SUPERCONDUCTING RF AND BEAM-CAVITY INTERACTIONS

Joseph J. Bisognano

Continuous Electron Beam Accelerator Facility
12000 Jefferson Avenue, Newport News, Virginia 23606

Introduction

Beam-cavity interactions can limit the beam quality and current handling capability of linear and circular accelerators. These collective effects include cumulative and regenerative transverse beam breakup (BBU) in linacs, transverse multipass beam breakup in recirculating linacs and microtrons, longitudinal and transverse coupled-bunch instabilities in storage rings, and a variety of transverse and longitudinal single-bunch phenomena (instabilities, beam breakup, and energy deposition). The superconducting radio frequency (SRF) environment has a number of features which distinguish it from room temperature configurations with regard to these beam-cavity interactions. Typically the unloaded Qs of the lower higher order modes (HOM) are at the 10^9 level and require significant damping through couplers. High gradient CW operation, which is a principal advantage of SRF, allows for better control of beam quality, which for its preservation requires added care with respect to collective phenomena. Gradients are significantly higher than those attainable with copper in CW operation but remain significantly lower than those obtainable with pulsed copper cavities. Finally, energy deposition by the beam into the cavity can occur in a cryogenic environment. In this note those characteristics of beam-cavity interactions which are of particular importance for superconducting RF cavities are highlighted.

Q Dependence of Collective Effects

The Q dependence of the various beam-cavity interactions provides a fundamental differentiation which is in many ways more basic in the underlying accelerator physics than whether the phenomena occur in a linear or circular accelerator. At one extreme are coupled-bunch effects (cumulative and multipass BBU and storage ring coupled-bunch instabilities) which are exacerbated by high-Q modes. For bunch-to-bunch coupling, wakefields must linger for at least the duration of the bunch spacing, and, moreover, a long or CW train of bunches can coherently excite a mode for a time of order Q/ω. A mode Z/Q is essentially a geometric property of a cavity and is insensitive to wall material or de-Q-ing efforts through couplers. It describes the single-pass excitation of a cavity mode by a beam bunch. On the other hand, $Z = (Z/Q)Q$ describes the coherent summation of multiple excitations over the filling time of the mode. Therefore, it is the impedance $Z = (Z/Q)Q$, not Z/Q, which determines the average current limits for tolerable emittance degradation or the onset of exponential growth, and the coupled-bunch phenomena are extremely sensitive to Q. At the other extreme are single-bunch, single-pass effects which are driven by the short-time behavior of wakefields.

For short bunches (nanoseconds to picoseconds) in a single pass, the inherently short sampling time results in lost frequency resolution (smearing of hundreds of megahertz to hundreds of gigahertz) which effectively averages frequency dependent, possibly high-Q impedances over the mode spectrum. In addition, for picosecond linac bunches, the wakefield at short times is dominated by the impedances at frequencies well above cutoff where modes can be expected to overlap. Thus, it is the impedance Z/Q, not Z, which determines the peak current limits for tolerable emittance degradation or the onset of exponential growth, and these limits are insensitive to Q. An intermediate regime is found in pulsed operation where transients are important for bunch trains shorter than the filling time of the higher order mode in question.

Transverse Single-Pass, Multi-Bunch BBU

A train of bunches passing offset through an accelerating structure can excite transverse deflecting modes through a longitudinal electric field which increases linearly off axis for such modes. For a low energy beam, the excitation can be strong enough to deflect the beam significantly during transit through a single structure into regions of higher longitudinal field and increased coupling. Therefore, there can be closure of a feedback loop within a single structure: an excited cavity mode deflects a beam which in turn further excites the mode. Stability comes from the damping of the fields through wall losses and couplers. This effect is known as regenerative beam breakup, and for a standing wave structure has a threshold current given by [1]

$$I_{\text{th}} = \frac{\pi^3 E k}{2Z'' L^2} \tag{1}$$

where Z'' is the transverse impedance per unit length, L is the length of the RF structure, E is the beam energy and k is the wavenumber of the HOM. Above threshold, there is exponential growth of the cavity excitation; any offset will ultimately grow to the point of destruction of the beam. Since SRF structures usually are only a few cells long to prevent the appearance of trapped modes, the dependence of the threshold current on the square of the length lessens the importance of this current limit in practice. At a few MeV, however, an undamped ($Q \sim 10^9$), gigahertz SRF cavity would be limited to currents typically of the order of a milliampere. Note that the threshold current depends on Z'', not Z''/Q, and is therefore inversely proportional to the damped Q.

