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Meson Exchange and the Relativistic Multiple Scattering Formalism

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Abstract

A relativistic multiple scattering theory is formulated in the context of meson exchange. The elastic scattering amplitude for a fermion projectile satisfies a Dirac equation with an optical potential derived from a relativistic multiple scattering series. It is shown that the two-body t-matrix associated with the optical potential is the one with the projectile on its mass-shell in all intermediate states.

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In almost all the relativistic (Dirac) projectile-nucleus scattering calculations done in the past few years, the optical potential used has been the relativistic impulse approximation (RIA),[1] which is the relativistic analogue of the non-relativistic first-order impulse approximation optical potential. The tacit assumption behind the use of RIA, together with the Dirac equation, in which the heavy target is taken to be on its mass-shell, is that there exists a multiple scattering series for the optical potential (in which the RIA is an approximation to the first term). However, neither the existence of the relativistic multiple scattering theory (RMST) nor the relation of it to the RIA has been consistently derived. The lack of a RMST not only prevents us from performing a systematic study of the higher order multiple scattering terms, but also from making other corrections, such as off-shell effects and Pauli blocking, in a consistent manner. Therefore it is highly desirable to have a RMST.

In this work we show that a RMST can be formulated in the context of a relativistic meson exchange model. In the following we will consider a scalar "nucleon "interacting with a spin, iso-spin zero A-body target through meson exchange. A minimal set of meson exchange diagrams required for any such theory is the set of ladder and crossed ladder diagrams. In the limit when the heavy target becomes infinitely massive, this set reduces to a one body equation for the lighter particle moving in an instantaneous potential produced by the heavier particle (the one body limit [2]), and at high energies gives the eikonal approximation to scattering.[3] We will assume that the theory we seek is one in which these relativistic ladder and crossed ladders are summed efficiently.

The contributions to second order in the projectile-meson interaction are box and crossed box diagrams, shown in Fig 1. Since the target is a complex system with (in general) many closely spaced energy levels and continuum states with different combinations of clusters, we assume that all of these states can contribute, at least in principle, to the intermediate states. These states will be labled by the index n, with $n = 0$ referring to the ground state. Included in this sum are states where one, two, and possibly many nucleons are knocked out of the target. The elastic scattering matrix for the two diagrams in Fig 1 is, for spin zero particles,

$$M_{box} = \sum_{n=0}^{\infty} ig^2 \int \frac{d^4k \; g_{0n}(P-p', P-k) g_{n0}(P-k, P-p)\Delta(k-p')\Delta(p-k)}{(2\pi)^4[e_k^2 - k_0^2 - i\epsilon][E_n^2(k) - (W-k_0)^2 - i\epsilon]} \quad (1)$$

$$M_{cross} = \sum_{n=0}^{\infty} ig^2 \int \frac{d^4k \; g_{0n}(P-p', P-q) g_{n0}(P-q, P-p)\Delta(k-p')\Delta(p-k)}{(2\pi)^4[e_k^2 - k_0^2 - i\epsilon][E_n^2(q) - (E_0(p)+E_0(p') - W - k_0)^2 - i\epsilon]} \quad (2)$$

where the total four momentum in the center of mass is $P = (W, 0)$, $q = p' + p - k$ and the meson propagators are

$$\Delta(k-p) = \frac{1}{\mu^2 - (k-p)^2 - i\epsilon} = \frac{1}{\omega_{p-k}^2 - (k_0 - p_0)^2 - i\epsilon}. \quad (3)$$

The meson-projectile vertices which couple the ground state to the n^{th} excited state are g_{0n} and g_{n0}, and

$$e_k = (m^2 + \mathbf{k}^2)^{1/2} \; ; \; E_n(k) = (M_n^2 + \mathbf{k}^2)^{1/2} \quad (4)$$

where m is the projectile mass and $M_n = M_0 + \Delta_n$ is the mass of the n^{th} excited state of the A nucleon system with excitation energy Δ_n, with $\Delta_0 = 0$.

The key to our derivation is the analysis of the singularities of the box and the crossed box diagrams in the complex k_0 plane. For this purpose we work at threshold where $\mathbf{p} = \mathbf{p}' = 0$ and $W = M_0 + m$. The singularities are shown in Fig 2.

(i) If the k_0 contour for the *box* diagram is closed in the upper half plane, the pole at $M_0 - E_n(k) + m$ dominates. This pole corresponds to restricting the target ground state to its positive energy mass-shell.

