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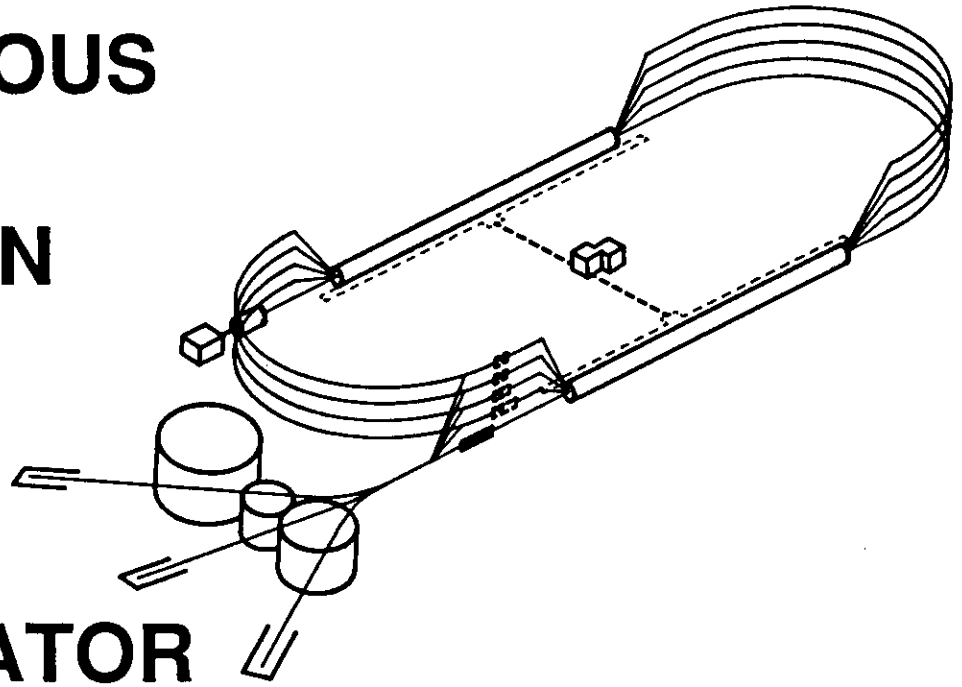

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Excited Charm Baryons in Semileptonic Λ_b Decay and their Contribution to a Bjorken Sum Rule*

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ABSTRACT

It has recently been shown that hadrons containing a single heavy quark exhibit a new flavor-spin symmetry of QCD. We exploit this symmetry to obtain model independent predictions for the form factors for the semileptonic decay of the Λ_b to the low-lying negative parity charm baryons. We discuss the connection between these results and the slope of the universal function ζ that determines the $\Lambda_b \to \Lambda_c$ weak form factors.

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It has recently been realized that in an effective theory constructed by taking the limit of QCD where the masses of heavy quarks ($m_{Q_i} >> \Lambda_{QCD}$) go to infinity [1-9] with their four-velocities fixed, new symmetries of the strong interactions appear and that these symmetries can be used to predict many properties of hadrons containing a single heavy quark. For N heavy quarks $Q_i, i = 1,..N$, this effective theory has an SU(2N) flavor-spin symmetry that arises because in the effective theory the couplings of heavy quarks are independent of their mass and spin.

The consequences of these symmetries for the weak semileptonic decays of $\bar{B}$ mesons to the ground state D and D^* mesons (with $J^P = 0^-$ and 1^-, respectively) were determined in refs. [1] where it was shown that all the form factors are expressible in terms of a single universal function of velocity transfer which is normalized to unity at zero recoil[†] . Weak semileptonic $\bar{B}$ meson decays to the low-lying positive parity charm mesons have also been considered, and their contribution to a Bjorken sum rule [11] for the slope of the universal function at zero recoil derived [12].

The implications of the heavy quark spin-flavor symmetries for the weak semileptonic decays of baryons were considered in refs. [13] and [14]. It was shown that the form factors which determine the properties of Λ_b semileptonic decay to the ground state $J^P = \frac{1}{2}^+ \Lambda_c$ baryon are expressible in terms of a single universal function $\zeta(w)$ of velocity transfer w. (Form factors for the weak semileptonic decays of bottom baryons containing strange quarks were also considered.) In this letter we perform for the Λ_b the calculations analogous to those done in ref. [12] for the $\bar{B}$: we consider the weak form factors for Λ_b semileptonic decay to the low-lying excited negative parity charmed baryons and derive their contribution to a Bjorken sum rule for the slope of the universal function ζ at zero recoil.