A second form of beam breakup, cumulative BBU, occurs in a multi-cavity accelerator [2]. Suppose bunches enter the first cavity offset. The first bunch experiences no additional deflection, but subsequent bunches experience deflection due to the excitation of preceding bunches. The level of excitation varies with time depending on the relative frequencies of the bunch train and the excited HOM. For a CW beam, the strongest level of excitation occurs when a harmonic of the bunch frequency is a half-width away from the HOM frequency. When this frequency relationship is not satisfied transient excitation can exceed that of the steady state; the cavity mode fills coherently until there is enough time to resolve the frequency difference between the bunching and the mode frequencies. After this time, the level of total excitation can decrease. Now consider a series of cavities. Excitation of, say, cavity n produces enhanced deflections at cavity $n + 1$; the resulting excitation at cavity $n + 1$ produces further enhanced deflections at cavity $n + 2$, and so on. A series of cavities, therefore, produces an amplification of an initial offset at the first cavity. Displacements between cavities can also be amplified. Note, however, that in this sequence there is no closure of a feedback loop; an excitation of cavity n enhances the field in cavity $n + 1$ and not in cavity n. There is no exponential growth in time. Reduction of initial offset or of cavity misalignments proportionally reduces the maximum beam displacement, both for pulsed and CW beams. Cumulative beam breakup is also present in multipass linear accelerators at currents below the multipass BBU threshold discussed in the next section. Figure 1 shows the displacement of bunches at the exit of a four-pass recirculating linac as a function of bunch number. Note the transient behavior which settles down to an equilibrium steady state. For this case, the mode frequencies and bunching frequencies are not harmonically related, and the transient behavior is considerably stronger than the steady-state equilibrium. Since this steady state is a forced oscillation of the cavity, the cavity excitation is at a harmonic of the bunching frequency for equally spaced, equally charged bunches. Thus, all bunches receive the same kick, the phase of which depends on the relationship between the bunch harmonic frequency and the HOM frequency. This kick in principle can be compensated with DC magnetic elements. Clearly, HOMs which are near harmonics (within a half-width) of the bunch frequency are of most concern. For user flexibility, bunches can be differentially loaded to provide, for example, different end stations of a linac with different bunch current [2]. For example, a 1500 MHz bunching could be differentially loaded at 500 MHz to form three classes of bunches (1,4,7,10,...), (2,5,8,11,...), and (3,6,9,12,...). Since the basic periodicity of the system is at 500 MHz, HOMs neighboring on harmonics of 500 MHz are now of particular importance. For 500 MHz harmonics which are not also harmonics of 1500 MHz, each of the three classes of bunches receives a different kick. The resulting offset cannot be corrected by DC elements, and thus there is an effective emittance increase during transport through the linac which requires an RF device (e.g., RF separators) to sort out. For average currents at the hundred microampere level, this effect is negligible. Similar considerations would apply to repeating trains of bunches as found in some linear collider scenarios.

Multipass Phenomena

When a linac beam is recirculated through the accelerator structure, the feedback system formed by the cavities and beam is closed. Suppose that the first cavity has an excited HOM which deflects the beam. Recirculation brings the beam back

