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PARITY VIOLATION WORKSHOP -- CEBAF

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I would like to welcome all of you. This is a treat for me because it is a chance to discuss physics. I want to talk about parity violation in the nuclear domain, and I will start by saying a little about electron scattering to set the framework [1].

In electron scattering there are three electron variables (Figure 1): the initial and final wave number K_1, and K_2, and scattering angle θ, or equivalently, the four momentum transfer squared q^2, the quantity $\nu = q.p/M_T$ which is simply the electron energy loss $\nu = \epsilon_1 - \epsilon_2$ in the laboratory frame, and the electron scattering angle θ. The S-matrix for this process

$$S_{fi} = -\frac{(2\pi)^4 i}{\Omega}\,\delta^{(4)}(K_1+p-K_2-p')T_{fi}$$

$$T_{fi} = \frac{4\pi\alpha}{q^2}\left[i\bar{u}(K_2)\gamma_\mu u(K_1)\langle p'|J_\mu^\gamma(0)|p\rangle\right] \tag{1}$$

takes the usual form, where the T-matrix is the product of the Møller potential created by the electron and the matrix element of the electromagnetic current for the hadronic system. I am going to use superscripts to denote isospin. Here γ just denotes the electromagnetic current. Given the T-matrix, one can compute the cross section, which is going to involve known electron variables and bilinear combinations of the current matrix elements for the target. This bilinear combination, when averaged over initial states and summed over final states, is a Lorentz tensor which can depend only on the two four-vectors p and q left in the problem. The electromagnetic current is conserved, and one can construct the general form of this Lorentz tensor.

$$W_{\mu\nu} = (2\pi)^3 \Omega \overline{\sum_i} \sum_f \langle i | J^\gamma_\nu(0) | f \rangle \langle f | J^\gamma_\mu(0) | i \rangle (E) \delta^{(4)}(p' - p + q)$$

$$= W^\gamma_1(q^2, q \cdot p) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \tag{2}$$

$$+ W^\gamma_2(q^2, q \cdot p) \frac{1}{M_T^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right)$$

It consists of two functions of the Lorentz invariants q^2 and q.p; their coefficients simply exhibit the Lorentz structure of the tensor. This is a well-known theorem. One can then calculate the double-differential cross section for electron scattering.

$$\left(\frac{d^2\sigma}{d\epsilon_2 d\Omega_2} \right)^{\mathrm{ERL}}_{ee'} = \sigma_M \frac{1}{M_T} \left[W^\gamma_2 + 2W^\gamma_1 \tan^2 \frac{\theta}{2} \right]$$

$$\sigma_M = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4\epsilon_1^2 \sin^4 \frac{\theta}{2}} \tag{3}$$

σ_M is the Mott cross section; it is the cross section for the scattering of a Dirac electron on a point charge. The structure of the target is contained in the two response surfaces which are functions of q^2 and q.p. If those variables are kept fixed and one makes a straight-line Rosenbluth plot against $\tan^2\theta/2$, the contribution of those two response surfaces can be separated.

If one looks at electron scattering to discrete states of the target,

$$p' = p - q$$
$$-M^{*2} = -M^2 - 2p.q + q^2 \tag{4}$$

then there is a relation between p.q and q^2 which is obtained by simply squaring the statement of conservation of four-momentum. M* is the mass of the final state. There is clearly only one independent variable left, and we take that to be q^2.

Let me give an example; Suppose we have a 0^+ target. The general form of the matrix element of the current for a 0^+ target can be written in the following form.

$$\langle p';0^+|J_\mu^\gamma(0)|p;0^+\rangle = \left(\frac{m^2}{\Omega^2 EE'}\right)^{1/2} F_0^\gamma(q^2)\,\frac{1}{m}\left(p_\mu - \frac{p.q}{q^2}\,q_\mu\right) \tag{5}$$

This follows simply from Lorentz invariance, current conservation, and parity. There is a single electromagnetic form factor which is a function of q^2, and the remaining terms exhibit the Lorentz structure of the matrix element. The response tensor can be constructed and then the cross section for scattering from the 0^+ target.

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_M}{1 + \dfrac{2\epsilon_1 \sin^2\frac{\theta}{2}}{m}}\,|F_0^\gamma(q^2)|^2 \tag{6}$$

One has the Mott cross section, the recoil factor, and the electromagnetic form factor which is a function of q^2. I give you my favorite example in Figure 2. This is the measurement of the elastic cross section for ^{48}Ca done at Saclay [2]; it is essentially the square of that form factor, as a function of the momentum transfer q; this is the diffraction pattern that you observe. Note the log scale which extends over 12 decades. The charge distribution in ^{48}Ca is obtained essentially by taking the Fourier transform of the form factor and the extracted charge distribution for ^{48}Ca is shown in Figure 3 as a function of distance from the center of the nucleus measured in Fermis [2, 3]. Recall $1F = 10^{-13}$cm. The band indicates the experimental uncertainty in this charge distribution. The heavy dashed curve gives you an indication of our current ability to calculate the charge density on theoretical grounds within the framework of what I call quantum hadrodynamics or QHD. By that I mean a relativistic quantum field theory of the nuclear system based on hadronic, baryon and meson, degrees of freedom [3]. Let me just say a little bit about how you do that calculation [4]. This result is calculated within the mean field approximation where the scalar and vector fields are replaced by their expectation values and the baryons move in these mean meson fields. This relativistic mean field theory gives you a very nice description of calcium. Eventually that description starts to break down at high q^2 where you look at the short distance structure of the charge density. How do you improve this approach? You include exchange currents. These are additional currents in the nuclear system arising because of the sub-nucleonic degrees of freedom. The mesonic degrees of freedom can create currents flowing between the baryons. From the nuclear physics point of view, the distribution of current and charge in this nuclear system arises from complex hadronic processes. In theory, we work harder and harder to try and get a better and better description of that charge and current distribution.

