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Schematic model of nuclear spin excitations

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A simple model to estimate the strength of spin and nonspin collective states is presented. The model was inspired by early schematic models based on energy-weighted sum rules and is a useful tool for interpreting experimental data without the complexities of realistic microscopic calculations. The strength of collective states is calculated by assuming that a single collective state completely exhausts the energy-weighted sum rule.

The study of the nuclear spin response has recently attracted a great deal of experimental attention¹⁻⁷. In particular, the results of recent proton scattering experiments have shown that spin-flip states in ^{40}Ca are strongly excited at energies above 10 MeV and that excitations at higher energy, near 35 MeV, are strongly enhanced in comparison with single particle model predictions. The trend is systematic⁵, with a suppression of the spin strength at low excitation and an enhancement at high energy also being observed in proton scattering experiments on ^{12}C , ^{44}Ca and ^{90}Zr . Experiments on ^{40}Ca using different proton beam energies have also established that this is truly a nuclear structure effect and not a consequence of the reaction dynamics at a particular beam energy^{5,11}. To assist in the theoretical interpretation of these results, a simple schematic model, valid in the low momentum transfer region, was proposed⁸. This model allowed an estimate of the strength of both spin-flip and non-flip states by assuming that a single collective state completely exhausted the energy-weighted sum rule (EWSR). While the importance of more realistic shell model⁹ and microscopic random phase approximation (RPA)^{10,11} calculations should not be understated, the schematic model has proven to be a useful tool^{4,5} in the analysis of subsequent experimental results. Since the initial introduction of the model focussed mainly on the relationship with the existing data and omitted the general structure of the formalism, it has not been readily apparent how it should be extended to accommodate the new experimental data as they have become available. The purpose of this Brief Report is, then, to provide a complete exposition of the schematic model framework in a manner sufficiently general to allow the calculation of any quantities that are (or are likely to become) relevant.

In both the schematic model and RPA calculations, the cross section has been calculated with the plane wave Born approximation. Within this approximation, the general form of the cross section depends on the spin-independent strengths,

$$\mathcal{M}_0^\lambda(\mathbf{q}) \equiv \sum_\nu \left| \langle 0 | \sum_k c^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda; \nu \rangle \right|^2, \quad (1)$$

and on the spin-dependent strengths,

$$\mathcal{M}_1^{\lambda J \hat{\xi}}(\mathbf{q}) \equiv \sum_\nu \sum_M \left| \langle 0 | \sum_k (\boldsymbol{\sigma}^k \cdot \hat{\xi}) e^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda, JM; \nu \rangle \right|^2. \quad (2)$$

In these expressions, the states are labelled by λ , the multipolarity of the collective excitation, and, in the case of the spin-dependent excitations, by J and M , the total and magnetic angular

momentum quantum numbers. All other quantum numbers necessary to specify the states uniquely are denoted by ν . The vector $\hat{\xi}$ is one of three basis vectors: one longitudinal vector in the direction of the momentum transfer (\hat{z}) and two transverse vectors (\hat{x} and \hat{y}), one of which lies in the scattering plane and one which is orthogonal to it. Although the recent proton scattering experiments³⁻⁷ have been concerned with polarizations orthogonal to the scattering plane, there can be spin-flip probabilities for any direction. The conventional notation is that $\sigma S_{\zeta\zeta}$ is the spin-flip cross section for the direction $\hat{\zeta}$. The contributions to this cross section then arise from the \mathcal{M}_1 terms associated with the two basis vectors orthogonal to $\hat{\zeta}$; conversely, the non-flip cross section arises from the \mathcal{M}_1 terms associated with the basis vector $\hat{\xi} = \hat{\zeta}$ and the nonspin cross section from the \mathcal{M}_0 terms. This is obvious from the fact that the operator $\sigma \cdot \hat{\zeta}$ induces a rotation in spin space through an angle π about the $\hat{\zeta}$ axis. In general, all of these strengths have both isovector and isoscalar contributions and can depend on matrix elements with an isospin operator, r_i^k ; if included, the strength corresponds to that of a resonance with an isospin change ($\Delta T = 1$) while, as written, Eqs. (1) and (2) would suggest that there is no isospin change ($\Delta T = 0$). The isospin operator has been suppressed because it has no influence on the schematic model calculation of the spin strength. The full calculation of the cross sections requires some reaction scheme, such as the Love-Franey t-matrix approach,^{12,13} to provide momentum-dependent coefficients for each of the terms. It is in these coefficients that the explicit isospin dependence of the cross section is to be found.