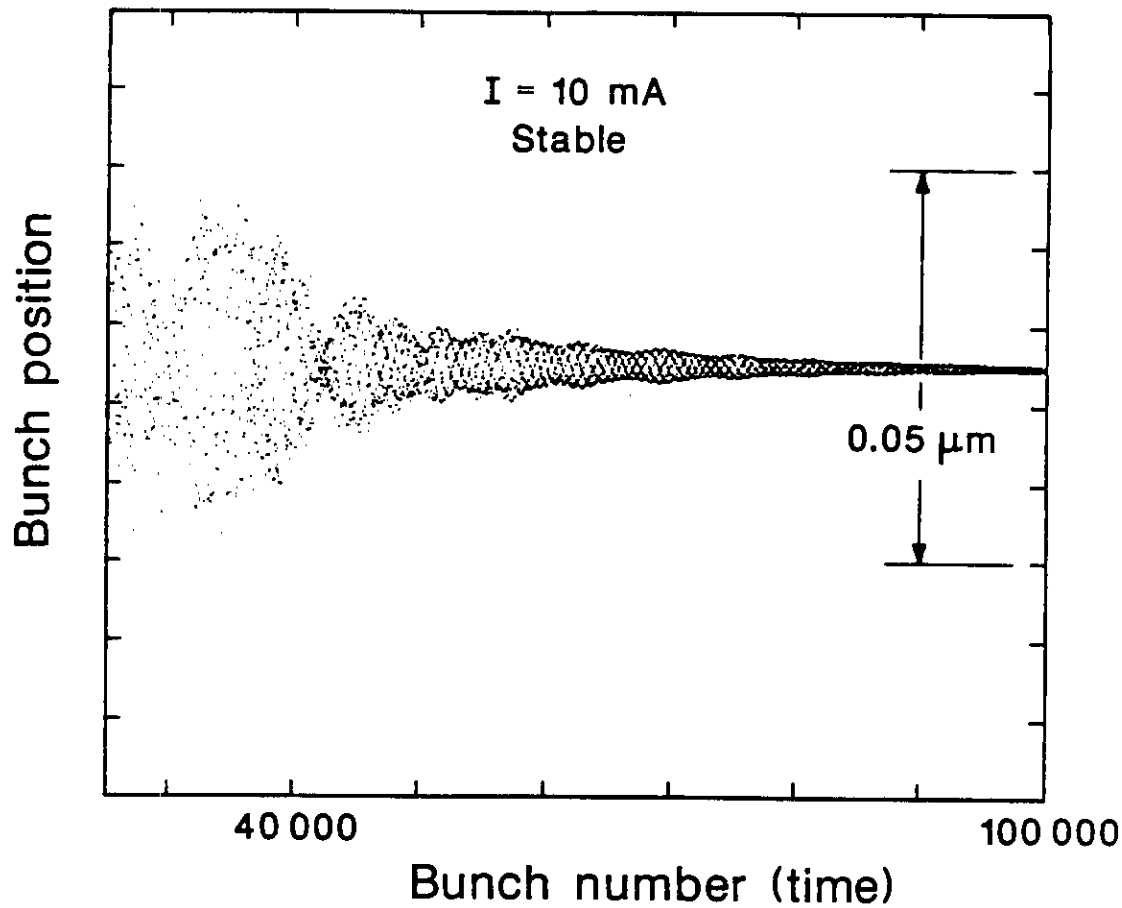

Figure 1 Transient and steady state beam excitation from cumulative BBU in a recirculating linac

through that first cavity for one or more passes. Since the lattice can translate the deflection into a transverse offset, the excited mode can now be enhanced as was the case in regenerative BBU. The feedback loop is closed and expontential growth can occur at high enough currents. In addition, many cavities can participate in this instability if HOM frequencies overlap; that is, there is also a cumulative aspect to this effect. We refer to this combination of regenerative and cumulative BBU as multipass beam breakup. Figure 2 shows again bunch displacement with bunch number, but now at a current above the threshold for multipass beam breakup.

Multipass beam breakup in linacs and coupled-bunch instabilities in storage rings are both caused by the interaction of the beam bunches with high-Q higher order modes. The basic difference is that multipass beam breakup involves an infinity of bunches each of which undergoes a finite number of passes, whereas the coupled-bunch instability involves a finite number of bunches each of which undergoes essentially an infinity of passes. Also, given the isochronicity of typical recirculating linac lattices, only transverse deflecting modes are important in linacs, whereas both longitudinal and transverse modes can drive instabilities in storage rings.