I am now going to talk about parity violation [1, 5]. As Stan Kowalski has already indicated, there is in addition to the electromagnetic interaction through the exchange of a virtual photon, a weak-neutral-current interaction of the electron through the exchange of the Z° (Figure 4). The effects of this weak-neutral-current interaction are completely masked at the energies that we are talking about (significantly below 100 GeV, the mass of the Z°) by the electromagnetic interaction. The effects are characterized by the Fermi coupling constant $G = 1.02 \times 10^{-5}/m_p^2$; they are very small unless you look at something that is dependent on the presence of the weak interaction, like parity violation.

The extension of the T-matrix in Eq. [1] which includes these two processes is:

$$T_{fi} \doteq \frac{4\pi\alpha}{q^2}\left[i\bar{u}(K_2)\gamma_\mu u(K_1)\langle f|J_\mu^\gamma(0)|i\rangle - \frac{Gq^2}{4\pi\alpha\sqrt{2}}\, i\bar{u}(K_2)[a\gamma_\mu+b\gamma_\mu\gamma_5]u(K_1)\langle f|\mathcal{J}_\mu^{(o)}(0)|i\rangle\right] \tag{7}$$

In addition to the Møller interaction with the electromagnetic current, there is an interaction with the Z° which can be replaced by a point coupling as long as we are working at energies well below 100 GeV. I have factored out an overall factor $4\pi\alpha/q^2$, so the weak interaction term gets multiplied by the inverse of that factor. This gives rise to the characteristic factor $Gq^2/4\pi\alpha\sqrt{2}$ that is going to characterize the strength of the interference structure. It has been assumed here, for generality, only that the weak-neutral-current interaction has a V-A structure. a is the strength of the vector coupling of the electron, b the strength of the axial-vector electron coupling, and the weak neutral hadronic current is assumed to have the form

$$\mathcal{J}_\mu^{(o)} = J_\mu^{(o)} + J_{\mu 5}^{(o)} \quad ; \text{ V-A Theory} \tag{8}$$

That is, it will be assumed that the weak neutral current is the sum of a Lorentz vector and a Lorentz axial vector.

In the Standard Model of Weinberg, Salam, and Glashow [6, 7, 8].

$$\begin{aligned} a &= -(1 - 4\sin^2\theta_w) && ; \text{ vector} \\ b &= -1 && ; \text{ axial-vector} \end{aligned} \tag{9}$$

As Stan indicated, a is very close to zero, and b is simply -1.

Now the Standard Model also says more about the weak neutral current in Eq. [8]. It says that it is composed of two terms, one of which is a different isovector component of the same current that give rise to charge-changing semi-leptonic processes.

$$\mathcal{J}_\mu^{(o)} = J_\mu^{V3} + J_{\mu 5}^{V3} - 2\sin^2\theta_W \, (J_\mu^S + J_\mu^{V3}) \quad ; \text{ nuclear domain} \tag{10}$$

The second term, mixed in through the Weinberg angle, is just the electromagnetic current.

$$J_\mu^\gamma = J_\mu^S + J_\mu^{V3} \tag{11}$$

The isospin structure of the weak neutral hadronic current in the Standard Model is now explicit. These relations hold in what I will call the nuclear domain, a term which will be defined more precisely later in the talk. This is the same expression that Stan was talking about.

Given a T-matrix, one can calculate a cross section. We are here interested in the interference term between the weak and electromagnetic interactions. That interference term can again be characterized by two tensors. One tensor is the interference between the electromagnetic current and the vector part of the weak neutral current, and the second is the interference with the axial-vector part. If one assumes that the vector part of the weak neutral current is conserved (this is not essential here, it just makes life a little simpler) then the general structure of this tensor can again be exhibited just as in Eq. [2].

$$\begin{aligned} W_{\mu\nu}^{int} \; \alpha \; & \langle i | J_\nu^{(o)}(0) | f \rangle \langle f | J_\mu^\gamma(0) | i \rangle \\ & + \langle i | J_\nu^\gamma(0) | f \rangle \langle f | J_\mu^{(o)}(0) | i \rangle \\ = {} & W_1^{int} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ & + W_2^{int} \frac{1}{M_T^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \end{aligned} \tag{12}$$

We can also exhibit the general form of the second tensor.

$$W_{\mu\nu}^{V-A} \;\alpha\; \langle i | J_{\nu 5}^{(o)}(0) | f \rangle \langle f | J_{\mu}^{\gamma}(0) | i \rangle$$

$$+ \langle i | J_{\nu}^{\gamma}(0) | f \rangle \langle f | J_{\mu 5}^{(o)}(0) | i \rangle \qquad (13)$$

$$= W_8^{int} \frac{1}{M_T^2} \epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma$$

Since this involves an axial current, it must be a pseudotensor; and there is only one pseudotensor which can be constructed from p and q.

