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Relativistic Few Body Calculations

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RELATIVISTIC FEW BODY CALCULATIONS *

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ABSTRACT

A modern treatment of the nuclear few-body problem must take into account *both* the quark structure of baryons and mesons, which should be important at short range, *and* the relativistic exchange of mesons, which describes the long range, peripheral interactions. A way to model both of these aspects is described. The long range, peripheral interactions are calculated using the spectator model, a general approach in which the spectators to nucleon interactions are put on their mass-shell. Recent numerical results for a relativistic OBE model of the NN interaction, obtained by solving a relativistic equation with one-particle on mass-shell, will be presented and discussed. Two meson exchange models, one with only four mesons $(\pi, \sigma, \rho, \omega)$ but with a 25% admixture of γ^5 coupling for the pion, and a second with six mesons $(\pi, \sigma, \rho, \omega, \delta, \eta)$ but pure $\gamma^5\gamma^\mu$ pion coupling, are shown to give very good quantitative fits to the NN scattering phase shifts below 400 MeV, and also a good description of the $\vec{p}\ {}^{40}Ca$ elastic scattering observables. Applications of this model to electromagnetic interactions of the two body system, with emphasis on the determination of relativistic current operators consistent with the dynamics and the exact treatment of current conservation in the presence of phenomenological form factors, will be described.

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1. INTRODUCTION

The major part of this talk will report on an approach to the relativistic treatment of nuclear systems which has grown out of work using relativistic equations with one particle off-shell. The essence of this approach is that the relativistic series of Feynman diagrams describing any nuclear process can always be reorganized so that only the particles which are interacting are off-shell, and all other particles, which are spectators to the interaction, can be put on-shell. For two and three nucleon systems this can always be done so that only one particle is off-shell, and the amplitudes required are either covariant vertex functions or covariant scattering amplitudes. With modifications, it appears that this general approach can be extended to many body systems, and I now refer to it as the spectator model.

The spectator model is designed to treat nuclear forces at large distances, from one to two fermis and beyond, where it is assumed that the forces are peripheral, and might be correctly described by relativistic meson exchange mechanisms. The minimal set of meson exchange diagrams which should be treated is the sum of all ladder and crossed ladder diagrams. Below the meson production threshold, this set will be regarded as sufficient, but above the production threshold self energies must be included, but only as necessary to insure three (and perhaps four) body unitary. Values of meson and baryon parameters, particularly masses, will be taken as much as possible from known, measured results and form factors, which describe the short range structure of the hadrons, will be treated phenomenologically. The examples developed in this talk are taken from this relativistic meson exchange picture.

At shorter distances, probably inside of a few tenths of a fermi, it is assumed that the physics is best described in terms of the underlying quark and gluon degrees of freedom. The phenomenological form factors, coupling constants and hadron masses used in meson exchange models would then be eventually replaced by microscopic calculations of these qualitities. Furthermore, these calculations will probably require a full treatment of QCD, or some realistic modeling of the non-perturbative (confining) forces in QCD. For the next decade, modeling will probably be necessary. It would be very desirable to develop a model which is compatable with the relativistic meson theory. This would require that both the relativistic nature of the quarks and gluons, and the relativistic *motion* of the composite mesons and baryons be treated together. A model which which satisfies these requirements is described briefly in section 2 below.

The remaining three sections of this talk deal with the spectator model. First, the concepts used in the spectator model will be reviewed, and it will be shown how they are applied to nuclear physics problems. Next, recent unpublished numerical results for NN scattering and $\bar{p}\,^{40}Ca$ will be presented. Finally, applications to electromagnetic interactions of the deuteron will be described in some detail.

2. QUARK STRUCTURE

The composite structure of mesons and baryons can be described by their corresponding relativistic wave function. In a simple approximation, the Fock space expansions for these wave functions may be limited to the first term describing the $q\bar{q}$ valence component for mesons, and the qqq valence component for baryons. To insure that the model is relativistic, and that it is compatable with the spectator model for peripheral interactions, momentum space relativistic Bethe Salpeter wave functions, derived from the relativistic bound state vertex functions, are used. This program has just been started in collaboration with Hiroshi Ito and Warren Buck of Hampton University, and I will refer to it as the relativistic quark cluster model (RQCM).