This distinction between the many versus infinite passes strongly affects the threshold character of the instability. For the transverse coupled-bunch instabilities in storage rings the growth rate is given by [3]

$$\Delta\omega^{\perp}_{s,a} = -i\,\frac{eI_b n_b c}{4\pi(E/e)\nu_{\perp}}\,\frac{(\sigma/R)^{2a}}{2^a\,a!}\left[Z_{\perp}\right]^{s,a}_{\text{eff}} \tag{2a}$$

where

$$\left[Z_{\perp}\right]^{s,a}_{\text{eff}} = \sum_{p=-\infty}^{+\infty}\left(pn_B+s+\nu_{\perp}-\frac{\xi}{\eta}\right)^{2a} e^{-\left(pn_B+s+\nu_{\perp}-\frac{\xi}{\eta}\right)^2(\sigma_\ell/R)^2}\, Z_{\perp}\left(\nu^{\perp}_p\omega_0\right) \tag{2b}$$

and

$$\nu^{\perp}_p = pn_b + s + \nu_{\perp} + a\nu_s \ . \tag{2c}$$

where I is the bunch current, n_b is the number of bunches, a is the internal mode number, s is the bunch-to-bunch mode number, σ is the rms bunch length, Z is the transverse impedance (Z''/k) and $\nu_{\perp}$ and ν_s are the betatron and synchrotron tunes respectively. If the Im $(\omega) > 0$ the system is unstable, irrespective of the magnitude of the current I_b. As long as the impedance has the correct frequency behavior, the threshold current is zero if there is no damping provided by synchrotron radiation or internal bunch frequency spread (Landau damping).

The growth rates and threshold current for multipass beam breakup are given by solution of an eigenvalue equation [5]