Given these tensors, the cross section can be computed. In particular, one can calculate a general relation for the electron scattering asymmetry, which is the difference in cross section for the scattering of right- and left-handed longitudinally polarized electrons (Figure 5) divided by the sum [1].

$$\left[\frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}\right] \cdot \left[W_2^\gamma \cos^2\frac{\theta}{2} + 2W_1^\gamma \sin^2\frac{\theta}{2}\right] =$$

$$\left\{ b\left[W_2^{int}\cos^2\frac{\theta}{2} + 2W_1^{int}\sin^2\frac{\theta}{2}\right] \right. \qquad (14)$$

$$\left. -a\left(\frac{2W_8^{int}}{M_T}\right)\sin\frac{\theta}{2}\left(q^2\cos^2\frac{\theta}{2} + \vec{q}^{\,2}\sin^2\frac{\theta}{2}\right)^{1/2} \cdot \right\} \frac{Gq^2}{4\pi\alpha\sqrt{2}}$$

Just as in electron scattering, there are three independent electron variables $\{q^2, q.p, \theta\}$. The target response surfaces are functions of q^2 and q.p. If those two variables are kept fixed, one can, in principle, separate the terms with a Rosenbluth-like plot against θ. Only the V-A structure of the weak neutral currents has been assumed, and there are three target response functions that one can obtain from a general asymmetry measurement.

Let us discuss this result. The first comment is the one just made; the structure functions are functions of q^2 and q.p. One can separate them by looking at the θ dependence. If one looks at a transition to a discrete state of the target, then again the structure functions are functions only of q^2; all other factors cancel in the ratio and only form factors appear in this expression. Steve Pollock is going to talk this afternoon about the expressions one gets for elastic scattering from the nucleon. What has been assumed in the derivation of Eq. [14]? A V-A structure for the weak neutral currents has been assumed, as has Lorentz invariance, one-photon exchange, and conservation of the vector weak neutral current (the latter is not an essential assumption). Two-photon exchange gives a correction of $O(\alpha)$ to this result. Good parity has been assumed for the hadronic structure. That is, since one is looking at an interference term which is proportional to the weak interaction, parity can be used to characterize the nuclear matrix elements in evaluating this term.

What good is the expression in Eq. [14]? Well, I can use it to study the structure of the hadronic weak-neutral-current interaction. The nucleus can be used as a laboratory to select and study various pieces of the current. For example, nuclear isospin selection rules can be used to select various pieces of the weak neutral current. Angular momentum selection rule can also be used to select various multipoles and pieces of the current. Furthermore, one can, in principle, use that expression to measure a and b.

A few other obvious comments are worth keeping in mind. Only the parity-violating part of the weak-neutral-current interaction can be measured in this experiment. There is a parity-conserving part of the weak-neutral-current interaction which is always going to be masked at our energies. The only way to get at that part in the nuclear domain is to do neutrino experiments. The electron-scattering asymmetry can therefore never replace neutrino experiments.

We now have the <u>ratio</u> of the two form factors characterizing the electromagnetic and weak-neutral currents. A priori, these currents have nothing to do with each other, and the two form factors characterizing the distributions of electromagnetic and weak-neutral charge would have nothing to do with each other. And think for a minute about the hadronic point of view where one tries to explain the charge distribution in a nucleus like ^{40}Ca in terms of sub-nucleonic hadronic degrees of freedom. It becomes a very complex situation; charge is carried by mesons and baryons. The weak-neutral charge is also carried in sub-nucleonic hadronic degrees of freedom. It is also carried by mesons and baryons. A priori those things have absolutely nothing to do with each other.

On the other hand, let us go to the Standard Model in the nuclear domain where we know that the weak nuclear current has the structure exhibited in Eqs. (10-11). Suppose one has elastic scattering from a nucleus with isospin 0 like ^{12}C or ^{40}Ca. Only the isoscaler part of the weak nuclear current can then contribute. The only place you have an isoscaler piece in the weak neutral current in the Standard Model shown in Eqs. (10-11) is in the electromagnetic current itself. The weak neutral current is precisely proportional to the isoscaler current in this case and hence to the electromagnetic current itself. That means that the weak neutral current and the electromagnetic current are precisely proportional for these transitions.

$$\begin{aligned} \mathcal{J}_{\mu}^{(o)} &\doteq -\, 2\sin^2\theta_w J_{\mu}^{S} \\ &\doteq -\, 2\sin^2\theta_w J_{\mu}^{\gamma} \end{aligned} \tag{17}$$

This, in turn, implies that the form factors are precisely proportional.

$$F_o^{(o)}\ (q^2) = -\, 2\sin^2\theta_w\ F_o^{\gamma}(q^2) \tag{18}$$

Which means that the ratio of form factors required in Eq. (16) is simply a constant. The expression for the asymmetry is then [9, 1].

$$\mathcal{A} = \frac{Gq^2}{\pi\alpha\sqrt{2}} \sin^2\theta_w \qquad \text{(Feinberg)} \qquad (19)$$

At Bates there is an experiment underway to measure this quantity for elastic scattering from ^{12}C.