The basic idea is to include quark structure in the manner illustrated in

Fig. 1. Fig. 1a shows the one pion exchange interaction with phenomenological form factors at the πNN vertex. In Fig. 1b, the same OPE term is shown, but now the πNN form factor is given in terms of the nucleon and pion relativistic vertex functions. If these vertex functions have been previously specified, as described below, then the form factor is determined. Other processes involving

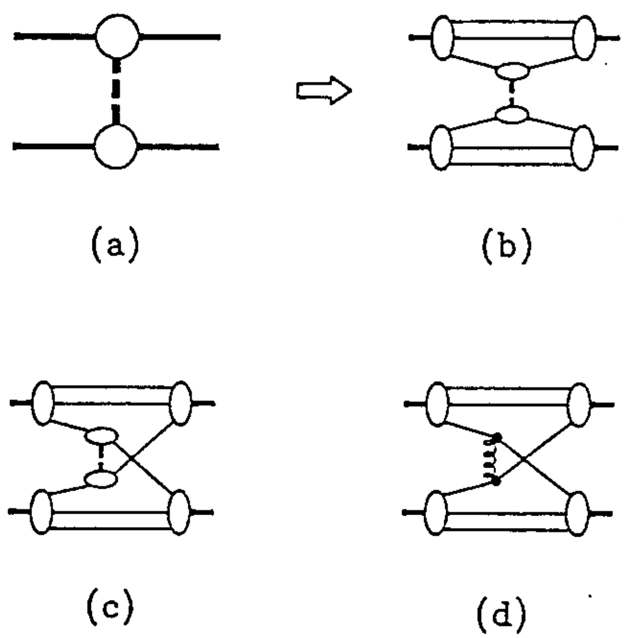

Figure 1: *(a)OPE interaction with phenomenological form factors can be replaced by (b)OPE interaction with strong form factors given by Nqqq and $\pi q\bar{q}$ vertices. Examples of other diagrams arrising from quark structure are (c)OPE followed by quark interchange and (d)one gluon exchange with quark interchange.*

quark *interchange* can now be calculated. In Fig. 1c, pion exchange followed by quark interchange is shown and Fig. 1d shows one gluon exchange. Both of these processes, which lie outside of the OBE model, can be estimated, and could provide detailed guidance as to where to look for important effects due to explicit quark degrees of freedom.

If quark exchange effects are large (and calculations indicate that they are) why is the OBE model so successful? One possible answer, shown in Fig. 2, is that quark interchange effects may be *dual* to OBE. In particular, a sum of a variety of quark interchange diagrams associated with OPE, one gluon exchange, and other short range effects may nearly equal the sum of OBE diagrams with

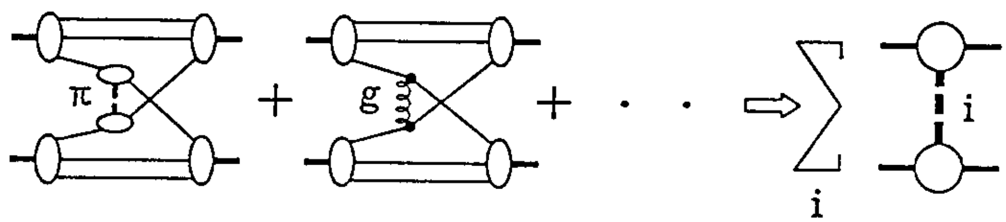

Figure 2: *Sum of quark interchange diagrams may be equivalent to a sum of OBE interactions. This is the concept of duality.*

effective form factors and masses. Perhaps the σ exchange arrises in such a fashion, or ρ and ω exchange may not be due to the exchange of actual ρ's and ω's, but may be an efficient way to represent the sum of quark interchange processes. This idea has been previously introduced by Weber and his collaborators[1], and by many others. The RQCM will permit the study of such effects in the context of the relativistic spectator model.

The Bethe Salpeter wave functions could be calculated from the Bethe Salpeter equation[2] by introducing phenomenological confining forces modeled after lattice gauge results, but this is a long program, and to get preliminary results soon we chose to start "in the middle" and parameterize the wave functions themselves, and fit the parameters to data. For the pion we chose a wave function of the form

$$\psi(P,q) = \frac{N[(1-\eta)\gamma^5 + \eta\frac{P\gamma^5}{2m_\pi}]C}{(\Lambda_1^2 - q^2)(\Lambda_2^2 - q^2)(\Lambda_3^2 - q^2)}$$