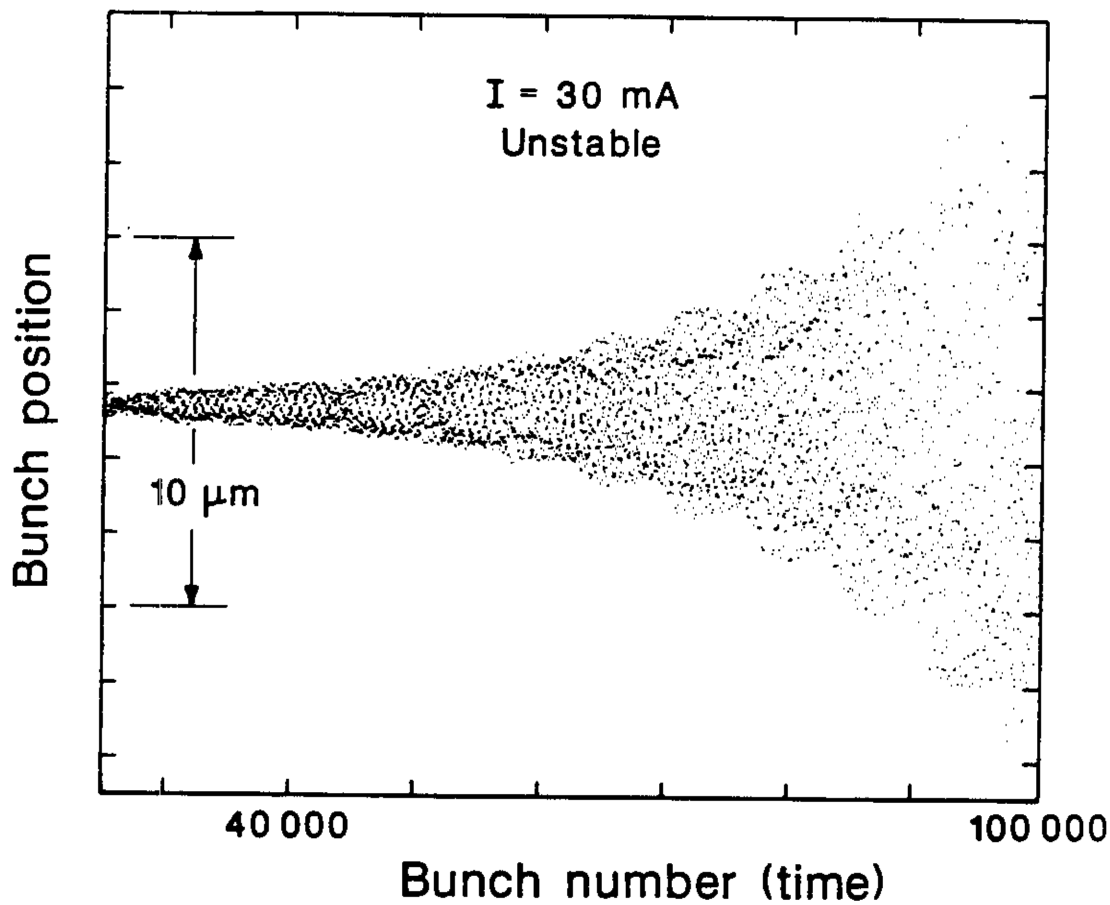

Figure 2 Exponential growth of beam excitation above the threshold of multipass beam breakup

$$
\begin{aligned}
D_i = I \sum_{p=2}^{n_p} \sum_{r<p} \sum_{\ell=1}^{n_0} \left(T_{i,\ell}^{pr}\right)_{12} e^{M_0 \Omega \tau (p-r)} Z_\ell h_\ell(\Omega) D_\ell \\
+ I \sum_{p=1}^{n_p} \sum_{\ell=1}^{i-1} \left(T_{i,\ell}^{pp}\right)_{12} Z_\ell h_\ell(\Omega) D_\ell
\end{aligned}
\tag{3a}
$$

where

$$
\begin{aligned}
h_n(\Omega) &= \frac{H_n(\Omega)\sin(\omega_n \tau)}{1 + H_n(\Omega)^2 - 2\,H_n(\Omega)\cos(\omega_n \tau)} \\
H_n(\Omega) &= e^{-\frac{\omega_n \tau}{2Q}} e^{-i\Omega\tau}
\end{aligned}
\tag{3b}
$$

and

$T_{i,\ell}^{pq}$ is the transfer matrix for (x, p_x) from pass ℓ to pass i

$Z_n = \dfrac{Z_n'' \ell e}{2Q f_b}$

Z_n'' = transverse impedance of cavity/unit length

M_0 = number of bunches in one recirculation

ℓ = cavity length

$f_b = \dfrac{1}{\tau}$ = bunching frequency

$s_k(\omega_n \tau) = e^{\frac{k\omega_n \tau}{2Q}} \sin(k\omega_n \tau)$

n_p = passes

n_0 = number of cavity sites

I = average beam current

The inverse of the threshold current appears as the eigenvalue. To find thresholds for instability, a plot of complex current eigenvalues has been found useful. First the coherent frequency Ω is swept in real frequency with an arbitrarily small imaginary part corresponding to growth. For n_0 cavities there are n_0 families of eigenvalues, with the actual threshold current corresponding to the smallest possible real value obtained. Figure 3 illustrates the simplest such plot for a single cavity in two passes. The lowest positive current crossing gives the threshold current. Note that there is now, in contrast to the storage ring situation, a finite value of current below which there is stability. Figure 4 illustrates the eigenvalue structure for a multicavity, multipass linac.

For an HOM of a single cavity, the threshold current is inversely proportional to the transverse impedance Z or the mode Q as was the case for the storage ring