Now Eq. (18) is really a remarkable result when you think about it. What does it say? This relation holds at all momentum transfers, hence it holds at all distance scales. Think in terms of the Fourier transform, the larger the momentum transfer, the shorter the distance scale one is probing. So what does the equality in Eq. (18) mean? It means that these two form factors, or equivalently the distribution of the electromagnetic charge and the distribution of the weak neutral charge, are precisely the same no matter at what distance scale you probe the system. This equality holds at long wavelengths, or low momentum transfers, where one sees only the gross features of nuclear structure; it holds at shorter distances or higher momentum transfers, where one sees neutrons and protons; it holds at still shorter distance scales or still higher momentum transfers, where one sees sub-nucleonic hadronic degrees of freedom, that is, where one sees the hadronic exchange currents; and, finally, it holds all the way down to the distance scales where you see quarks. Those two charge distributions are identical in all the complicated nuclear degrees of freedom. Let me put it another way - in a more dramatic fashion. If one could measure the parity-violating form factor for ^{48}Ca through this experiment, one should see precisely the result exhibited in Figure 1. What has been assumed? I have assumed Lorentz invariance, isospin invariance, and the Standard Model. I mean, it is really a spectacular result. Nowhere in this expression is there any nuclear structure.

Nowhere does it matter where the charge is distributed in that system. Nowhere does it matter what this diffraction pattern looks like. It is just spectacular!

Isospin invariance has been assumed. How good is this in the ground state of carbon? It is probably good to a small fraction of a percent for this state in carbon. I do not know anything about the q^2 dependence of the isospin breaking. There are several corrections at the 1% level, some of which are very difficult to calculate.

Let us take another example. Let us look at an inelastic transition to an unnatural parity state where the initial state has isospin T=0 and the final state T=1 (Figure 6). An isovector transition selects the isovector piece of the weak neutral current, and the parity-violating asymmetry can then be written as a known term, plus another term which now depends on nuclear structure.

$$\mathcal{A} = \frac{Gq^2}{2\pi\alpha \quad 2} \left\{ -(1-2\sin^2\theta_W) + \zeta_{ns}(q^2)(1-4\sin^2\theta_W) \frac{\sin\frac{\theta}{2}\left(\frac{q^2}{\vec{q}^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)^{1/2}}{\left[\frac{q^2}{2\vec{q}^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right]} \right\} \tag{20}$$

The nuclear structure enters in the ratio of the axial vector electric matrix element to the corresponding electromagnetic magnetic transition amplitude.

$$\zeta_{ns}(q^2) = \mathrm{Re}\left[\frac{\langle J||T_J^{el5}(q)||0\rangle}{\langle J||T_J^{mag}(q)||0\rangle}\right] \tag{21}$$

Again, the nuclear state has here been assumed to be an eigenstate of parity. There is now one function characterizing the nuclear structure, and again in an experiment one can, in principle, separate this term by keeping q^2 fixed and varying the electron scattering angle θ. In fact, these two are measurable. Again, the first term is a known expression independent of nuclear structure in the Standard Model. Consider a specific example: let us look at the excitation of the 15.1 MeV 1^+,1 level in ^{12}C (Figure 7). First let me show you what the magnetic dipole form factor looks like for this transition in ^{12}C in Figure 8 [1]. This is the square of the magnetic dipole matrix element.

$$F_T^2 = |\langle 1^+ || T_1^{mag}(q) || 0^+ \rangle|^2 \tag{22}$$

The solid curve is based on a very simple model. If one assumes that ^{12}C is a closed p 3/2 - shell and that this is a particle-hole excitation in the p- shell (Figure 9), then the form factor for that transition is just the single-particle matrix element of the magnetic dipole operator, which one can calculate within the traditional framework of nuclear physics and electron scattering [1, 10]. In fact, that picture is too simple. If one does an open-shell RPA calculation, then that single-particle matrix element is essentially reduced by a factor ξ.

$$\langle 1^+,1 \vdots\vdots \hat{\mathcal{T}}_{1,1}(q) \vdots\vdots 0^+,0 \rangle = \frac{1}{\xi} \underbrace{\langle (1p_{1/2}) \tfrac{1}{2} \vdots\vdots \mathcal{T}_{1,1}(q) \vdots\vdots (1p_{3/2}) \tfrac{1}{2} \rangle}_{\longleftarrow \text{ traditional nuclear physics } \longrightarrow} \tag{23}$$

And the solid curve in Figure 9 is calculated within the traditional nuclear physics approach, using harmonic oscillator wave functions with an oscillator parameter fit to elastic charge scattering, and the parameter ξ is varied to obtain the solid curve [1]. Now I can use exactly the same approach to calculate the transition matrix element of the axial vector current which, in this simple approximation, is just proportional to the spin matrix element for the nucleus. So now one can calculate the parity-violating asymmetry within exactly the same approach. Figure 10 is a plot of the figure-of-merit, which is the square of the asymmetry times the cross section. Other people will speak in more learned fashion about the figure-of-merit. It is relevant because in order to measure the difference in cross sections one has to measure the spin-up and spin-down cross sections individually to a specified statistical accuracy. The dashed curve in Figure 10 is the figure-of-merit for this inelastic transition in ^{12}C [11]. Note that there is now interesting nuclear structure in this figure-of-merit.