Beginning with the spin-independent terms, \mathcal{M}_0 , the classical EWSR takes the form¹⁴

$$S_1(r^\lambda Y_{\lambda 0}) \equiv \sum_{\nu} (\omega_{\nu} - \omega_0) \left| \langle \lambda; \nu | \sum_k r_{\lambda}^k Y_{\lambda 0}(\hat{\mathbf{r}}_k) | 0 \rangle \right|^2 = \frac{\hbar^2}{2m} \frac{A(2\lambda + 1)\lambda}{4\pi} \langle r^{2\lambda-2} \rangle, \quad (3)$$

and the energy-weighted density sum rule¹⁵⁻¹⁷ is given by

$$\begin{aligned} S_{\rho}(r^\lambda Y_{\lambda 0}) &\equiv \sum_{\nu} (\omega_{\nu} - \omega_0) \langle 0 | \hat{\rho}(\mathbf{r}) | \lambda; \nu \rangle \langle \lambda; \nu | \sum_k r_{\lambda}^k Y_{\lambda 0}(\hat{\mathbf{r}}_k) | 0 \rangle = -\frac{\hbar^2}{2m} \nabla \cdot \left\{ \rho^{(0)}(\mathbf{r}) \nabla [r^\lambda Y_{\lambda 0}(\hat{\mathbf{r}})] \right\} \\ &= -\frac{\hbar^2}{2m} \lambda r^{\lambda-1} \frac{d\rho^{(0)}}{dr} Y_{\lambda 0}(\hat{\mathbf{r}}). \end{aligned} \quad (4)$$

The ground state density, $\rho^{(0)}(\mathbf{r})$ appears in this expression and is needed to calculate the radial expectation values appearing in Eq. (3). A simple Woods-Saxon shape¹⁸ shape has been found useful but there is nothing to prohibit a more realistic choice. The fundamental assumption of

the schematic model, that a *single collective state with energy ω_λ completely exhausts the EWSR*, drastically facilitates the calculation. The transition matrix element can be found immediately from Eq. (3),

$$\langle 0 | \sum_k r_k^\lambda Y_{\lambda 0}(\hat{\mathbf{r}}_k) | \lambda; \nu \rangle = \sqrt{\frac{S_1(r^\lambda Y_{\lambda 0})}{\omega_\lambda}}, \quad (5)$$

and used to calculate the density matrix element from Eq. (4),

$$\langle 0 | \hat{\rho}(\mathbf{r}) | \lambda; \nu \rangle = -\frac{\hbar^2}{2m} \lambda r^{\lambda-1} \frac{d\rho^{(0)}}{dr} \frac{Y_{\lambda 0}(\hat{\mathbf{r}})}{\sqrt{\omega_\lambda S_1(r^\lambda Y_{\lambda 0})}}. \quad (6)$$