$$= S(\frac{1}{2}P + q)\Gamma(P,q)S^T(\frac{1}{2}P - q) \tag{1}$$

where P and q are the total and relative quark four momenta, respectively, C is the dirac charge conjugation matrix, S is the free quark propagator and Γ is the $\pi q\bar{q}$ vertex function. The constant N is fixed by normalization, and the four parameters Λ_i and η are determined from a calculation of the pion form factor, pion decay constant f_π, and $\pi \to 2\gamma$ decay rate. Note that the wave function (1) has no poles at $(\frac{1}{2}P \pm q)^2 = m_q^2$, so that it can be regarded as including the renormalization and shift of the quark mass (from "current" to "constituent" values) associated with the bound state interaction. Because of this fact, the fifth parameter, the "free" mass of the light quark m_q, enters only into the normalization of the pion form factor, where it is absorbed in N, and into the $\pi \to 2\gamma$ decay rate, where it enters into the free propagator S which connects the 2γs in the triangle diagram. We find that the choices $\Lambda_1 = 0.75 fm^{-1}$, $\Lambda_2 = 3.15 fm^{-1}$, $\Lambda_3 = 3.62 fm^{-1}$, $\eta = 0.075$, and m_q=8.4 MeV give excellent fit to f_π, the pion form factor and rms charge radius, and do reasonably well with the 2γ decay rate (3.13 eV is obtained while the experimental value is 7.37 ± 1.5 eV).[3] This model also gives a good description of the process $\gamma + \gamma \to 2\pi$.[3] After the nucleon wave functions have been determined, we will be able to estimate the diagrams in Fig. 1, and (perhaps) insert them as driving terms into the relativistic spectator model described below.

There are many questions of principle which remain to be answered. These include the problem of how to avoid double counting, incorporate qauge invariance and chiral symmetry, and how to include a dynamical model of confinement in a consistent fashion.

3. OVERVIEW OF THE SPECTATOR MODEL

This section will review the concepts used in the spectator model, and describe their applications.

3.1 Concepts

There are three types of amplitudes which are sufficient for most applications: bound state vertex functions, scattering amplitudes, and off-shell form factors.

When bound states are present, the relativistic vertex function with one particle off-shell is required. This vertex function is related to a matrix element of the interacting field between the bound state and the spectator particles in the final state, which, for two body bound states, is

$$\Gamma^{(2)}(p, P_B) \simeq S^{(-1)}(p_2) < p_1|\phi(0)|P_B > \tag{2}$$

where $P_B = p_1 + p_2$ and $p = \frac{1}{2}(p_1 - p_2)$ and $S(p_2)$ is the propagator of the off-shell particle 2. Blankenbecker and Cook were the first to introduce this covariant vertex for the deuteron[4], and a complete discussion of its relation to relativistic deuteron wave functions has been given by Remler[5] and Buck and myself.[6] For three body bound states a similar amplitude is needed[7]

$$\Gamma^{(3)}(p, q, P_B) \simeq S^{-1}(p_3) < p_1 p_2|\phi(0)|P_B > \tag{3}$$

and I believe that the idea can be generalized, with some complications, to four or more bodies. For systems with A nucleons, a vertex can be defined in a similar way

$$\Gamma^{(A)}(p_A) \simeq S^{-1}(p_A) < A - 1|\phi(0)|A > \tag{4}$$

where nuclear matter can be treated by letting $A \to \infty$. These amplitudes are drawn schematically in Fig. 3a,b, and c.