Now let me come back to the point I mentioned before. In fact, nuclear states are not eigenstates of parity if the weak interactions are fully taken into account. In the hadronic sector of the weak interactions, there is a parity-admixing piece. The role of this term in the nuclear domain is model dependent. The semi-leptonic interaction can be handled in a model-independent fashion, but one must make a model to analyze the role of the purely hadronic weak interaction. The solid curve in Figure 10 is the result of a very simple model calculation. This was done by Brian Serot as part of this thesis. He took a one-π exchange potential, and made one vertex parity-conserving and the other parity-violating one-π vertex was taken from parity experiments that have been done in nuclear physics. Given this potential, one can do perturbation theory and every nuclear level will have a small wrong-parity piece mixed in. Now in addition to the interference term between γ and Z° exchange (Figure 4) there will be a familiar electromagnetic multipole transition matrix element that connects the different-parity pieces of the wave function. The solid curve in Figure 10 is the calculation of Serot of that piece for this particular transition in ^{12}C for 1GeV electrons [11]. Over most of the region of q^2 it is a completely negligible affect. It is also true that this contribution falls rapidly with q^2 whereas the interference term relatively grows with q^2. So over most of the regime, at least for this case, admixing wrong-parity states in the nuclear wave function does not effect the previous interpretation over the relevant range of q^2 to several orders of magnitude. However, if one completely measures this figure-of-merit as a function of q^2, it is evident from Figure 10 that one has the possibility of, in fact, extracting that parity admixture in the nuclear wave function from this same experiment! You have to do this at very low q^2 where this contribution is the dominant term; you have to measure at very forward angles.

For the remainder of this talk I want to concentrate on quarks and quantum chromodynamics (QCD) [12]. Although you do not see quarks as asymptotically free particles, if one introduces quarks as fields or particles and assigns the quantum numbers indicated in Table 1, where the first column is nuclear isospin, the second the third-component of nuclear isospin, the third charge, the fourth baryon number, the fifth and sixth strangeness and charm, and the final hypercharge, then one can explain the existing multiplets and supermultiplets of hadrons. The baryons are assumed to have the quantum numbers of the three quark system (qqq) and the mesons have the quantum numbers of bound $(q\bar{q})$ systems.

Furthermore, if one assumes that the electroweak currents can be constructed from point quark Dirac fields, then a marvelously simple and predictive description of these currents is obtained. This is an amazing assumption - to assume that one can construct the currents from point Dirac fields of the quarks. What is the electromagnetic current, for example? Well it is simply the Dirac current for each of the quark fields, multiplied by the electromagnetic charge on the quarks.

$$J_\mu^\gamma = i\left[\underline{\underline{\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d}} - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c\right] \tag{24}$$

The charge-changing weak current is also constructed from the quarks.

$$\begin{aligned} \mathcal{J}_\mu^{(+)} &= \underline{i\bar{u}\gamma_\mu(1+\gamma_5)\ \left[d\cos\theta_c\right.} + s\sin\theta_c\Big] \\ &\quad + i\bar{c}\gamma_\mu(1+\gamma_5)\left[-d\sin\theta_c + s\cos\theta_c\right] \end{aligned} \tag{25}$$

The first term takes a down (d) to an up (u) quark, the second a strange (s) to a u quark, and the third and fourth take d and s quarks to a charmed (c) quark. These terms happen to appear in this particular combination. It is slightly screwy. They are mixed up a little bit by what is called the Cabbibo angle θ_c. The weak neutral current in this Standard Model is composed of terms that are diagonal in the "flavors" of the quarks; there are no off-diagonal quark terms in that weak neutral current.

$$\mathcal{J}_{\mu}^{(o)} = \frac{i}{2}\Big[\underline{\underline{\bar{u}\gamma_{\mu}(1+\gamma_5)u - \bar{d}\gamma_{\mu}(1+\gamma_5)d}} - \bar{s}\gamma_{\mu}(1+\gamma_5)s + \bar{c}\gamma_{\mu}(1+\gamma_5)c\Big] - 2\sin^2\theta_w \underline{J_{\mu}^{\gamma}} \tag{26}$$

The last term is our old friend the electromagnetic current which is mixed in through $\sin^2\theta_w$. Now I have underlined the terms that involve the u and d quarks in Eqs. (24-26). The other terms involve the heavier strange s and charmed c quarks.

Let me now more precisely define what I call the <u>nuclear domain</u>. The nuclear domain is that sector of the Hilbert space that contains only u and d quarks and any numbers of <u>pairs</u> of those quarks. One can clearly make all non-strange and non-charmed baryons and mesons in that domain.

Let me make another crucial point. QCD is a local gauge theory based on color as an internal intrinsic degree of freedom. It is an internal degree of freedom like isospin. QCD is a local gauge theory of the strong interactions binding quarks into the observed hadrons. The force in this theory is mediated by gluons, which are massless quanta similar to the photon in quantum electrodynamics (QED). In the Standard Model of the strong and electroweak interactions based on $SU(3)_c \times SU(2)_w \times U(1)_w$, the gluons are absolutely neutral to the electroweak interaction. That is, when you go to QCD, the electroweak currents are still given by Eqs. (24-26) (summed over quark colors). This is a crucial point. These are still the electroweak currents in the Standard Model.