In the leading logarithmic approximation the relationship between heavy quark bilinears $\bar{Q}_j \Gamma Q_i$ in QCD and the bilinears $\bar{\mathbf{Q}}_j \Gamma \mathbf{Q}_i$ in the effective theory is

$$\bar{Q}_j \Gamma Q_i = C_{ji}(w) \bar{\mathbf{Q}}_j \Gamma \mathbf{Q}_i \tag{1}$$

where [7,8,9]

$$C_{ji}(w) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(m_{Q_j})}\right]^{a_I} \left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(\mu)}\right]^{a_L} . \tag{2}$$

For the case $Q_i = b$ and $Q_j = c$

$$a_I = -6/25 \tag{3}$$

and

$$a_L = \frac{8\left[wr(w) - 1\right]}{27} \tag{4}$$

with

$$r(w) = \frac{1}{\sqrt{w^2 - 1}} \ell n \left(w + \sqrt{w^2 - 1}\right) . \tag{5}$$

[†] See also ref. [10] for the application of the symmetry which arises in the "Shifman-Voloshin" limit in which $m_b >> m_b - m_c$ and $m_c >> \Lambda_{QCD}$.

Here $w = v \cdot v'$ is the dot product of the four-velocity of Q_i, v^μ, with the four-velocity of Q_j, v'^μ. (Recall that in the effective theory interactions of a heavy quark with gluons don't alter its four-velocity.) For $\Gamma = \sigma_{\mu\nu}, 1$, or γ_5 the quark bilinears are renormalized in the full theory and in these cases it is understood that the left side of eq. (1) is evaluated at a scale equal to m_{Q_i}.

In the limit $m_Q \to \infty$ the spin of the heavy quark $\vec{S}_Q$ and the spin of the light degrees of freedom $\vec{S}_\ell = \vec{S} - \vec{S}_Q$ are separately conserved by the strong interaction so baryons containing a single heavy quark can be simultaneously assigned the quantum numbers s_Q, m_Q, s_ℓ, and m_ℓ. Since the dynamics depend only on s_ℓ, the baryons will appear in degenerate multiplets of the total spin s that can be formed from combining s_Q and s_ℓ. It is therefore convenient in the heavy quark limit to classify states by s_ℓ and π_ℓ, the parity of the light degrees of freedom.

The heavy quark symmetry cannot, of course, tell us anything about the spectroscopy of the light degrees of freedom: it can only predict relationships between heavy quark systems involving a given state of these degrees of freedom. We can, however, be reasonably confident of the qualitative characteristics of this spectroscopy given the phenomenological success of the nonrelativistic constituent quark model for baryons [15]. This model predicts that the lowest-lying I=0 excited states of Λ_Q with valence quark content udQ will be seven negative parity levels (three $J^P = \frac{1}{2}^-$ states, three $J^P = \frac{3}{2}^-$ states, and one $J^P = \frac{5}{2}^-$ state).* To understand the structure of the quark model predictions it is convenient to first consider the harmonic oscillator model within which the three body problem can be solved analytically. Then as $m_Q \to \infty$ with $\vec{r} \equiv \vec{r}_u - \vec{r}_d$ and $\vec{s} = \frac{1}{2}(\vec{r}_u + \vec{r}_d)$ the internal Hamiltonian with a harmonic potential $\frac{1}{2}k(\vec{r}_i - \vec{r}_j)^2$ between quarks i and j separates into the sum of H_r and H_s with $H_r = \frac{p_r^2}{m} + \frac{3}{4}kr^2$ and $H_s = \frac{p_s^2}{4m} + ks^2$. H_s represents the motion of the center-of-mass of the u and d quarks (with mass 2m), while H_r represents the relative motion of the u and d quarks (with reduced mass $\frac{1}{2}$m). In this model the Λ_Q has both H_s and H_r in their ground states. Since $\omega_r = \left(\frac{3k}{m}\right)^{1/2}$ and $\omega_s = \left(\frac{k}{m}\right)^{1/2}$, the first excited state is a negative parity state with $(\ell r, \ell s) = (0,1)$ while the second is a negative parity state with $(\ell r, \ell s) = (1,0)$. It is easy to show that the latter has an excitation energy equal to that of ordinary nucleon excitations (since in the harmonic limit $\vec{r}$ decouples from $\vec{r}_Q$) while $\omega_s = \frac{\omega_r}{\sqrt{3}}$. Since in the lower-lying $(\ell_r, \ell_s) = (0,1)$ state the ud pair must have spin zero so that it has isospin zero, this state has $s_\ell^{\pi_\ell} = 1^-$. The higher state conversely must have the ud pair in a spin one state; when combined with the orbital angular momentum, this gives states with $s_\ell^{\pi_\ell} = 0^-, 1^-$, and 2^-. When combined with $s_Q^{\pi_Q} = \frac{1}{2}^+$, these states of the light degrees of freedom give the spectroscopy described above. Within the context of the nonrelativistic constituent quark model, these qualitative features are not dependent on the harmonic oscillator model. To see this, first consider the single quark "orbitals" that would exist in the absence of the interaction V_{ud} between the u and d quarks. In this limit the ground state would be $|s_u s_d\rangle$ and there would be two degenerate first excited states $|s_u p_d\rangle$ and $|p_u s_d\rangle$, where s_q and p_q are single particle orbitals for quark q with $\ell = 0$

