CEBAF-PR-88-009

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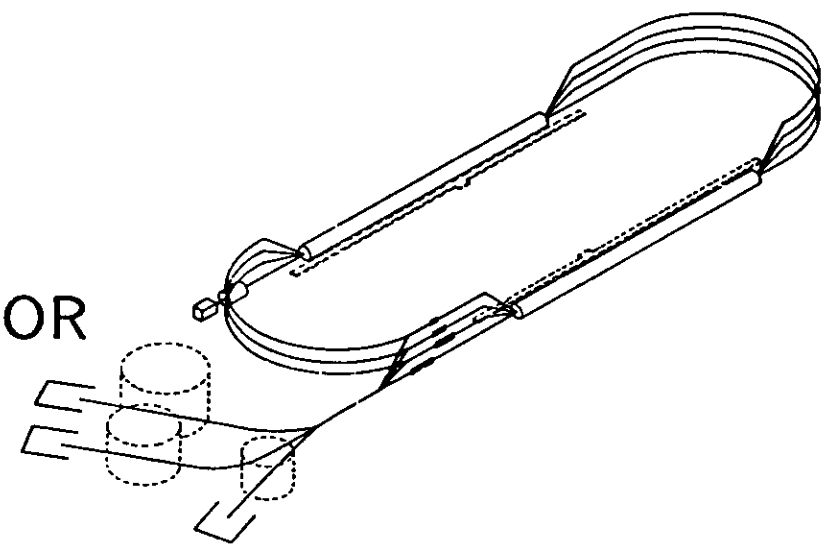

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The Southeastern Universities Research Association (SURA) operates the Continuous Electron Beam Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150.

NONLINEAR SPIN ACCEPTANCE IN ELECTRON STORAGE RINGS*

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It is shown that the degree of polarization of a beam in electron storage rings can be limited by nonlinear effects. The strength of a nonlinear effect depends on the amplitude of the particles. Particles performing synchrotron and betatron oscillations with high amplitudes can contribute over proportionally to depolarization. As a result, the emittance of a beam is not allowed to exceed certain boundaries, otherwise the beam will become more and more depolarized. This limit is called nonlinear spin acceptance.

PACS numbers: 41.80-y, 29.20 Dh, 29.75+x

In electron storage rings the spins of the circulating electrons become aligned opposite the field direction in the bending magnets (Sokolov–Ternov effect).[1] The maximum degree of polarization in a ring without vertical bendings is 92.4 percent, under certain circumstances even higher.[2] The degree of polarization is, in general, reduced by depolarizing effects caused by synchrotron and betatron oscillations. These oscillations are excited and damped by the emission of synchrotron photons.

The spin motion in a homogenous field is described by the so–called BMT equation[3]

$$\frac{d\vec{s}}{dt} = \vec{s} \times \vec{\Omega} \tag{1}$$

$\vec{s}$ is the spin vector and

$$\vec{\Omega} = \frac{c}{E}\left[(\gamma a + 1)\vec{B} - a\frac{\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{B})\cdot\vec{\beta} - \left(\gamma a + \frac{\gamma}{\gamma+1}\right)(\vec{\beta}\times\vec{E})\right] \tag{2}$$

* Submitted to Physical Review Letters on July 20, 1988.

E is the particle energy, γ is the Lorentz factor, $\beta = \vec{v}/c$, a is anomalous magnetic moment of the electron, $\vec{B}$ is the magnetic field, and $\vec{E}$ the electric field. This equation is usually solved by finding in a first step a closed solution for the spin along the closed orbit.[4] This closed solution is called $\vec{n}$ axis. In a second step the direction of the spins of particles performing oscillations around the closed orbit is calculated:

$$\vec{s} = \sqrt{1-\alpha^2-\beta^2}\,\vec{n} + \alpha\vec{l} + \beta\vec{n} \tag{3}$$

α and β describe the deviations from the $\vec{n}$ axis and $\vec{l}$ and $\vec{m}$ are vectors orthogonal to the $\vec{n}$ axis and orthogonal to each other.