Let us work in the nuclear domain where one has only up (u) and down (d) quarks. Let us combine those up and down quarks into a field Ψ.

$$\Psi \doteq \binom{u}{d} \quad ; \text{ nuclear domain}$$
$$G^{(\pm)} \equiv G\cos\theta_c \tag{27}$$

This field transforms as an isodoublet under isospin. Let us further combine the Cabbibo angle with the Fermi constant to give an effective charge-changing weak coupling constant $G^{(+)}$ as indicated in Eq. (27); since $\cos\theta_c \simeq 0.97$ this is a 3% effect. The electromagnetic current can be rewritten in terms of this isospiner Ψ in the following form.

$$J_\mu^\gamma = i\left[\bar{\Psi}\gamma_\mu \frac{1}{2}\tau_3\Psi + \frac{1}{6}\bar{\Psi}\gamma_\mu\Psi\right] \tag{28}$$

And now you recognize the first quantity as the third component of an isovector and the second as an isoscaler. So the electromagnetic current has precisely the structure we assumed before in the Standard Model. The first piece of the charge-changing weak current can also be written in this fashion.

$$\mathcal{J}_\mu^{(\pm)} = i\bar{\Psi}\gamma_\mu(1+\gamma_5)\tau_\pm\Psi \tag{29}$$

You can see that these are simply the raising and lowering components of an isovector operator. And the first part of the weak neutral current can similarly be written in terms of the field Ψ in the following fashion.

$$\mathcal{J}_\mu^{(0)} = i\bar{\Psi}\gamma_\mu(1+\gamma_5)\frac{1}{2}\tau_3\Psi - 2\sin^2\theta_W J_\mu^\gamma \tag{30}$$

You can see that the first term is the third component of an isovector; it is the sum of a Lorentz vector and axial-vector current. The second term is the same electromagnetic current that appeared in Eq. (27).

So now within the nuclear domain, and within the Standard Model, the electroweak currents have all the transformation properties assumed previously in Eqs. (10-11). And all the results we obtained on identity of form factors, and on cancellation of form factors in ratios, depended only on the general structure of the currents.

Let me just note in passing that this analysis neglects the following piece of the current.

$$\delta \mathcal{J}_{\mu}^{(o)} = \frac{i}{2}\left[-\bar{s}\gamma_{\mu}(1+\gamma_{5})s + \bar{c}\gamma_{\mu}(1+\gamma_{5})c\right] \tag{31}$$

It is a piece of the current that involves strange quarks and charmed quarks. It is a pure isoscaler because charmed and strange quarks have no nuclear isospin. It is a pure isoscaler that depends on the presence of heavy quarks. Now one knows that there is no net strangeness or charm in the nucleus to a very good approximation, but there may be pairs of these quarks in the nucleus. There are, in fact, "sea-quarks" in the nuclear system, and thus, to a certain extent, there are $\bar{s}s$ pairs in the nucleus. From the hadronic point of view, if there are any ϕ mesons present, then to the extent that these are $\bar{s}s$ pairs, one has an additional mechanism for introducing these pairs in the nuclear system. So this is also a correction to everything said previously. I expect its role to be comparable to that of isospin breaking, but this is just a gut feeling which I cannot back up with quantitative calculations.

Let me say a little about the SLAC parity-violation experiment [13]. Vernon Hughes was intimately involved in that experiment. It was an experiment on deuterium (^{2}H) in the deep-inelastic region [14]. What do I mean by the deep inelastic region? It is the region where q^2 gets very large, and the energy loss ν gets very large.

$$q^2 \to \infty \quad ; \quad x \equiv \frac{q^2}{2m\nu} \quad \underline{\text{fixed}} \tag{32}$$
$$\nu \to \infty$$

This is the so-called scaling region where the ratio is fixed. In fact, in this region one is pulling the nucleus apart. We have just blown it to pieces; hadrons are coming out. In this regime the theory of QCD is asymptotically free [12]. What does that mean? It means that the strong interactions become very weak. At short-distances, or at very high momenta, the strong interaction becomes very weak, and, in fact, in the SLAC experiments, the strong interaction provides a perturbative correction to the interpretation in terms of simple non-interacting quarks.

To make life simple, let us assume that the electron scattering angle is small as in the SLAC experiment, that means $\theta \to 0$. Let us also assume that $\sin^2\theta_w \equiv 1/4$ in which case the nuclear axial-vector term drops out of Eq. (14).

$$\theta \to 0$$
$$a = -\ (1-4\sin^2\theta_w) \equiv 0 \tag{33}$$

So life is really simple, in fact all we need now to get the parity-violating asymmetry is the ratio of the (νW_2) for the interference term between the electromagnetic and vector weak neutral current and for the electromagnetic current itself. That is all that is left in the general formula.

Let us work in the nuclear domain, which I have now precisely defined. Let us further assume that the proton is made up of three quarks, (uud) and any number of gluons as shown in Figure 11. Recall that it is the nonlinear interaction of the gluons that gives rise to confinement, and nobody really knows how to do this part of the problem.

Let us use the quark-parton model for the required ratio of the structure functions [14]. I am not going to go into detail on the quark-parton model, but the important point is that if all you what is a ratio, and if the gluons are absolutely neutral, then all of the gluon dynamics cancels in the ratio, and the only thing that enters is the ratio of the charges of the quarks.

$$\left[\frac{\nu W_2^{int}}{\nu W_2^{\gamma}}\right]_{{}^2H} = \frac{\langle 2\sum_i Q_i^{\gamma} Q_i^{o}\rangle^{n}_{quark} + (n \rightleftharpoons p)}{\langle \sum_i (Q_i^{\gamma})^2\rangle^{n}_{quark} + (n \rightleftharpoons p)} = \frac{4}{5} \tag{34}$$

So all one has to do now is to look up the electromagnetic and weak neutral charges of the quarks, which one can get from Table 1 and Eqs. (28, 30). You add the contribution from the neutron and the proton incoherently in the theoretical analysis of this experiment. This expression tells us what the parity-violating asymmetry for the deuteron in the SLAC experiment should be.

$$\mathcal{A}_{{}^2H} = -\frac{Gq^2}{4\pi\alpha\sqrt{2}} \cdot \frac{4}{5} \tag{35}$$

Figure 12 compares this result with the results of the SLAC experiments. This is the world's supply of data on the asymmetry measured at SLAC for deep inelastic scattering from deuterium. The theoretical result in Eq. (35) depends solely on q^2; after you divide by q^2 it has no dependence on the other variables in the problem, for example, on the electron scattering energy loss. The expression in Eq. (35) clearly provides a very good representation of the data.