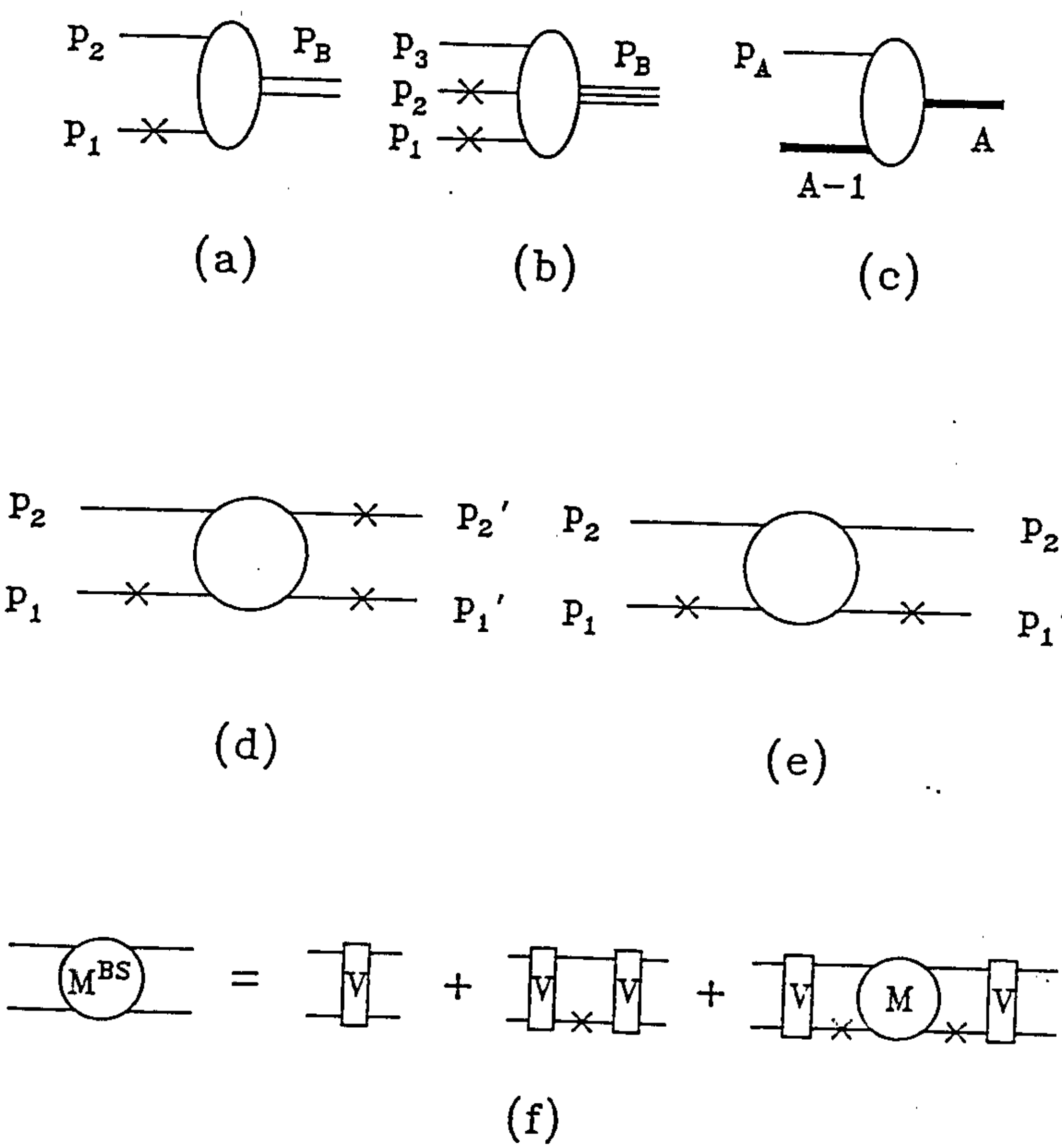

Figure 3: *Concepts in the spectator model. In all diagrams, the cross indicates that the particle is on-shell, and all bound states and A or A-1 particle systems (denoted by a heavy dark line) are also on-shell. Two body bound state vertex functions (a), three body bound state vertex functions (b), vertex function for one off-shell particle in an A body system (c), half off-shell (d) and fully off-shell (e) scattering amplitudes, and equation for the BS amplitude consistent with the spectator model (f).*

For scattering problems, the off-shell scattering amplitude is needed. For two body scattering, the *half* off-shell amplitude is a generalization of (2):

$$M(p,p';P) \simeq S^{-1}\left(\frac{1}{2}P - p\right) < \frac{1}{2}P + p|\phi(0)|p', P > \tag{5}$$

where p' and p are the relative momenta of the two particles in the initial and final state, and the notation is meant to imply that both particles are on-shell in the initial state, but that only particle 1 is on-shell in the final state. Hence

$$\left(\frac{1}{2}P + p\right)^2 = m^2 \tag{6}$$

which becomes a contraint on p_o. In the CM frame, $P = (W, \vec{0})$, this constraint becomes

$$p_o = E_p - \frac{1}{2}W \tag{7}$$

This amplitude is illustrated diagramatically in Fig. 3d. Sometimes the fully off-shell amplitude is required, which is shown diagramatically for the two body system in Fig. 3e. Knowledge of these amplitudes implies knowledge of the relativistic kernel V from which these amplitudes can be calculated by solving the spectator wave equation[8], which in the CM for two spin zero particles is

$$M(p,p';P) = V(p,p';P) + \int \frac{d^3k}{(2\pi)^3}\frac{V(p,k;P)M(k,p';P)}{2E_kW(2E_k - W)} \tag{8}$$

The equation for the bound state vertex function $\Gamma^{(2)} = \Gamma$ can be derived from Eq.(8) by using the fact that the existance of the bound state implies a pole in M at $P^2 = P_B^2 = M_B^2$

$$M(p,p';P) = -\frac{\Gamma(p,P)\Gamma^+(p',P)}{M_B^2 - P^2} + R \tag{9}$$

where R is non singular at $P^2 = M_B^2$. Inserting (9) into (8), and demanding that it hold in the vicinity of the pole, gives the bound state equation.

$$\Gamma(p,P_B) = \int \frac{d^3k}{(2\pi)^3}\frac{V(p,k)\Gamma(k,P_B)}{2E_kM_B(2E_k - M_B)} \tag{10}$$