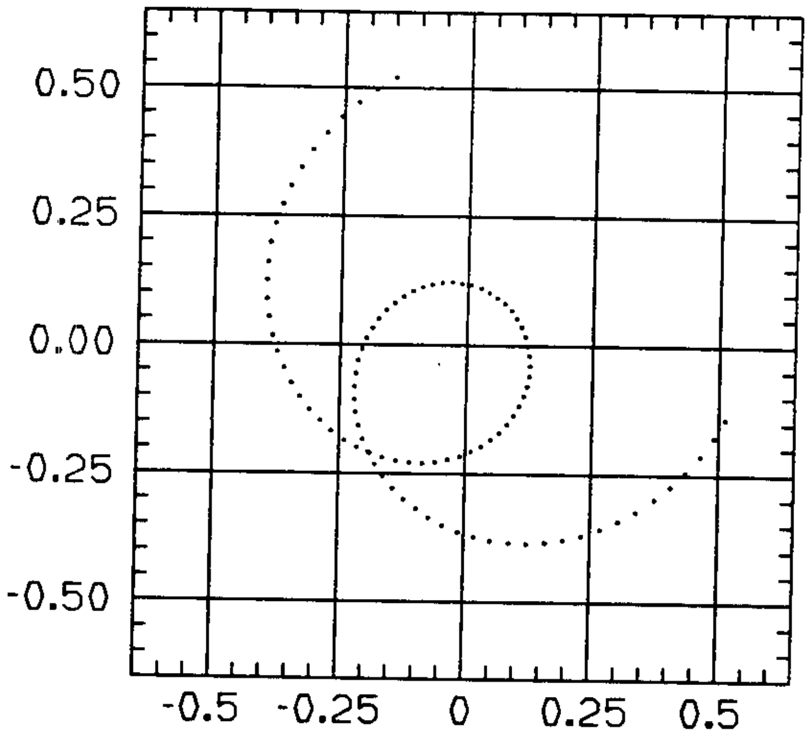

Figure 3 Complex current plot for the determination of threshold current of multipass beam breakup driven by a single cavity in a two-pass configuration.

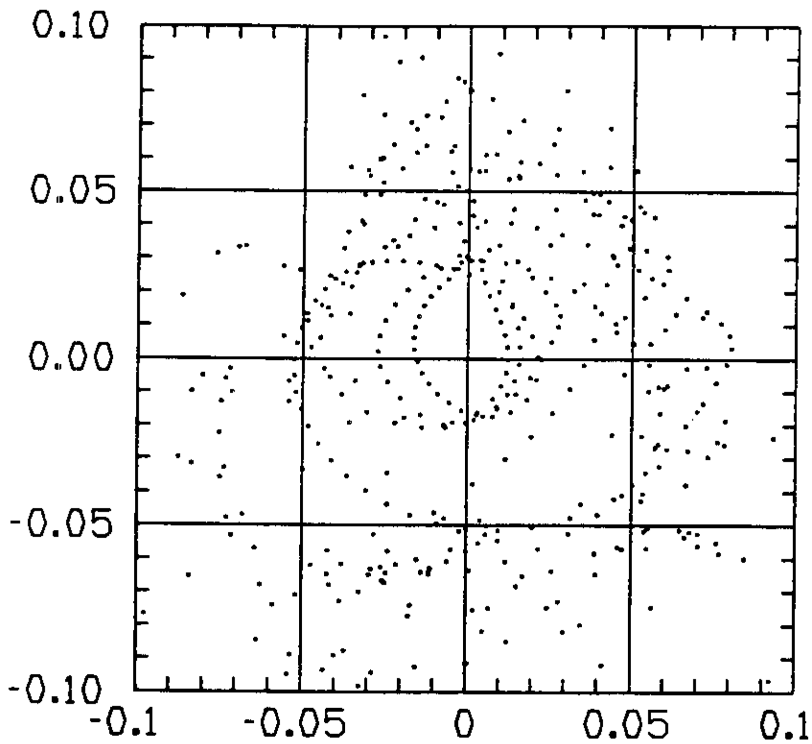

Figure 4 Complex current plot (amperes) for the determination of threshold current of multipass beam breakup driven by 400 cavities in a four-pass configuration

coupled-bunch instability. In contrast to the storage ring there is a non-zero threshold current for multipass beam breakup in linacs even when there is no synchrotron radiation damping or Landau damping.

Single-Bunch, Single-Pass Effects

In addition to the coupled-bunch phenomena which have been described so far, there is a large class of single-bunch, single-pass effects which limit the peak current (more precisely, bunch charge and length) handling capabilities in storage rings and linacs. In a single pass through an RF structure, there is insufficient time for the bunch to experience the long-term ringing associated with the high Qs characteristic of superconducting cavities. Thus, these current limits are not particularly sensitive to the success or failure of damping the HOM Qs. However, there do remain some features of the collective phenomena which distinguish superconducting cavities from room temperature ones.