So what do we learn from all of this? I just want to draw my conclusions.

What does the nucleus look like? Figure 13 gives a schematic sketch of the nucleus. The nucleus is a collection of hadrons with a fixed baryon number. The baryons are confined quark triplets. It contains, in addition, sub-nucleonic hadronic degrees of freedom; there are mesons present in the nuclear system. These represent quark-antiquark pairs. Each of these hadrons is confined by the nonlinear interaction of the gluons.

If I consider an electroweak interaction with this system, through the exchange of a photon, a Z°, or a $W^\pm$, the electroweak interaction sees only the quarks. The gluons are absolutely neutral. Electroweak interactions are also colorblind; the color of the quarks does not matter.

Now you can see the reason for the simplicity of those previous relations on the currents and on form factors. It does not matter if it is a photon, or a Z°, or what have you; they all see the quarks. They see the same quark distribution, no matter how complicated it is, whether in mesons or baryons, and the different electroweak interactions simply look at different isospin components of those currents. These have precisely the same spatial distribution, no matter how it is distribution in the hadronic degrees of freedom. From the nuclear physics point of view, that is a miracle. From a high-energy physics point of view, it is obvious! It does not matter how complicated the hadronic structure is for those relations to hold.

So in <u>summary</u>: Why do I like the parity experiments? I like the parity experiments because they test the full structure of the Standard Model. That is, they test the structure of the strong interactions, the local gauge theory QCD based on color, and the unified gauge theory of the electroweak interactions. They test the full theory of the Standard Model based on $SU(3)_c \times SU(2)_W \times U(1)_W$ in the <u>nuclear domain</u> where the strong interactions are strong, and not asymptotically free. And these are truly <u>nuclear physics</u> experiments, for to quote Nathan Isgur;

"Nuclear physics is the study of the strong interaction, confinement aspects of QCD". [N. Isgur, CEBAF Summer Workshop (1984)].

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FIGURES

Fig. 1 Kinematic situation for target response in semileptonic processes.

Fig. 2 Elastic (e,e) cross section for ^{40}Ca vs. momentum transfer. The scattering here is from the charge distribution.

Fig. 3 Experimental charge density with estimated uncertainty from elastic electron scattering (solid lines and shaded area) and relativistic Hartree calculation of this quantity within the framework of QHD (heavy dashed line). Taken from Ref. [4].

Fig. 4 Parity violation in electron scattering (e,e′).

Fig. 5 Scattering of right- and left-handed longitudinally polarized electrons. The particle-violating asymmetry depends on the difference of these cross sections.

Fig. 6 Unnatural parity, isovector nuclear transitions.

Fig. 7 Magnetic dipole, isovector transition in ^{12}C.

Fig. 8 Transverse magnetic dipole form factor squared [Eq. (22)] for the $0^+,0 \rightarrow 1^+,1$ (15.11 MeV) state in ^{12}C [1].

Fig. 9 Simple particle-hole picture of the transition in Figure 7.

Fig. 10 Figure of merit for parity-violating asymmetry for $0^+,0 \rightarrow 1^+,1$ (15.11 Mev) transition in ^{12}C (dashed curve). The solid curve shows the contribution of the wrong-parity admixtures in the wave functions. These calculations are due to Serot [11]. An incident electron energy of 1 GeV is assumed here.

Fig. 11 Model of the nucleon.

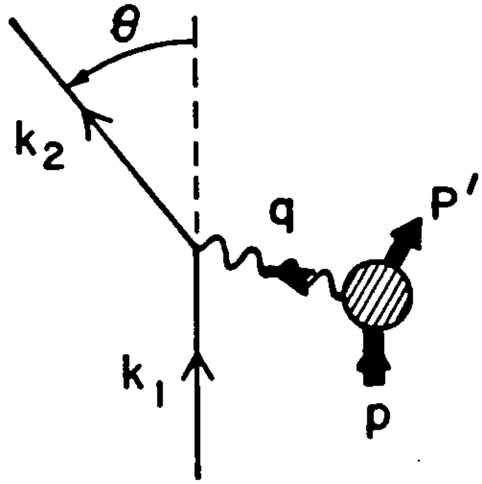
Fig. 12 Result in Eq. (35) compared with SLAC data for parity-violation asymmetry in deep inelastic (e,e′) from ^2H [13].

Fig. 13 Picture of the nucleus in the Standard Model.

Tables

Table I. Quark quantum numbers

Field/Particle	T	T_3	Q	B	S	C	Y=B+S+C
u	1/2	1/2	2/3	1/3	0	0	1/3
d	1/2	-1/2	-1/3	1/3	0	0	1/3
s	0	0	-1/3	1/3	-1	0	-2/3
c	0	0	2/3	1/3	0	1	4/3



$$q^2 = (k_2 - k_1)^2$$
$$\nu = p \cdot q / M_T$$
$$\theta$$

Figure 1. Kinematic situation for target response in semileptonic processes.