* We are only interested in I=0 states since the $b \to c$ current will not change isospin.

and 1. When the $u-d$ interaction is turned on, $|s_u s_d\rangle$ will be perturbed into the ground state wavefunction, while $|s_u p_d\rangle$ and $|p_u s_d\rangle$ will be perturbed into the $(\ell_r, \ell_s) = (0,1)$ and (1,0) states. In fact under the influence of V_{ud} the new $\ell = 1$ eigenstates will have the structure

$$|\psi_s\rangle \sim \sqrt{\frac{1}{2}}(|s_u p_d\rangle + |p_u s_d\rangle)$$
$$|\psi_r\rangle \sim \sqrt{\frac{1}{2}}(|s_u p_d\rangle - |p_u s_d\rangle)$$

which are respectively symmetric and antisymmetric under $\vec{r}_u \leftrightarrow \vec{r}_d$ (and correspond to the harmonic oscillator states (ℓ_r, ℓ_s)=(0,1) and (1,0), respectively). Since V_{ud} is an attractive interaction, $|\psi_s\rangle$ will have a lower energy than $|\psi_r\rangle$; this is in accord with the conclusion based on the harmonic oscillator model. The physics of this system is analogous to that of a helium atom with inverted statistics (and with residual repulsion instead of attraction).

In addition to this orbital splitting of order 150 MeV, quark spin-spin interactions are expected to further split the states $|\psi_s\rangle$ and $|\psi_r\rangle$: these interactions are highly attractive in the spin zero state of $|\psi_s\rangle$ and repulsive in the spin one state of $|\psi_r\rangle$ (*cf.* the $\Delta - N$ splitting). The models thus give a total splitting between the $s_\ell^{\pi\prime} = 1^-$ state and the center of gravity of the $s_\ell^{\pi\prime} = 2^-, 1^-, 0^-$ spin-orbit multiplet of about 350 MeV. We thus can expect the $|\psi_s\rangle$-type $s_\ell^{\pi\prime} = 1^-$ state to be the first negative parity excited state encountered above the Λ_Q by a considerable margin.

The weak decays of Λ_{Q_i} to these excited states of Λ_{Q_j} are determined by the matrix elements

$$\langle {}^{s\prime}\Lambda_{Q_j}^{(n)} J^P(\vec{v}', s')|\bar{\mathbf{Q}}_j \Gamma \mathbf{Q}_i|\Lambda_{Q_i}(\vec{v}, s)\rangle. \tag{6}$$

(We have simply denoted the ground state isospin zero udQ_i baryon by Λ_{Q_i}; the excited states discussed above are defined in an obvious notation where the superscript (n) denotes a state belonging to the nth multiplet with light quantum numbers $s_\ell^{\pi\prime}$.) The helicity analysis of Politzer [16] is useful in constraining which states have nonzero matrix elements (6) in the heavy quark limit. For each s_ℓ there is one helicity zero amplitude. By parity if $\pi_\ell = +$ only states with even s_ℓ have a nonzero matrix element (6) and for $\pi_\ell = -$ only states with odd s_ℓ have matrix elements which are nonzero. So of the low lying negative parity baryons considered above only the $s_\ell^{\pi\prime} = 1^-$ states contribute to the weak decay of Λ_{Q_i}. We will see later that states with these quantum numbers for the light degrees of freedom are the *only* resonant contributions to Bjorken's sum rule for the derivative of ζ at threshold.