It can be shown that the fully off-shell 2 body amplitude shown in Fig. 3e is sufficient to obtain solutions to the relativistic three body Faddeev equations (a three body force term may also be needed), but it appears that a systematic treatment of four or more particles may require two body amplitudes with *both* particles in the initial and/or final state off-shell. Such BS amplitudes can be calculated consistently within the framework of the spectator model provided the kernel V is known for all particles off-shell, which will be assumed. The equation, represented diagramatically in Fig. 3f, is

$$M^{BS}(p,p';P) = V(p,p';P) + \int \frac{d^3k}{(2\pi)^3} \frac{V(p,k;P)V(k,p';P)}{2E_k W(2E_k - W)}$$

$$+ \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{V(p,k_1;P)M(k_1,k_2;P)V(k_2,p';P)}{4E_{k_1}E_{k_2}W^2(2E_{k_1} - W)(2E_{k_2} - W)} \tag{11}$$

This definition of M^{BS} is consistent in that $M^{BS} = M$ when one particle is on-shell in both the initial and final state, and the off-shell extrapolation defined in (11) is precisely the amplitude which arises in cases where the spectator model does not uniquely define spectators in either the initial or final state.

The last concepts needed in the spectator model are the vertex functions which describe how probes interact with nucleons or mesons. For nucleon scattering, the probe is the nucleon itself, and the amplitudes needed are the M matrices just discussed. For pion scattering, the πNN vertex function, and the πN scattering amplitudes are needed. The πNN vertex function is contained in V, and work is underway to apply the spectator model to πN scattering. Finally, for electron scattering, the off-shell γNN, $\gamma\pi\pi$, and other current "operators" are needed. Recently, a way has been found to introduce the currents in such a way that gauge invariance is satisfied exactly, and pheonomological form factors (both electromagnetic and strong, such as those used at the πNN vertex) may be used without constraints.[9] This will be discussed further in section 5 below.

3.2 Applications

Figure 4 illustrates how the three concepts discussed above are used in applications. Figs. 4a and b show the relativistic impulse approximation (RIA) to the deuteron form factor and the three-body form factor (in the three body case, the $pd\ ^3He$ vertex is also required). The relativistic bound state vertex functions and the off-shell nucleon form factors are required, and the spectators to the electromagnetic interactions are on-shell. I originally viewed these diagrams as an approximation to the full diagrams with all internal particles off-shell[10], but I now believe that these should be viewed as one term (probably the largest) in the exact current operator, the structure of which is largely determined by the dynamical content of the two body interaction kernel V (see section 5 below).

Figures 4c, d, and e show various contributions to electrodisintegration. Fig. 4c is particularly amusing; this impulse diagram requires precisely the bound state amplitude calculated in the spectator model! Figure 4d is a meson exchange contribution (MEC), and 4e is the final state interaction (FSI). In Fig. 4e, the spectator is again on shell, and the half off-shell amplitude calculated in Eq. (8) is just what is required for the rescattering. The full MEC contains many terms, and will be discussed in Section 5.

Figure 4f shows how spectators can be uniquely identified if three body scattering is regarded as a succession of two body scatterings. Using this analyses, relativistic three body Faddeev equations, driven by the off-shell amplitude shown in Fig. 3e, can be derived.[7] In Fig. 4g the self consistent equations for an A body bound state are written down diagramatically. The kernel used in this equation is identical to the optical potential[11] required for relativistic proton nucleus scattering, and results using such a potential will be reported in the next section. In the limit when $A \to \infty$, if the two body scattering ampli-

