations at o.

$$\begin{pmatrix} \Delta\beta_x/\beta_o^x \\ \Delta\beta_y/\beta_o^y \\ \Delta\psi_x \\ \Delta\psi_y \end{pmatrix} = \begin{pmatrix} -\mathrm{S}_1^x & -\mathrm{S}_2^x & \mathrm{S}_1^x & \mathrm{S}_2^x \\ \mathrm{S}_1^y & \mathrm{S}_2^y & -\mathrm{S}_1^y & -\mathrm{S}_2^y \\ \mathcal{S}_1^x & \mathcal{C}_2^x & \mathcal{S}_1^x & \mathcal{C}_2^x \\ -\mathcal{S}_1^y & -\mathcal{C}_2^y & -\mathcal{S}_1^y & -\mathcal{C}_2^y \end{pmatrix} \begin{pmatrix} k_{2,1}\delta \\ k_{2,2}\delta \\ k_{2,3}\delta \\ k_{2,4}\delta \end{pmatrix} \qquad (3)$$

Here we have defined

$$\mathrm{S}_j^n \equiv 4\beta_j^n \eta_j^x l \sin 2(\psi_o^n - \psi_j^n),$$

$$\mathcal{S}_j^n \equiv (2/\pi)\beta_j^n \eta_j^x l \sin^2(\psi_o^n - \psi_j^n), \text{ and} \qquad (3')$$

$$\mathcal{C}_j^n \equiv (2/\pi)\beta_j^n \eta_j^x l \cos^2(\psi_o^n - \psi_j^n), \quad n = x \text{ or } y.$$

The following procedure then corrects the linear variations of β and ψ with momentum. First, specify a starting point and input betatron parameters, and compute the "natural" linear variation with momentum of β and ψ at a downstream observation point. Next, insert the *negative* of the variations so established into (3). Finally, invert (3) to determine the $k_{2,1}$, $k_{2,2}$, $k_{2,3}$, and $k_{2,4}$ values required for the desired compensation.

Two-Family Correction

For lattices satisfying conditions outlined here, this is a subset of four-family correction. It is defined by the conditions $k_{2,1} = k_{2,3}$ and $k_{2,2} = k_{2,4}$. Recall that $\Delta\beta$ and $\Delta\alpha$ *are driven by* $k_{2,1} - k_{2,3}$ *and* $k_{2,2} - k_{2,4}$; we thus cannot modify β and α aberrations using this method. To implement two-family correction, proceed with the inversion of (3) as above. However, instead of substituting $-(\Delta\beta_{x,y}/\beta_o^{x,y})_{natural}$ into (3), use 0 for the target envelope function variation.

APPLICATION TO THE CEBAF TRANSPORT SYSTEM

Chromatic aberrations may be corrected over various sections of the CEBAF transport system (such as arcs only, complete recirculation channels, or globally). The region of interest is defined through appropriate selection of initial point and observation point. To illustrate the technique, we will consider the correction of chromatic effects over a complete recirculation channel.

DIMAD[5] has been used to analyse the behavior of all nine recirculation lines. The results for the linear chromatic aberrations for nominal tunings of the 31 March 1990 reference lattice, as established using a "detailed chromatic analysis", are summarized in Table 1.

Table 1

Linear Chromatic Aberrations in CEBAF Recirculation Channels

E (GeV)	$\left(\frac{\Delta\beta_x}{\beta_o^x}\right)$	$(\Delta\alpha_x)$	$(\Delta\psi_x)$	$\left(\frac{\Delta\beta_y}{\beta_o^y}\right)$	$(\Delta\alpha_y)$	$(\Delta\psi_y)$
0.445	20.3δ	-20.2δ	-11.8δ	0.2δ	-12.5δ	-10.7δ
0.845	21.0δ	-77.7δ	-9.8δ	-12.0δ	-7.8δ	-11.9δ
1.245	6.4δ	-19.5δ	-10.6δ	-0.3δ	-15.2δ	-10.3δ
1.645	-2.0δ	-26.6δ	-11.6δ	-15.4δ	-99.6δ	-9.2δ
2.045	-11.2δ	-12.4δ	-11.2δ	-5.3δ	-45.5δ	-9.6δ
2.445	-5.6δ	-82.5δ	-11.1δ	-9.9δ	-100.6δ	-9.0δ
2.845	11.5δ	10.0δ	-14.4δ	5.5δ	-48.2δ	-9.6δ
3.245	11.2δ	-77.6δ	-11.3δ	2.8δ	-129.3δ	-8.8δ
3.645	6.2δ	-26.0δ	-9.5δ	3.1δ	-72.5δ	-8.4δ

These results were generated under the following conditions: the initial point in each channel is the beginning of the spreader, design phase ellipse parameters and phase advances are propagated forward from the initial point, and the observation point is the injection point of the downstream linac.

Equation (3) has been applied to each channel to establish four- and two-family corrections. The resulting chromatic behavior is, for either solution and in all lines, adequate to achieve CEBAF design goals.

As an example, the full momentum dependences of various lattice parameters at the end of the 0.445 GeV line are are displayed in Figure 1 for uncorrected and corrected (both two and four family) cases. The momentum range of interest, $\pm 5 \times 10^{-3}$, reflects the physical acceptance of the transport system rather than the $\pm 5 \times 10^{-5}$ beam energy spread. (The system has a peak dispersion of 2.5 m within beam pipes with an interior diameter of 1″.) We remark that only β^*_o, α^*_o, and ψ^*_o only are displayed; the vertical variations are similar in character and magnitude. Both $x(\delta)$ and $y(\delta)$ are given, as the variation in y is an order of magnitude larger than the variation in x. This is because the x variation arises within the periodic arc structure where it is suppressed by symmetry, while the y arises in the nonsymmetric spreaders and recombiners. The system is therefore more subject to vertical orbit variation than horizontal. Path length variation δl is included to illustrate the momentum dependence of this critical parameter. The nominal bunch length is 0.5 mm; without sextupoles, the path length varies by a full bunch length over the momentum aperture. With sextupoles, the variation falls to 0.25 mm. Finally, no significant nonlinear coupling is observed, even at the limit of the acceptance, so no related parameters are presented.