(ii) If the k_0 contour for the box diagram and the crossed box diagrams are closed in the upper half plane, the the double poles at $k_0 = m - \omega$ nearly cancel. The sum of these two terms is propotional to

$$\frac{1}{E_n - M_0 + \omega} - \frac{1}{M_0 - E_n + \omega} \approx \frac{2(M_0 - E_n)}{\omega^2 - (M_0 - E_n)^2}, \quad (5)$$

so that when $n = 0$ and $M_0 \to \infty$, the pole calculated in (i) above gives the *exact* answer (the negative energy poles are also negligible in this case).

(iii) For *excited* states of the target nucleus, the pole in the box diagram at $M_0 - E_n(k) + m$ may overlap the double meson pole or even the negative energy nucleon pole at $-e_k$. (This happens when $\Delta_n \approx \omega$ or $2m$.) Such overlaps, which seem to be a manifestation of dissolution singularities,[4] introduce spurious singularities into the equation. However, all of these spurious singularities are eliminated if the contour is closed in the *lower* half plane. Keeping the pole at $k_0 = e_k$ still permits us to separate out the leading term from the box, but in this case the cancellations in (ii) will go like $(m - e_k)/\omega^2$, which does not approach zero as $A \to \infty$. However, this loss of convergence can be accepted since the contributions from excited states are generally smaller anyway.

These observations lead us to write the equation for the projectile target t-matrix in the following operator form

$$T = V + VG_0^T T + VG_{n\neq 0}^p T \tag{6}$$

where G_0^T is the propagator for the projectile and the target in its ground state, with the *target* on its mass-shell, and $G_{n\neq 0}^p$ is the propagator for projectile and excited states of the target, with the *projectile* on the mass-shell. In this equation

$$V = \sum_i v^i + V' \tag{7}$$

where v^i are the OBE diagrams describing the interaction of the projectile with the i^{th} nucleon in the target and V' is the sum of all irreducible terms remaining from the full ladder and crossed ladder sum. Points (i)-(iii) imply that V' is very small, and if A and m both approach infinity the leading OBE terms are exact.[2] In general Eq(6) sums ladders and crossed ladders exactly if V' is included.

While Eq(6) is an exact formulation of the problem, it is too complicated to be useful. We need a philosophy for identifying leading contributions which will be summed exactly and others which will be treated perturbatively. The philosophy we use is familiar from non-relativistic multiple scattering theories.[5] The leading effects are assumed to arise from multiple scattering through the intermediate states in which the projectile interacts repeatedly with the same target nucleon. This is important compared to the terms where the projectile is interacting with two or more different target particles since the matrix elements of the latter are propotional to (small) correlation functions involving two or more particles. It is not our intention to improve on these basic assumptions, but rather to describe how they can be implemented in a relativistically covariant manner.

For elastic scattering, a convenient first step is to introduce an effective potential U (the optical potential). In operator form, the t-matrix in terms of the potential U is

$$T = U + UG_0^T T \tag{8}$$

where the equation for the optical potential follows from Eq(6)

$$U = V + VG^p_{n\neq 0}U = V + UG^p_{n\neq 0}V \tag{9a}$$

$$U = V + VG^p_{n\neq 0}V + VG^p_{n\neq 0}UG_{n\neq 0}V \tag{9b}$$

Eq(9a) sums all inelastic contributions; (9b) is convenient for projecting the result onto the elastic subspace needed in Eq (8). For large A, Eq(8) is an effective one particle equation for the projectile moving in a fixed, instantaneous field generated by the target. If the projectile has spin 1/2, Eq(8) is a Dirac equation.

To take into account all the leading effects from rescattering from the same nucleon, which controls the strong short range NN interaction, we introduce the multiple scattering series as discussed above. To this end, separate out V' from V, introduce a new propagator g_i such that

$$t^i = v^i + v^i g_i t^i \tag{10}$$

$$\bar{v}^i = V'^i + v^i g_i \bar{v}^i \tag{11}$$

where $V'^i \equiv V'/A$. Note that g_i describes the propagation of the i^{th} nucleon in the nucleus, but is otherwise unspecified. With these definitions, Eq(9a) becomes

$$U^i = t^i + \sum_j t^i(G^p_{n\neq 0} - g_i\delta_{ij})U^j + \bar{v}^i(1 + G^p_{n\neq 0}\sum_j U^j) \tag{12a}$$

where

$$U = \sum_i U^i \tag{12b}$$

Eq(12) is our final result for the optical potential. It gives the exact result for the sum of all ladders and crossed ladder diagrams, and in the nonrelativistic limit ($m \to 0$), $\bar{v}^i \to 0$.