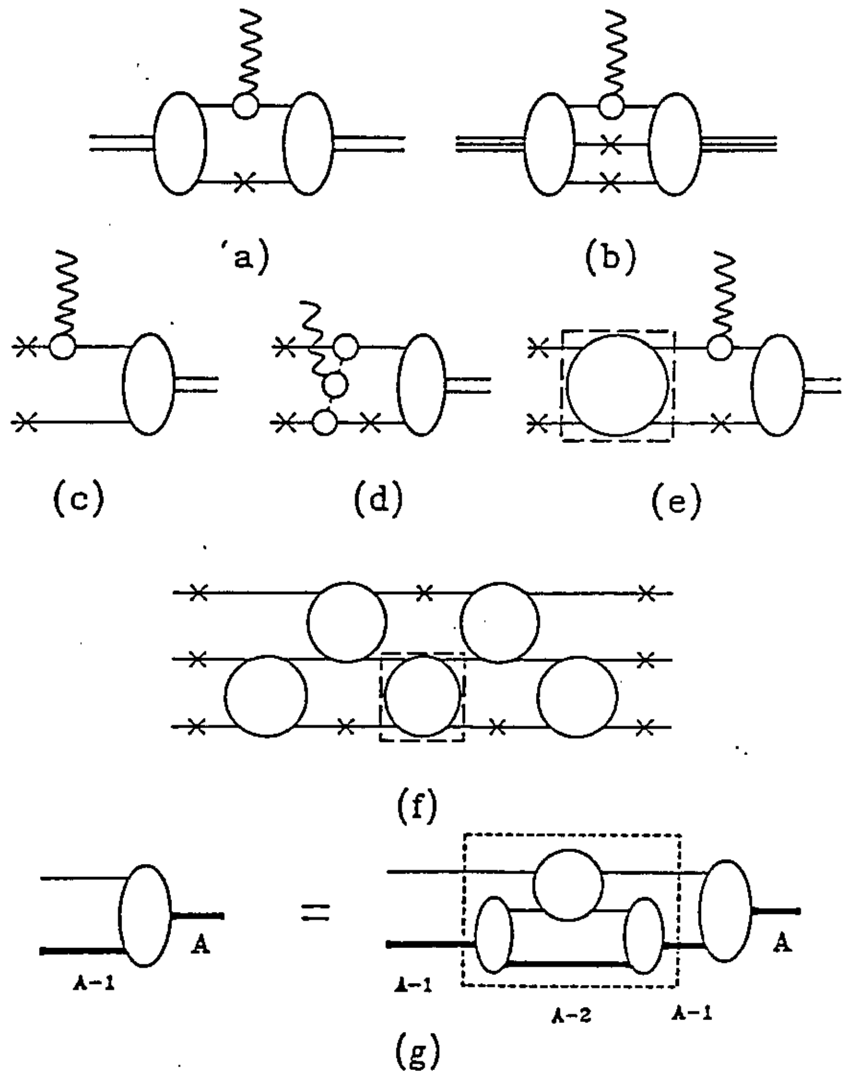

Figure 4: *Applications of concepts in the spectator model. Symbols are described in figures or the text. Two (a) and three (b) body form factors in the impulse approximation; d(e, e′ p)n diagram for the RIA(c), MEC(d), and FSI(e) with the half off-shell amplitude shown in the dashed box; typical sequence of two particle scattering which drives the three body amplitudes are shown in (f) with fully off-shell two body amplitude shown in the dashed box; equation for A nucleon bound state shown in (g) with potential for p-nucleus scattering shown in the dashed box.*

tude M is approximate by its born term V, it can be shown that only σ and ω exchanges survive, and the mean field results of Serot and Walecka can be obtained.[12] Hence the concepts of the spectator model can be applied to few and many body problems in a consistent fashion.

3.3 Assessment

The discussion presented in this section has been very heuristic, but it is possible to develop the discussion in a more formal and rigorous manner. The advantages of the spectator model are

(i) it is manifestly covariant; the transformation properties of all amplitudes under the Lorentz group can be written down explicitly, and all amplitudes conserve energy and momentum, as required by space-time translational invariance;

(ii) there is a close connection to field theory through its expansion in Feynman diagrams, permiting the dynamics of meson exchange to be introduced in a natural way;

(iii) the non-relativistic limits of all amplitudes can be obtained naturally in the $m \to \infty$ limit, establishing a close correspondence with non-relativistic theory and facilitating interpretation of all quantities;

(iv) it can be shown[13] that the kernel V is rapidly convergent in the $m \to \infty$ limit, providing a smooth transition from two body equations to one body equations; and

(v) there is cluster separability; for example, the 3 body equations are driven by the same two body amplitudes calculated in the two body problem.[7]

There are two disadvantages of the spectator approach, only one of which is serious, in my opinion. The non-serious disadvantage is that the equations appear to be unsymmetric because only one of the two particles is on-shell. When dealing with identical particles, where symmetry is required, it can be obtained by explicitly symmetrizing the kernel, as illustrated in Fig. 5a for the OBE model. Once this is done, it can be shown that the two body amplitude is fully symmetric, as shown in Fig. 5b. Alternatively, with a symmetrized kernel it can

Figure 5: *(a)Symmetrization of the OBE kernel. The first term is the direct term, the second the exchange term. (b) Diagramatic representation of the symmetry relation for the scattering amplitude which results from the symmetrized kernel.*

be shown that an equivalent form of the equation can be written in which the propagator is an equal mixture of terms with particle one on-shell and terms with particle two on-shell. Hence, while the equations may look unsymmetric, they are in fact fully symmetric for identical particles, and the Pauli principle for two identical spin $\frac{1}{2}$ nucleons is satisfied exactly.