The field $\vec{\Omega}$ (Eq. (2) can be split into two terms: $\vec{\Omega}_{CO}$ decribes the field along the closed orbit and $\vec{\omega}$ the field along the actual trajectory relative to the closed orbit. As a consequence, Eq. (3) can be developed into a series and afterwards rewritten in the following form

$$\alpha' = \left(1 - \frac{1}{2}\cdot\left(\alpha^2+\beta^2\right)\right)\vec{\omega}\vec{m} + \beta\vec{\omega}\vec{n}$$

$$\beta' = \left(1 - \frac{1}{2}\cdot\left(\alpha^2+\beta^2\right)\right)\vec{\omega}\vec{l} - \alpha\vec{\omega}\vec{n} \tag{4}$$

where α' is the derivative with respect to the particle trajectory. In the standard linear description of polarization in storage rings, this equation is simplified to[5]

$$\alpha' = -\vec{\omega}\vec{m}$$

$$\beta' = \vec{\omega}\vec{l} \tag{5}$$

and solved. Equation (5) leads to depolarizing resonances of the simple form $a\gamma = n \pm Q_{x,y,s}$, where n is an integer and Q is the horizontal, vertical or longitudinal Q–value of the storage ring, respectively. The physical consequences of this simplification are as follows:

1. The omitted terms describe an additional spin rotation. This additional rotation makes the spin motion dependent on the amplitude of the oscillation.
2. With the approximation $\sqrt{1-\alpha^2-\beta^2} \approx 1$ the spin motion is considered to be commutative. In general, this assumption is not valid for rotations.
3. In addition to these points most of the standard programs consider only linear fields. Sextupoles, higher order multipoles, and the beam–beam force can cause higher order spin resonances.

All three effects lead to depolarizing resonances of the form

$$a\gamma = n \pm kQ_x \pm lQ_y \pm mQ_s \tag{6}$$

where k, l, m and n are integers.

A further complication stems from the fact that these higher order effects act differently on the different particles. It is in the nature of a nonlinear effect that the strength of this resonance depends on the amplitude of the particle oscillation. In order to calculate the depolarization of the whole beam, the strength of the depolarization as a function of the amplitude and the probability to reach this amplitude must be evaluated. With increasing bunch dimensions this effect becomes more and more severe. This limitation for the storage of polarized beams is called nonlinear spin acceptance.

In the following this concept is discussed in more detail. The bunch shape is assumed to be Gaussian in all three dimensions[6]

$$N(x_1,x_2,x_3) = N_0 \prod_{j=1,3} \frac{1}{\sqrt{2\pi}\cdot\sigma_j} \cdot exp\left[-\left(\frac{x_j^2}{2\sigma_j^2}\right)\right] = N_0 \prod_{j=1,3} \frac{1}{\sqrt{2\pi}\cdot\sigma_j} \cdot exp(-n_j)\ . \quad (7)$$

This distribution is caused by particles performing oscillations with the amplitude x_j:

$$x_j = A_j\sqrt{\beta}_j\ exp(-i\omega_j t)\ , \quad (8)$$

β_j describes the property of the machine.

In the following the depolarization is explained first in a one dimensional model. Afterwards it will be extended to three dimensions. As explained above, the time constant for depolarization depends on the distance x_j from the bunch center

$$P(t,x_j) = P_0 \cdot exp\left(-\frac{t}{\tau_i(x_j)}\right) \quad (9)$$

where P is the degree of polarization and τ_j is the depolarization time constant in the interval $x_j, x_j + \Delta x_j$.

When the time in which the particle passes through this interval is small compared to the depolarization time constant, the change in polarization can be written in the following way:

$$\frac{dP(t,x_j)}{dt} = -\frac{1}{P_0}\left(\frac{dP}{dt}\right)_{t=0} = \frac{1}{\tau_j} = \frac{dt}{(dn/dt)\tau_j} \quad (10)$$

n is defined in Eq. (7).

The contribution of a certain interval to depolarization is given by the normalized flux through this interval and its depolarization

$$d\left(\frac{1}{\tau_d}\right) = dP \cdot J \quad (11)$$

The normalized flux through a certain interval is according to Eq. (7)

$$J = (dn/dt) \cdot exp(-n) \tag{12}$$

and the depolarization time for the whole beam becomes (by combining equations (10), (11) and (12))

$$\frac{1}{\tau_D} = \int_0^\infty dn \frac{exp(-n)}{\tau_i(n)} \ . \tag{13}$$

Equation (13) can be expanded into 3 dimensions by introducing the standard polar coordinates η, ψ and θ

$$\frac{1}{\tau_D} = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} \int_0^\pi d\eta \; d\psi \; d\theta \; \eta^2 \sin\theta \; \frac{exp(-\eta)}{\tau_j(\eta, \psi, \theta)} \tag{14}$$