The propagator g_i should be chosen to minimize the contribution from inelastic channels, so that the second term on the right hand side of Eq(12a) can be treated perturbatively. If these contributions are dominated by one nucleon knockout processes as discussed above and illustrated in Fig 3, choosing g_i to be the propagator with *both* the heavy $A-1$ cluster *and* the projectile on mass-shell will *exactly cancel* the dominant inelastic contributions from the second term in Eq(12a) and ensure that they are exactly accounted for in the summation of Eq(10) which produces t^i. Restricting the $A-1$ cluster to its mass-shell[6] ensures cluster separability of the remaining two nucleon system.[7] Eq(10) for t^i then reduces, in the NN subspace, to the one particle on shell (spectator) equation previously introduced by one of us,[8] the only change being the shift in the total energy of the two-body subspace due to the motion of the $A-1$ cluster.

Finally U can be projected onto the elastic subspace using Eq(9b). This leads to an effective two-body t-matrix for the first term on the right hand side of Eq(12a) (denoted by $\hat{t}$) which has both nucleons in the initial *and* final states off shell and is obtained by quadrature from the spectator amplitude t

$$\hat{t}^i = \hat{v}^i + \hat{v}^i g_i \hat{v}^i + \hat{v}^i g_i t^i g_i \hat{v}^i. \tag{13}$$

Here $\hat{v}$ is the OBE potential with all four legs off shell (unless one of the legs is projected on-shell by g_i). In applications the first term in Eq(12a) is usually simplified by using the $t\rho$ approximation and is referred to as the RIA. Our derivation suggests that the full first term of Eq(12a) with g_i defined as in Fig 3 is a more precise definition.

In conclusion, we would like to emphasize that, in the context of the meson exchange model, the projectile-nucleus t-matrix does not readily assume the form convienient for multiple scattering analysis. In order to obtain a more manageable kernel and the corresponding t-matrix, we need to consider the explicit cancellations of meson poles between the box diagram and the crossed-box diagram. Once the integral equation for the t-matrix is obtained, the optical potential can be derived in a straight forward manner. The optical potential can then be expressed as a multiple scattering series, Eq(12), and in the impluse approximation the t-matrix associated with the optical potential is found to be the one with one particle on its mass-shell.

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Figure Captions

Fig.1.

Feynman diagrams for elastic scattering of a projectile (solid line) from a nucleus (double solid line) in its ground state. These diagrams are second order in the meson (dash line) projectile coupling g, but contain interactions of the A-body target nucleons to all order in g. The index n denotes an arbitrary excited state of the target, with $n = 0$ the ground state.

Fig.2.

Singularities of the box and crossed box diagrams in the complex k_0 plane for $W = M_0 + m$, and $\mathbf{p} = \mathbf{p}' = 0$. The circle around the meson poles are to indicate that they are double poles when $\mathbf{p} = \mathbf{p}'$. All the poles move as $|\mathbf{k}|$ varies. Cuts comming from the structure of the couplings g_{0n} and g_{n0} are not shown.

Fig.3

Diagramatic representation of second order rescattering of the projectile with the same target nucleon through intermediate states in which one nucleon is knocked out and the remaining $A-1$ system is in some excited state n_{A-1}. Choosing g_i as shown ensures that these leading terms are exactly included in the first term of Eq(12a), and that the other terms are small.