For $s_\ell^{\pi\prime} = 1^-$ the most general form for the matrix elements (6) consistent with the heavy quark spin symmetries is [14]

$$\langle {}^1\Lambda_{Q_j}^{(n)} \frac{1}{2}^- (\vec{v}', s')|\bar{\mathbf{Q}}_j \Gamma \mathbf{Q}_i|\Lambda_{Q_i}(\vec{v}, s)\rangle = \frac{\sigma^{(n)}(w)}{\sqrt{3}} \bar{u}(\vec{v}', s')\gamma_5(\not{v} + w)\Gamma u(\vec{v}, s) \tag{7a}$$

$$\langle {}^1\Lambda_{Q_j}^{(n)} \frac{3}{2}^- (\vec{v}', s')|\bar{\mathbf{Q}}_j \Gamma \mathbf{Q}_i|\Lambda_{Q_i}(\vec{v}, s)\rangle = \sigma^{(n)}(w)\bar{u}_\mu(\vec{v}', s')v^\mu \Gamma u(\vec{v}, s). \tag{7b}$$

Bjorken's sum rule follows from a consideration of the quantity

$$h^X(w) = \frac{1}{2}\sum_{s,s'}\langle\Lambda_{Q_i}(\vec{v},s)|\bar{Q}_i\bar{\Gamma}Q_j|X_{Q_j}(\vec{v}',s')\rangle\langle X_{Q_j}(\vec{v}',s')|\bar{Q}_j\Gamma Q_i|\Lambda_{Q_i}(\vec{v},s)\rangle, \tag{8}$$

with w of order unity. It is based on the observation that if one sums the contributions for all hadronic states with masses from $m_{\Lambda_{Q_j}}$ to $m_{\Lambda_{Q_j}}+\mu$, then so long as $\mu >> \Lambda_{QCD}$ this inclusive quantity can be computed in perturbative QCD as an infrared safe heavy quark transition. On the other hand if $\mu << m_{Q_j}$ the quantity h^X can also be computed for each exclusive channel using the heavy quark effective theory. Thus

$$\frac{|C_{ji}(\mu)|^2}{2}Tr\left\{\frac{(\not{v}'+1)}{2}\bar{\Gamma}\frac{(\not{v}+1)}{2}\Gamma\right\} = \sum_{m_{X_{Q_j}}-m_{\Lambda_{Q_j}}<\mu} h^X(w). \tag{9}$$

In eq. (9) the factor $|C_{ji}(\mu)|^2$ arises from perturbative strong interaction corrections to the heavy quark transition summed in the leading logarithmic approximation.

In refs. [13,14] it was shown that the spin and flavor symmetries imply that

$$\langle\Lambda_{Q_j}(\vec{v}',s')|\bar{\mathbf{Q}}_j\Gamma\mathbf{Q}_i|\Lambda_{Q_i}(\vec{v},s)\rangle = \zeta(w)\bar{u}(\vec{v}',s')\Gamma u(\vec{v},s) \tag{10}$$

where $\zeta(1)=1$. Using this and eqs. (7) we can include in (9) the contributions of baryon resonances X_{Q_j} with $s_\ell^{\pi'}=0^+$ and 1^-. Matrix elements of states with $s_\ell^{\pi'}=0^+$ that are not the ground state Λ_{Q_j} can easily be included since for the nth such state

$$\langle{}^0\Lambda_{Q_j}^{(n)}\frac{1}{2}^+(\vec{v}',s')|\bar{\mathbf{Q}}_j\Gamma\mathbf{Q}_i|\Lambda_{Q_i}(\vec{v},s)\rangle = (w-1)\zeta^{(n)}(w)\bar{u}(\vec{v}',s')\Gamma u(\vec{v},s). \tag{11}$$