Since the density operator is merely a sum of δ -functions, $\hat{\rho}(\mathbf{r}) = \sum_k \delta(\mathbf{r} - \mathbf{r}_k)$, the plane wave expansion in spherical harmonics,

$$e^{i\mathbf{q}\cdot\mathbf{r}} = 4\pi \sum_{lm} i^l j_l(qr) Y_{lm}(\hat{\mathbf{q}}) Y_{lm}^*(\hat{\mathbf{r}}), \quad (7)$$

can be readily used to determine the relevant matrix element,

$$\langle 0 | \sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda; \nu \rangle = \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \langle 0 | \hat{\rho}(\mathbf{r}) | \lambda; \nu \rangle = \frac{\hbar^2}{2m} \frac{4\pi\lambda}{\sqrt{\omega_\lambda S_1(r^\lambda Y_{\lambda 0})}} Y_{\lambda 0}^*(\hat{\mathbf{q}}) \int dr r^{\lambda+1} j_\lambda(qr) \frac{d\rho^{(0)}}{dr}, \quad (8)$$

with j_λ a spherical Bessel function. After substituting the expression for the EWSR value, Eq. (3), and simplifying, the total spin-independent strength is found to be

$$\mathcal{M}_0^{\lambda(\neq 0)}(q) = \left(\sum_\nu \right) \left| \langle 0 | \sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda; \nu \rangle \right|^2 = \frac{\hbar^2}{2m} \frac{(4\pi)^2 \lambda}{\omega_\lambda A(r^{2\lambda-2})} \left[\int dr r^{\lambda+1} j_\lambda(qr) \frac{d\rho^{(0)}}{dr} \right]^2. \quad (9)$$

Eq. (9) has been obtained after dividing by a factor of 4π that arises from an average over momentum orientations, $\hat{\mathbf{q}}$. The expression has been written with parentheses around the summation to serve as a reminder that in the schematic model, the sum is over only the single degenerate state. The result for the monopole state must be determined separately because the collective excitation operator is r^2 and is not of the form assumed in Eqs. (3) and (4). These two equations are modified so that

$$S_1(r^2) = \sum_\nu (\omega_\nu - \omega_0) \left| \langle 0^+; \nu | \sum_k r_k^2 | 0 \rangle \right|^2 = \frac{2\hbar^2}{m} A(r^2) \quad (10)$$

and

$$S_\rho(r^2) = \sum_\nu (\omega_\nu - \omega_0) \langle 0 | \hat{\rho}(\mathbf{r}) | 0^+; \nu \rangle \langle 0^+; \nu | \sum_k r_k^2 | 0 \rangle = -\frac{\hbar^2}{m} \left[3\rho^{(0)}(r) + r \frac{d\rho^{(0)}}{dr} \right]. \quad (11)$$

With exactly the same manipulations, assuming that a single collective state exhausts the total sum rule value and averaging over momentum transfer orientations, the monopole strength is found to be

$$\mathcal{M}_0^0(q) = \left(\sum_{\nu} \right) \left| \langle 0 | \sum_k e^{i\mathbf{q} \cdot \mathbf{r}_k} | 0^+; \nu \rangle \right|^2 = \frac{\hbar^2}{2m} \frac{(4\pi)^2}{\omega_0 + A(r^2)} \left[\int dr r^3 j_2(qr) \frac{d\rho^{(0)}}{dr} \right]^2. \quad (12)$$

The calculation of the spin-dependent strengths proceeds in substantially the same way, except that the density operator need be replaced by the spin density operator, $\hat{\rho}_{\mu}(\mathbf{r}) = \sum_k \sigma_{\mu}^{k\dagger} \delta(\mathbf{r} - \mathbf{r}_k)$ and the sum rules evaluated with operators for collective *spin states*. The relevant sum rules then take the following explicit forms: the classical EWSR value of Eq. (3) has precisely the same value,

$$S_1(r^{\lambda} [Y_{\lambda} \otimes \sigma]_{JM}) = \sum_{\nu} (\omega_{\nu} - \omega_0) \left| \langle \nu | \sum_k r_k^{\lambda} [Y_{\lambda}(\hat{\mathbf{r}}_k) \otimes \sigma]_{JM} | 0 \rangle \right|^2 = \frac{\hbar^2}{2m} \frac{A\lambda(2\lambda+1)}{4\pi} \langle r^{2\lambda-2} \rangle, \quad (13)$$