Because of the efficiency of the cavities (maximization of shunt impedance is not as crucial as with room temperature cavities) and the requirement of coupling through the beam pipe to avoid sites for multipactoring and breakdown, a typical RF cavity has a large aperture. This increase in opening reduces the coupling impedance of higher order modes; for transverse modes this coupling can be an order of magnitude below that which would be expected for an optimized room temperature cavity. Secondly, although the gradient achieved in SRF cavities is larger than that which can be achieved in CW room temperature cavities (less than an order of magnitude), it remains significantly less than that achieved in pulsed room temperature cavities. Thus, for CW operation, SRF machines have a clear advantage from an impedance point of view over room temperature machines, since the SRF impedance per megavolt is lower. However, in comparison to pulsed, room temperature linacs, the situation is not as clearcut where the SRF cavities offer less impedance per unit length, but depending on gradient, may in fact offer more impedance per megavolt.

The longitudinal single-pass loss factor $k_{\parallel}$ is given by

$$k_{\parallel} = \frac{1}{\pi} \int_{0}^{+\infty} d\omega \;\; e^{\frac{-\omega^2}{c^2}\sigma^2} Z_{\parallel}(\omega) \tag{4}$$

where σ is the rms bunch length. Since the higher order mode impedance is integrated, the loss factor depends on the Z/Q (which is proportional to the area under the resonance) rather that Z (which is the peak value of the resonance). For sequences of bunches, the integral is replaced by a sum over the harmonics of the bunching frequencies multiplied by $e^{\frac{-\omega^2}{c^2}\sigma^2}$. For high frequency modes which overlap the results of the summation and integration are comparable. The contribution of high-Q, nonoverlapping modes, however, becomes quite sensitive to the harmonic relation between the HOM and bunching frequencies. The power deposited in a single pass is given by

$$P = I_b Q_b k_{||} \tag{5}$$

where Q_b is the bunch charge, and I_b is the average beam current.

For the CEBAF 5 cell 1500 GHz cavity this loss factor is estimated to be about 9 V/pC [6] for a $\sigma = .25$ mm bunch. For the typical bunched beam at 800 μA (200 μA by 4 passes) there is a loss of 4.8 V and an associated average power deposition of 3.8 mW. The peak current in this case of about 250 mA. For such a cavity used in a single-pass FEL configuration, however, peak currents of 500 A would be required. The voltage loss would then be 9375 V with a power loss to the cavity of 15 kW if the 1500 MHz bunching were retained. Clearly, we would then be dealing with power losses that cannot be released in a cryogenic environment. A small fraction of this power dissipated in stainless steel would not be acceptable. Higher order modes would need to be extracted, and care would have to be taken to pin down where HOM power extracted through beam pipes would be deposited.

In addition to energy loss, energy spread is induced by the variation of the wakefield across the bunch. At 5 MV/m gradient, the induced energy spread is less than 1% for a 1 mm long bunch at peak currents of 500 A in a typical 1500 MHz cavity. With adjustment of the RF phase to remove the linear part of the variation, residual spread at the 0.1% level can be expected. Improvement of achieved gradient directly improves the residual energy spread.

There are analogous transverse emittance degradation effects in linacs, and fast head-tail instabilities in storage rings. In linacs, alignment errors are amplified; for example, for a 1500 MHz cavity it is found that such amplification can be less than unity at 500 A peak current without the introduction of stabilization through energy spread. Given the smooth, large aperture design of SRF cavities, the contribution of bellows, valves, and other discontinuities gains relatively more importance than for small aperture, room temperature cavities, and care must be taken to minimize the strength of such impedance sources. For storage rings, current limits are improved with the substitution of superconducting cavities for room temperature cavities, as discussed above, because of their higher gradients and wider opening, which results in considerably lower transverse impedance per unit gradient.

References

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