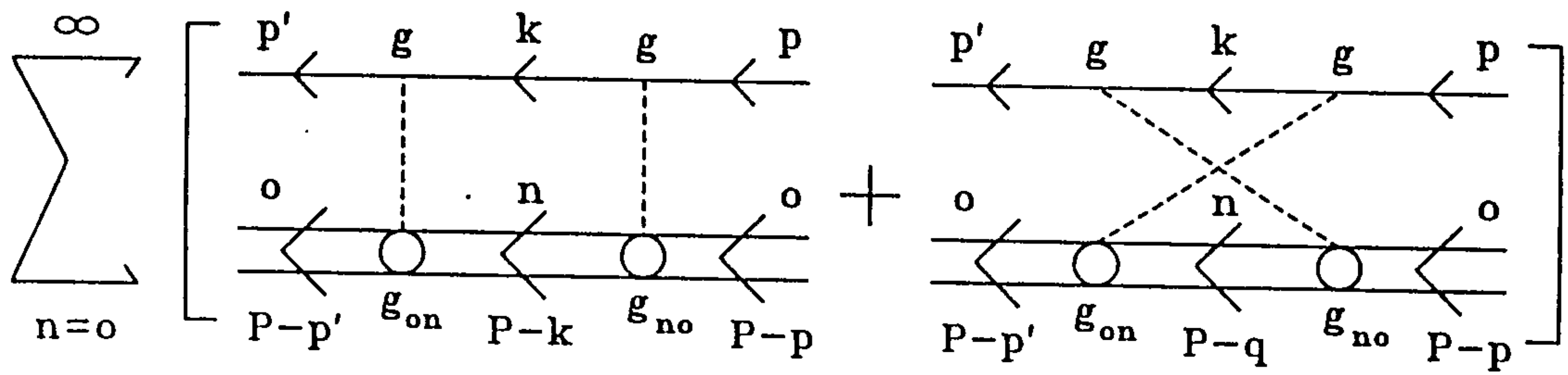
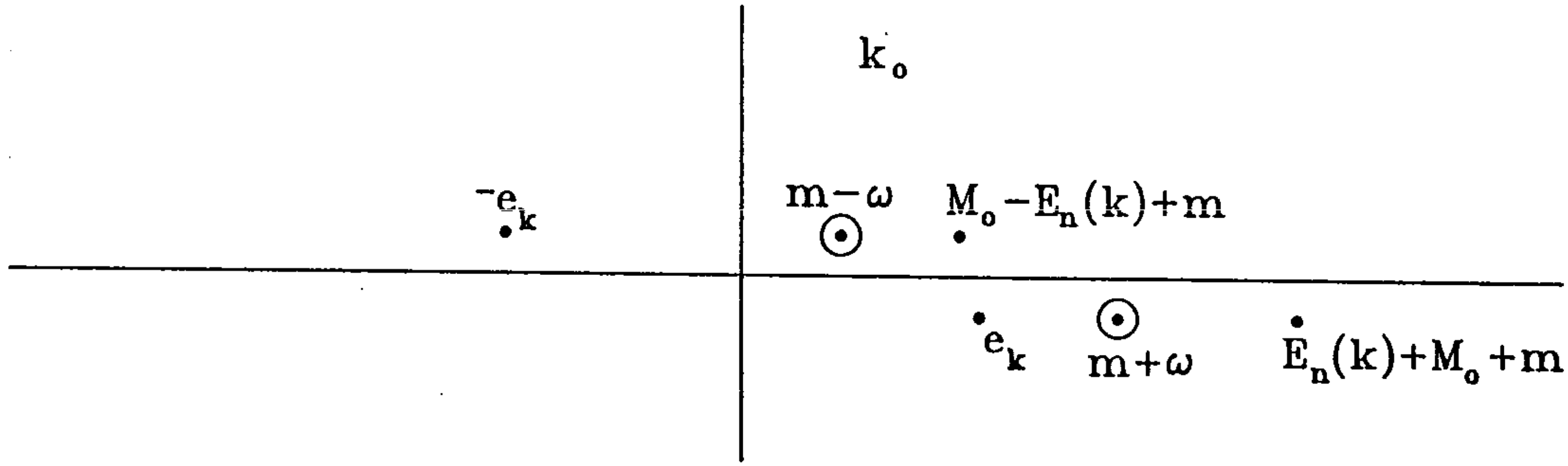
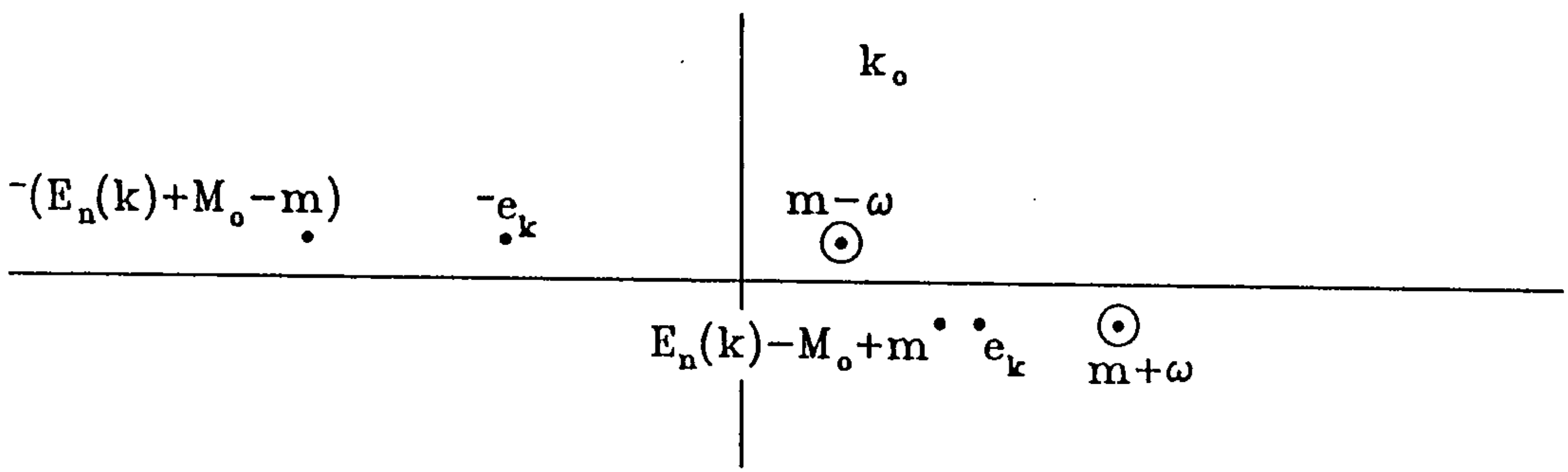