The magnitude τ_j is calculated by the program SITROS[7]. SITROS simulates the particle and spin motion up to the second order: the storage ring is divided into several parts and the transformation of the vector

$$\vec{X} = (x, x', y, y', s, \delta, x^2, xy, xx', \cdots, s\delta, \delta^2) \tag{15}$$

by a 6×27 matrix is calculated (x, x', y, y' are the transverse particle coordinates and their derivatives, s is the longitudinal bunch coordinate and δ is the energy deviation). The rotation of spin in a given section is described by a spinor. The components of the spinor are obtained by multiplying the vector $\vec{X}$ with a 4×27 matrix. An ensemble of particles starts with spins parallel to the polarization axis. The particles are traced over several damping times. During this time the emission of synchrotron radiation photons is simulated. The decay of the polarization is observed in order to calculate the depolarization time constant.

The figure shows an example for the storage ring HERA in the vicinity of a beam energy of 29 GeV. Two of the standard rotators[8] are installed and the standard linear procedures to obtain the maximum possible degree of polarization (spin match)[9] are applied.[10] In the linear approach the degree of polarization is high (dotted line A). The lower solid line B shows the polarization for particles between the forth and fifth standard deviation in the longitudinal direction. These particles become almost totally depolarized. The contribution of this effect to the polarization of the whole beam is shown in the curve C between the two previous curves.

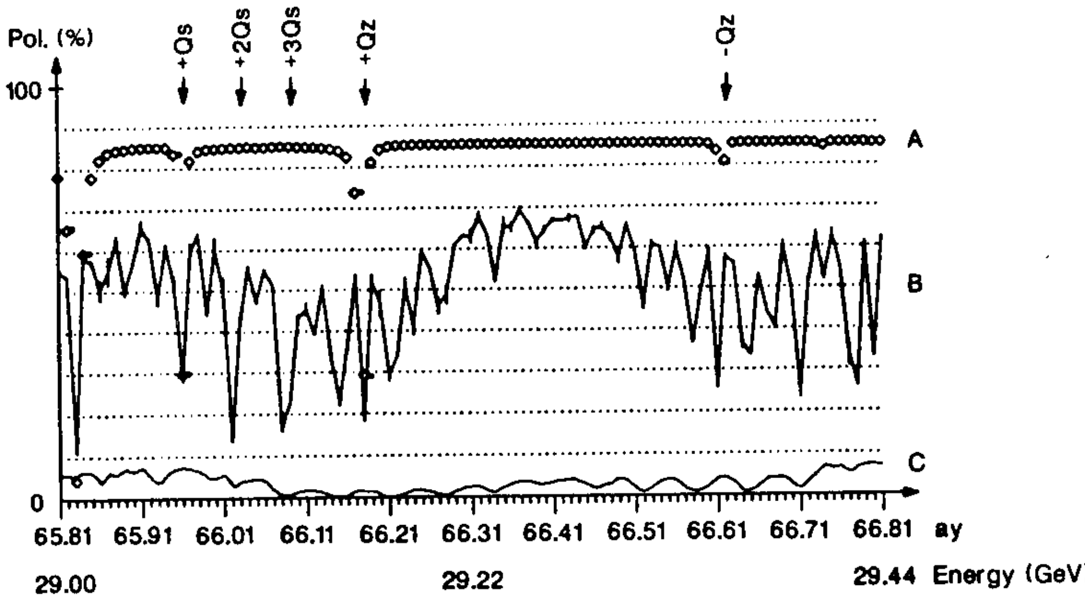

Figure Caption

The degree of polarization in HERA. The linear theory (A) shows a high degree of polarization. If the beam would only consist of particles with four to five standard deviation in the longitudinal direction and second order effects are taken into account, the polarization would be near zero (C). For a real beam with a normal Gaussian distribution, this effect reduces the degree of polarization significantly (B).

Acknowledgement

The authors wish to thank Prof. Dr. G.–A. Voss for support and encouragement, Dr. D. Barber, Dr. F. Willeke from DESY, and Prof. Dr. Strohbusch from the University of Hamburg for many stimulating discussions.

This work was partly supported by the U. S. Department of Energy under contract DE–AC05–84ER40150.

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