(The factor of $(w-1)$ in eq. (11) arises because at zero recoil (*i.e.*, $w=1$) $\bar{\mathbf{Q}}_j\Gamma\mathbf{Q}_i$ is related to a generator of the spin flavor symmetry and the states $|{}^0\Lambda_{Q_j}^{(n)}\frac{1}{2}^+\rangle$ are orthogonal to the ground state $|\Lambda_{Q_j}\rangle$.) Using

$$\sum_s u_\mu(\vec{v},s)\bar{u}_\nu(\vec{v},s) = \frac{2}{3}\left[\frac{\not{v}+1}{2}\right]\left[-g_{\mu\nu}+\frac{1}{4}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)+\frac{1}{2}(\gamma_\mu v_\nu-\gamma_\nu v_\mu)+v_\nu v_\mu\right] \tag{12}$$

to do the spin sum in the spin $\frac{3}{2}$ case, eq. (9) implies

$$1 = |\zeta(w)|^2 + (w-1)\sum_{n=1}^{n_{max}(\mu)}\left\{(w-1)|\zeta^{(n)}(w)|^2+(w+1)|\sigma^{(n)}(w)|^2\right\}+\ldots \tag{13}$$

where the ellipsis denotes the contributions of baryon resonances with quantum numbers for the light degrees of freedom other than $s_\ell^{\pi'}=0^+$ and 1^- and the contribution of the

nonresonant continuum. In eq. (13) we have cancelled a factor of $|C_{ji}(\mu)|^2$ from both sides of eq. (9) so the form factors $\zeta(w), \zeta^{(n)}(w)$ and $\sigma^{(n)}(w)$ are evaluated at μ.

We can dramatically reduce the number of states contributing to (13) by expanding in a power series about $w = 1$. This gives $\zeta(1) = 1$ and Bjorken's sum rule for the "charge radius" ρ^2 of $\zeta(w)$. Defining

$$\zeta(w) = 1 - \rho^2(w-1) + \ldots \qquad , \tag{14}$$

eq. (13) yields

$$\rho^2 = \sum_{n=1}^{n_{max}(\mu)} |\sigma^{(n)}(1)|^2 + \ldots \qquad . \tag{15}$$

Unlike the meson case, there is no $\frac{1}{4}$ term in this sum rule for the slope of ζ, and therefore no lower bound on ρ^2. Note that the μ dependence of ρ^2 (which can be inferred from eqs. (1)-(5)) is now contained in the cutoff in excitation energy μ. In eq. (15) the ellipsis denotes possible nonresonant contributions: there are no other resonant contributions. If a resonance is to contribute it must be produced in an S-wave or a P-wave: otherwise its contribution would be of higher order than $(w-1)$. If $f_S(w)$ is any S-wave form factor of an excited state then it must vanish at least as fast as $(w-1)$ as $w \to 1$ since all excited states of the light degrees of freedom are orthogonal to the ground state at $w = 1$. Thus excited states contributing to the right hand side of eq. (15) must have P-wave matrix elements, *i.e.*, ones proportional to $\vec{v}'$ in the frame where $\vec{v} = 0$. To proceed further it is convenient to consider the heavy quarks to be spinless, as we may because of the spin symmetry. We then want to know the conditions on $s_\ell^{\pi'}$ for $\langle s_\ell^{\pi'}|j(0)|0^+\rangle$ to be proportional to $\vec{v}'$. There are two possibilities depending on whether $j(0)$ is proportional to $\vec{v}'Q_j^\dagger Q_i$ or simply $Q_j^\dagger Q_i$. In the former case the matrix element of $Q_j^\dagger Q_i$ must not vanish as $w \to 1$, which means that the light degrees of freedom have remained in their ground state: this case corresponds to a P-wave "elastic" form factor proportional to $\zeta(w)$. In the latter case $s_\ell^{\pi'}(\vec{v}')$ must be a P-wave state with $J^P = 0^+$, *i.e.* $s_\ell^{\pi'} = 1^-$. Thus all resonant contributions to Bjorken's sum rule (15) are of the type considered in this paper.

The results derived here apply as $m_{Q_j} \to \infty$, but in the case of Λ_b decays they should be used with care. The predictions in eq. (7) receive corrections of order $\alpha_s(m_Q)/\pi$ and Λ_{QCD}/m_Q. Most of the perturbative corrections can be included by using the results of refs. [9] and [17]; they are not large. However, the excitation energies of the charmed baryons considered here are not small compared to the mass of the charm quark and we must therefore *a priori* expect large Λ_{QCD}/m_Q corrections [18,19].

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