subject to the assumption that the nucleus is filled to an LS closed shell so that the conmutation $\langle 0 | [\sigma_{\mu}, \sigma_{\mu'}] | 0 \rangle = 0$ can be used. The energy-weighted density sum rule of Eq. (4) must be modified by including a sum over Clebsch-Gordon coefficients to handle correctly the contributions from different magnetic substates,

$$\begin{aligned} S_{\rho}(r^{\lambda} [Y_{\lambda} \otimes \sigma]_{JM}) &= \sum_{\nu} (\omega_{\nu} - \omega_0) \langle 0 | \hat{\rho}_{\mu}(\mathbf{r}) | \lambda, JM; \nu \rangle \langle \lambda, JM; \nu | \sum_k r_k^{\lambda} [Y_{\lambda}(\hat{\mathbf{r}}_k) \otimes \sigma]_{JM} | 0 \rangle \\ &= -\frac{\hbar^2}{2m} \sum_{\mu'} (\lambda \mu' 1 \mu | JM) \nabla \cdot \left\{ \rho^{(0)}(\mathbf{r}) \nabla [r^{\lambda} Y_{\lambda\mu'}(\hat{\mathbf{r}})] \right\} \\ &= -\frac{\hbar^2}{2m} \sum_{\mu'} (\lambda \mu' 1 \mu | JM) \lambda r^{\lambda-1} \frac{d\rho^{(0)}}{dr} Y_{\lambda\mu'}(\hat{\mathbf{r}}). \end{aligned} \quad (14)$$

In the same way as for the spin-independent matrix elements, the assumption that only a single collective state completely exhausts the EWSR allows an immediate evaluation of the transition matrix element from Eq. (13),

$$\langle \lambda, JM; \nu | \sum_k r_k^{\lambda} [Y_{\lambda}(\hat{\mathbf{r}}_k) \otimes \sigma]_{JM} | 0 \rangle = \sqrt{\frac{S_1(r^{\lambda} [Y_{\lambda} \otimes \sigma]_{JM})}{\omega_{\lambda}}}. \quad (15)$$

This result, in combination with the energy-weighted density sum rule of Eq. (14), permits an evaluation of the density matrix element,

$$\begin{aligned} \langle 0 | \hat{\rho}_{\mu} | \lambda, JM; \nu \rangle &= \langle 0 | \sum_k \sigma_{\mu}^{k\dagger} \delta(\mathbf{r} - \mathbf{r}_k) | \lambda, JM; \nu \rangle \\ &= -\frac{\hbar^2}{2m} \sum_{\mu'} (\lambda \mu' 1 \mu | JM) \frac{\lambda r^{\lambda-1}}{\sqrt{\omega_{\lambda} S_1(r^{\lambda} [Y_{\lambda} \otimes \sigma]_{JM})}} \frac{d\rho^{(0)}}{dr} Y_{\lambda\mu'}(\hat{\mathbf{r}}). \end{aligned} \quad (16)$$

Because the structure of the spin density operator is also simple, the plane wave expansion, Eq. (7), can again be used to derive the relevant matrix element,

$$\langle 0 | \sum_k \sigma_\mu^{k\dagger} e^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda, JM; \nu \rangle = -\frac{\hbar^2}{2m} \sum_{\mu'} (\lambda \mu' 1 \mu | J M) \frac{4\pi\lambda i^\lambda}{\sqrt{\omega_\lambda S_1 (r^\lambda [Y_\lambda \otimes \sigma]_{JM})}} Y_{\lambda\mu'}(\hat{\mathbf{q}}) \int dr r^{\lambda+1} j_\lambda(qr) \frac{d\rho^{(0)}}{dr}. \quad (17)$$

Until now, the spherical tensor properties of the spin operator, σ_μ , have been exploited; it is more useful, however, for the matrix elements to be expressed in the form suggested by Eq. (2), using the Cartesian components of the vector spin operator. These components are

$$\sigma_x = -\frac{1}{\sqrt{2}}(\sigma_{+1} - \sigma_{-1}), \quad \sigma_y = \frac{i}{\sqrt{2}}(\sigma_{+1} + \sigma_{-1}) \quad \text{and} \quad \sigma_z = \sigma_0. \quad (18)$$