A second disadvantage is more serious: the process of putting particles on-shell introduces spurious singularities into the interaction kernels. (These are not singularities associated with particle production, which are expected, but singularities which have no physical origin.) It can be shown[7] that such singularities arise from the way in which Feynman diagrams are divided into spectator and non-spectator pieces, and that when all pieces are added together, these singularities cancel. This cancellation may therefore be used to justify dropping the imaginary parts of these singularities. It does not appear that the real parts (principal values) of the singularities can be discarded without greatly adding to the complexity of the equations, but as they occur only when at least one particle is off-shell, and therefore appear only in virtual intermediate states (which are integrated over), and as they occur only at rather large momenta, they seem to have a negligible effect on the numerical results and can be accepted as one of the features of this phenomenology. Their numerical influence is presently being studied in detail in collaboration with J.W. Van Orden.

4. RECENT NUMERICAL RESULTS

This section will report on recent fits to the NN scattering phase shifts and their first application: the predictions for $\vec{p}\ {}^{40}Ca$ scattering observables.

4.1 NN phase shifts

Work using Eq.(10) (suitably generalized to describe two spin $\frac{1}{2}$ particles) to describe the deuteron and NN scattering phase shifts has been underway for some time. The non-relativistic limit of this equation was studied some time ago[14], and numerical solutions for the deuteron in an OBE model have also been obtained[6]. The present work began in collaboration with K. Holinde (Julich) who brought an early version of the Bonn phase shift code to Williamsburg. Recently, J.W. VanOrden has made substantial improvements in the code, and we now can automatically vary the OBE parameters to obtain a best fit to the phase shifts, scattering lengths, effective ranges, and deuteron binding energy. This work is still in progress, and there will be small changes in the results I will report on here, but the essential features are clear at this time.

The OBE models presented here have the following features:

(i) The coupling of pseudoscalar mesons (π and η) includes an off-shell mixing parameter λ_m

$$g_m \left[\lambda_m \gamma^5 + (1-\lambda_m)\frac{(\not{k}_f - \not{k}_i)}{2m}\gamma^5\right] \tag{12}$$

defined in such a way that the coupling is independent of λ_m when both the initial and final nucleon are on-shell.

(ii) The coupling of vector mesons (ρ and ω) also includes an off-shell mixing parameter λ_m

$$g_m \left\{ [1+\kappa_m(1-\lambda_m)]\gamma^\mu + \lambda_m \frac{\kappa_m}{2m} i\sigma^{\mu\nu}(p_f - p_i)_\nu \right.$$
$$\left. -(1-\lambda_m)\kappa_m \frac{(p_f+p_i)^\mu}{2m} \right\} \tag{13}$$

defined in such a way that the coupling is independent of λ_m when both the initial and final nucleon are on-shell. Note that this mixing parameter gives off shell sensitivity only when the tensor coupling κ_m is non-zero. Since the tensor coupling of the ω meson is small, λ_ω was fixed at unity.

(iii) All meson nucleon vertices have the same phenomenological form factor

$$f_m(q^2) = \left(\frac{\Lambda^2 - \mu_m^2}{\Lambda^2 - q^2} \right)^{\frac{3}{2}} \tag{14}$$

where Λ is an adjustable parameter (the same for all mesons), μ_m is the meson mass, and q^2 is the square of the 4-momentum carried by the meson.

(iv) The off-shell nucleon carries a form factor of the form

$$f_N(p^2) = \left(\frac{\Lambda_N^2 - m^2}{\Lambda_N^2 - p^2} \right)^4 \tag{15}$$

This form factor is essential for convergence of the equations.

(v) Both the direct and exchange terms shown in Fig. 5a use the form of the four vector q^2 appropriate to the direct term

$$q^2 = (p_f - p_i)^2 = (E_f - E_i)^2 - (\vec{p}_f - \vec{p}_i)^2 \tag{16}$$

New fits currently being prepared will relax the restriction given in (v), and use the q^2 appropriate to each diagram. This will also require new form factors. Some fits of this kind have already been obtained, and the results do not differ significantly from those to be presented here.

Two OBE models have been found which fit the NN observables very well. The fits to NN phase shifts below 400 MeV are shown in Figure 6. While there are some differences between the fits, these differences are small, and it is not misleading to regard both models as fitting the phase shifts equally well. Yet the two models differ significantly in their dynamical content. Model 1 includes

Table 1: Parameters for Model 1 (upper) and Model 2 (lower). All masses and cutoff parameters are in MeV.

meson	$\frac{g_m^2}{4\pi}$	κ_m	λ_m	μ_m
π	13.83		0.25	
	14.11		0.00*	
σ	4.26			491
	4.65			510
ρ	0.40	7.22	0.97	
	0.65	6.20	0.77	
ω	7.49	0.26		
	8.79	0.02		
η	–		–	
	5.26		0.47	
δ	–			–
	0.41			520