Fig. 1



Box



Crossed Box

Fig. 2

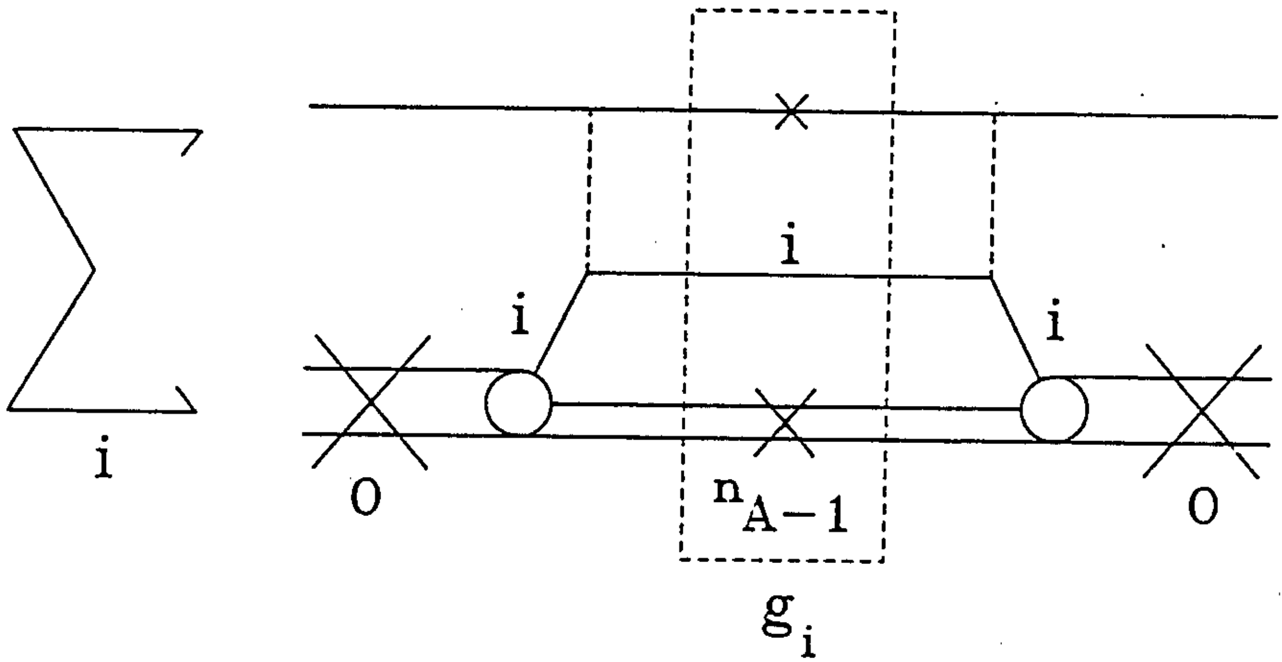


Fig. 3