Taking the appropriate combinations of Eq. (17), and substituting the expression for S_1 from Eq. (13), the general result for the strength of the collective spin excitations is

$$\begin{aligned} \mathcal{M}_1^{\lambda(\neq 0)J\hat{\xi}}(q) &= \left(\sum_\nu \right) \sum_M \left| \langle 0 | \sum_k (\boldsymbol{\sigma}^k \cdot \hat{\boldsymbol{\xi}}) e^{i\mathbf{q}\cdot\mathbf{r}_k} | \lambda, JM; \nu \rangle \right|^2 \\ &= \frac{\hbar^2}{2m} \frac{(4\pi)^2 \lambda}{\omega_\lambda A \langle r^{2\lambda-2} \rangle} \left[\int dr r^{\lambda+1} j_\lambda(qr) \frac{d\rho^{(0)}}{dr} \right]^2 \mathcal{F}_\xi^{\lambda J}. \end{aligned} \quad (19)$$

The factor $\mathcal{F}_\xi^{\lambda J}$ has been introduced as the *fraction of the total strength* for a state with multipolarity λ and total angular momentum J . Although the total strength for a given multipolarity is independent of the direction $\hat{\boldsymbol{\xi}}$, the fractional contribution depends on whether the transverse ($\hat{\boldsymbol{\xi}} = \hat{\mathbf{x}}$ or $\hat{\boldsymbol{\xi}} = \hat{\mathbf{y}}$) or longitudinal ($\hat{\boldsymbol{\xi}} = \hat{\mathbf{z}}$) matrix elements are of concern. The sum over magnetic quantum numbers, μ and M , is contained totally in the definition of the fractional contributions:

$$\mathcal{F}_z^{\lambda J} \equiv \frac{4\pi}{2(2\lambda+1)} \sum_M \left| \sum_\mu [(\lambda \mu 1 1 | J M) - (\lambda \mu 1 -1 | J M)] Y_{\lambda\mu}^*(\hat{\mathbf{q}}) \right|^2, \quad (20)$$

$$\mathcal{F}_y^{\lambda J} \equiv \frac{4\pi}{2(2\lambda+1)} \sum_M \left| \sum_\mu [(\lambda \mu 1 1 | J M) + (\lambda \mu 1 -1 | J M)] Y_{\lambda\mu}^*(\hat{\mathbf{q}}) \right|^2 \quad (21)$$

and

$$\mathcal{F}_x^{\lambda J} \equiv \frac{4\pi}{2\lambda+1} \sum_M \left| \sum_\mu (\lambda \mu 1 0 | J M) Y_{\lambda\mu}^*(\hat{\mathbf{q}}) \right|^2, \quad (22)$$

It is easy to read off the total strength for multipolarity λ from Eq. (19) since the fractional coefficients are conveniently normalized: $\sum_J \mathcal{F}_\xi^{\lambda J} \equiv 1$. The expressions in Eqs. (20)-(22) can be

simplified and put into closed form by using the properties of the Clebsch-Gordon coefficients and the spherical harmonics:

$$\begin{aligned}\mathcal{F}_{\hat{x}}^{\lambda J} = \mathcal{F}_{\hat{y}}^{\lambda J} &= \frac{1}{2} [(\lambda 0 1 1 | J 1)^2 + (\lambda 0 1 -1 | J -1)^2] \\ &= \frac{1}{2(2\lambda + 1)} \times \begin{cases} \lambda - 1 & J = \lambda - 1 \\ 2\lambda + 1 & \lambda \\ \lambda + 2 & \lambda + 1 \end{cases}\end{aligned}\quad (23)$$

and

$$\mathcal{F}_{\hat{z}}^{\lambda J} = (\lambda 0 1 0 | J 0)^2 = \frac{1}{2\lambda + 1} \times \begin{cases} \lambda & J = \lambda - 1 \\ 0 & \lambda \\ \lambda + 1 & \lambda + 1 \end{cases}.\quad (24)$$

There are evidently some interesting features: first, natural parity transitions are always purely transverse; only the 0^- state is purely longitudinal while all other unnatural parity states are mixed longitudinal/transverse transitions. Second, the strength of transverse transitions is always equally divided between natural parity and unnatural parity states.