ACKNOWLEDGEMENTS

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REFERENCES

[1] B. Bowling *et.al.*, "The CEBAF Beam Transport System Lattice Design", these proceedings.

[2] K.L. Brown, "A 2^{nd}-Order Magnetic-Optical Achromat," I.E.E.E. Trans. Nuc. Sci., vol. NS-26, pp. 3490-3492, June 1979.

[3] T. Collins (private communication); D. Douglas, A. Garren, and F. Dell, "The Collins Scheme for the Local Correction of Chromatic Aberrations Due to Insertions", LBL Report LBL-17984, June 1984.

[4] See, *e.g.*, eq. (3.6) of K.L. Brown and R.V. Servranckx, in *Phys. of High Ener. Part. Accel.*, M. Month *et.al.*, eds., New York: A.I.P. Conf. Proc. 127, 1985, p. 101.

[5] R.V. Servranckx *et.al.*, "Users' Guide to the Program DIMAD", SLAC Report 285 UC-28 (A), May, 1985.

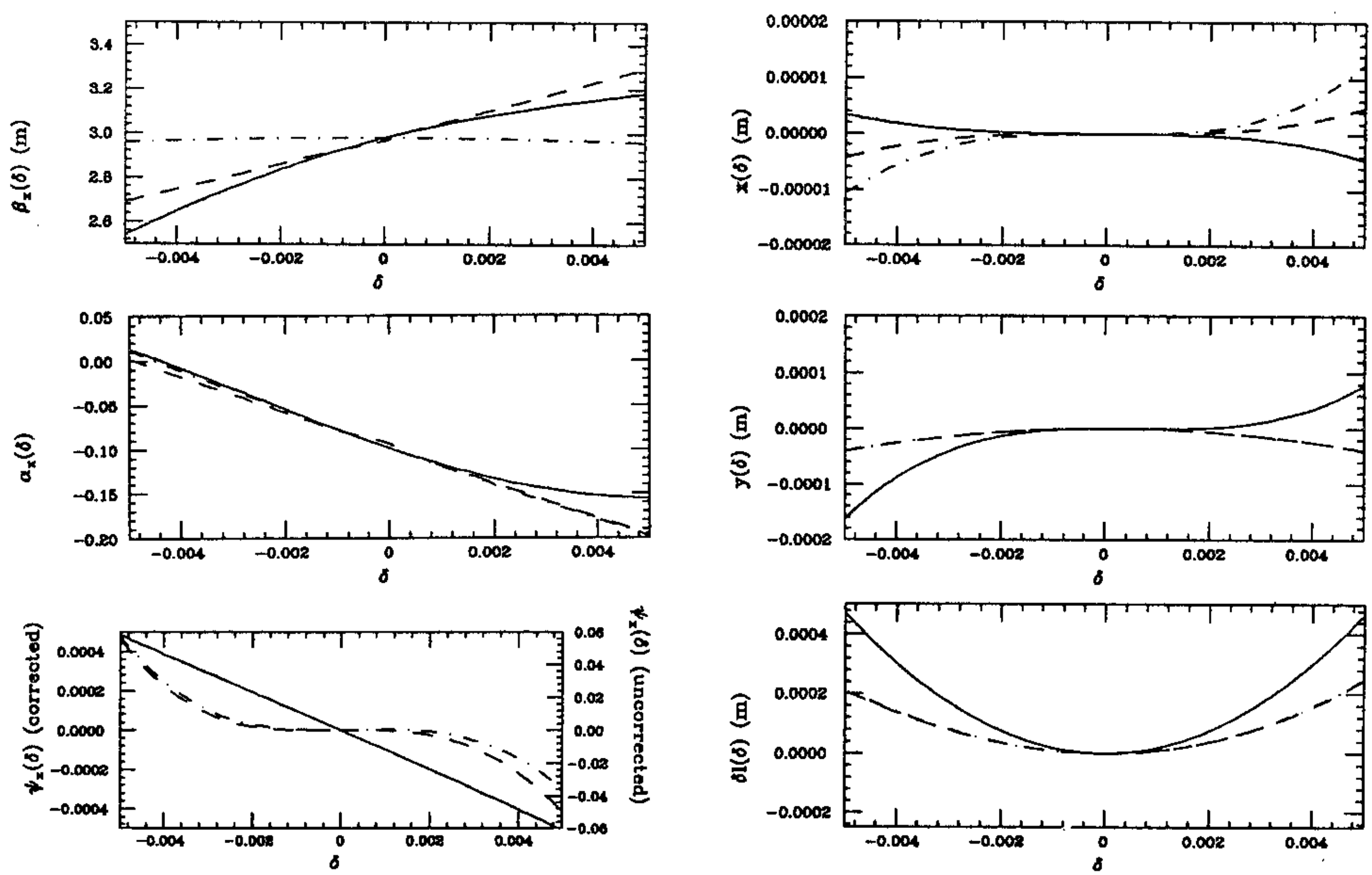

Figure 1. Momentum dependence of lattice functions over physical acceptance of 0.445 GeV recirculation channel. Solid curves denote uncorrected values, dashed curves represent values corrected with two families of sextupoles, dot-dash curves indicate values corrected with four families.