$\Lambda =$	2760	$\Lambda_N =$	1930
	2250		2000

*Constrained in Model 2

the exchange of only the four mesons essential to any OBE description of nuclear forces: π, σ, ρ, and ω. The mixing parameter $\lambda_\pi = 0.25$, corresponds to a 25% admixture of γ^5 coupling for the pion. Model 2 constrains $\lambda_\pi = 0$, giving a pure $\gamma^5\gamma^\mu$ coupling for the pion which many physicists believe is required by pair suppression and chiral symmetry. To obtain an equally good fit to the phase shifts, this model requires an additional η and δ meson. The δ meson is needed

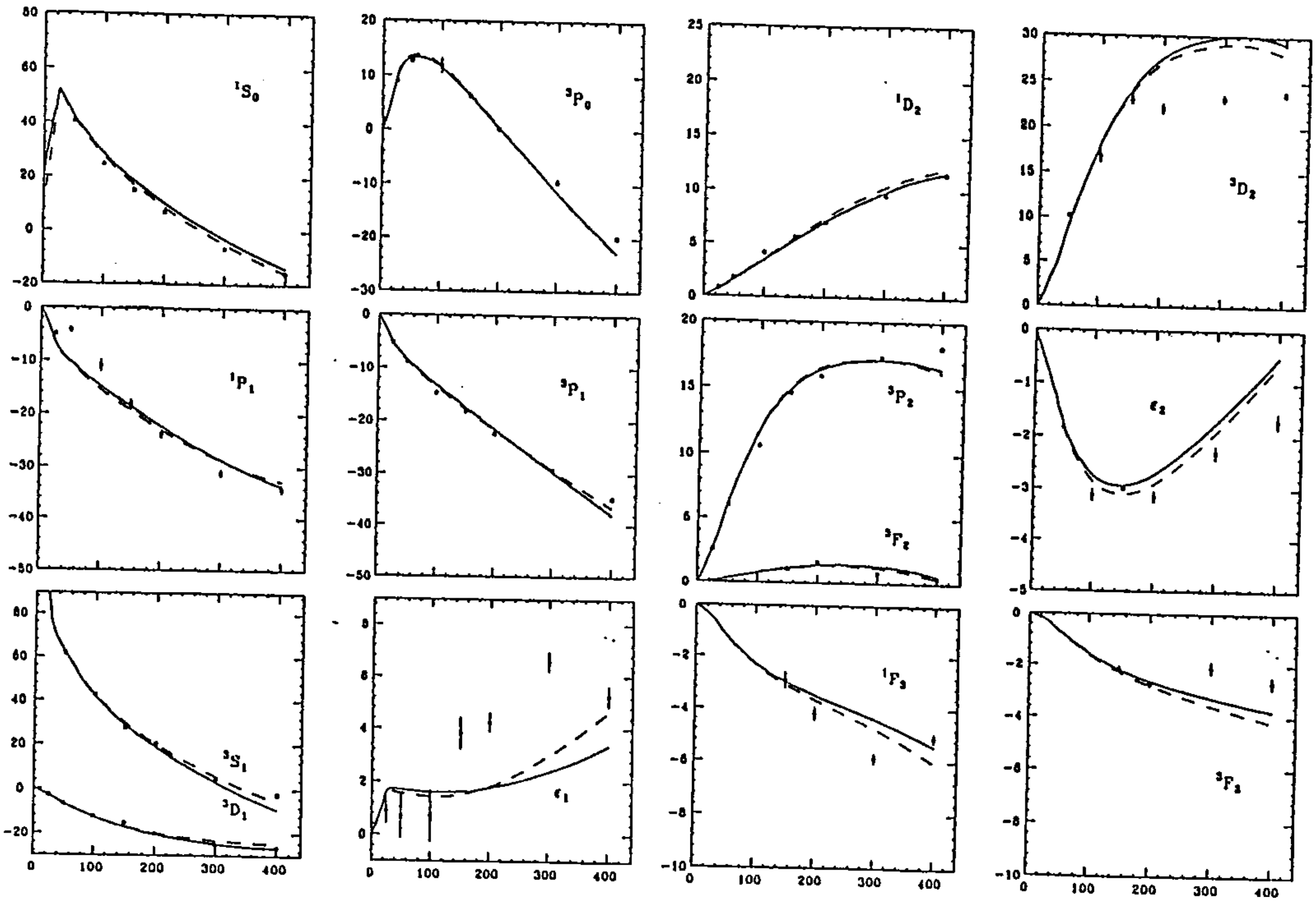

Figure 6: *Fits to the NN phase shifts (in degrees). Solid lines are model 1, and dashed lines are model 2.*