Again, because the spin monopole state is described with a different collective excitation operator than the form assumed in Eqs. (13) and (14), it must be calculated independently. The introduction of the spin operator into Eqs. (10) and (11) does not substantially alter their form, providing that the nucleus satisfies LS closure properties. The right-hand-sides are both multiplied by $\delta_{\mu\mu'}$ where μ and μ' are tensor indices; in Eq. (10), the indices arise from each of the two spin monopole excitation operators, $r^2\sigma_\mu$, that appears, and in Eq. (11) one index arises from the density operator, $\hat{\rho}_\mu$, and the other from the collective excitation operator. Consequently, the expression for the strength is the same, in form, as for the nonspin monopole state, Eq. (12),

$$\mathcal{M}_1^{01\xi}(q) = \left(\sum_\nu \right) \left| \langle 0 | \sum_k (\sigma^k \cdot \hat{\xi}) e^{iq \cdot r_k} | 1^+; \nu \rangle \right|^2 = \frac{\hbar^2}{2m} \frac{(4\pi)^2}{\omega_1 + A\langle r^2 \rangle} \left[\int dr r^3 j_2(qr) \frac{d\rho^{(0)}}{dr} \right]^2. \quad (25)$$

Thus, it is evident that the fundamental assumption of this schematic model, that the total energy-weighted sum rule strength is exhausted by a resonance at a single energy, leads to analytic expressions for the strength of *all* spin-independent and spin-dependent collective states. In practice, this assumption has proven to be reasonable when compared with the experimental data^{3,5,8} where distinct peaks are observed in the regions where giant resonances are expected; a general tendency towards such features is also borne out by fully microscopic RPA calculations¹⁰⁻¹¹. Indeed, it is well known¹⁸ that systematic trends, as a function of the nuclear mass, exist and describe the location of giant resonances that exhaust substantially large fractions of the EWSR. The model is

not well suited to a description of low-lying states and has only found use above about 10 MeV. The reason for this is clear: low-lying discrete states are highly fragmented and exhaust only a small fraction of the EWSR.

In any calculation of the cross section, the calculated strength must be combined with a method of determining the energy distribution of the full state. Systematic trends have also been deduced from experiment to describe the width of collective resonances but assume the distribution to be symmetric about the peak energy¹⁸. The schematic model presented here can use such trends but more realistic calculations of an energy-dependent width¹⁹ that creates an asymmetric distribution have been preferred. The results of calculations of this energy-dependent width, determined explicitly from the coupling of two-particle-two-hole states to one-particle-one-hole states, have been used in both schematic model and RPA calculations^{5,8-11}. The asymmetry has the effect of pushing strength from lower to higher excitation energies when compared with symmetric distributions and has been critical in explaining the observed enhancement of spin strength at large excitation. By including a realistic description of the width, the model is quite successful at describing well-localized strong collective states.

In summary, the schematic model has proven, since its introduction, to be a useful tool in understanding the distribution of spin strength observed in recent proton scattering experiments. While it cannot replace more realistic shell model and RPA calculations, the analytic forms that result make it very attractive. The principal results of this report are contained in Eqs. (9), (12), (19) and (23)–(25), which summarize the analytic expressions needed to calculate the schematic model strength for *any* collective state. This paper has provided the derivation of these expressions in a form sufficiently general that it is possible to extract from the model any quantities that have already found some use or are likely, in the near future, to become relevant.

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