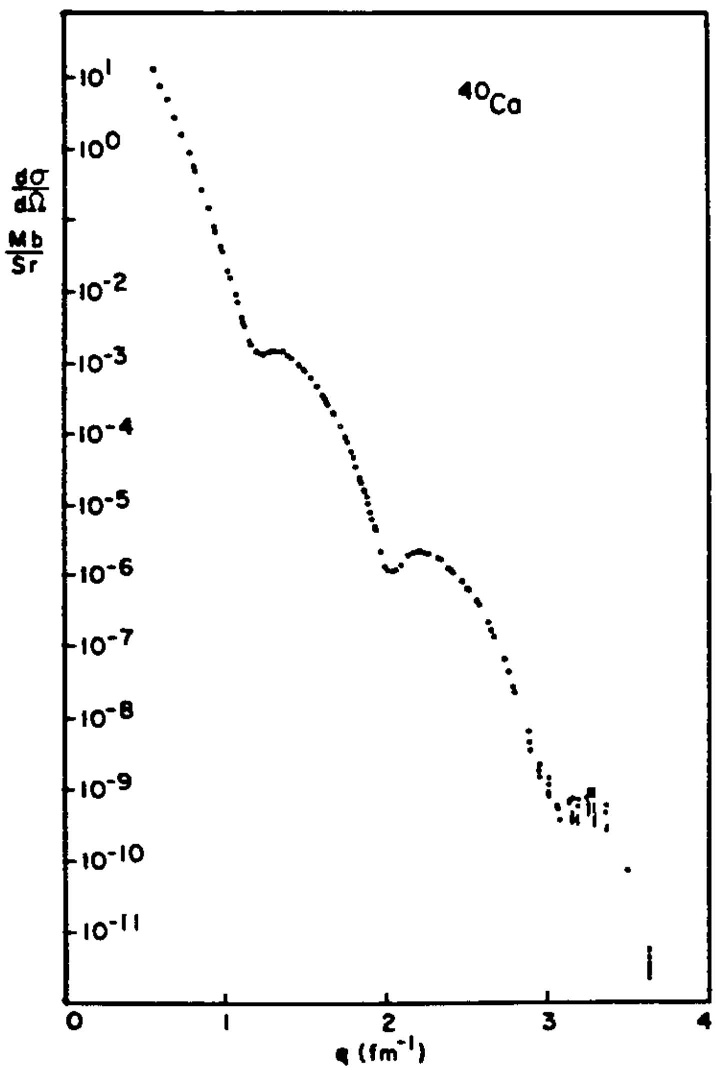


Figure 2. Elastic (e,e) cross section for ^{40}Ca vs. momentum transfer [2]. The scattering here is from the charge distribution.

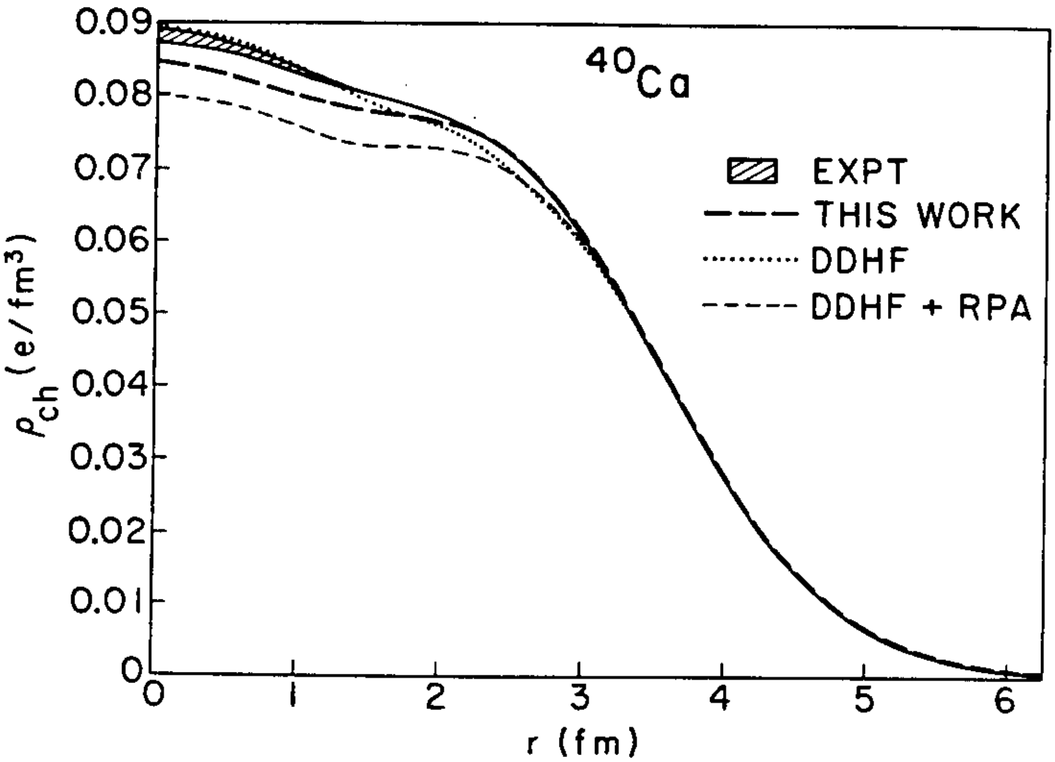


Figure 3. Experimental charge density with estimated uncertainty from elastic electron scattering (solid lines and shaded area) and relativistic Hartree calculation of this quantity within the framework of QHD (heavy dashed line). Taken from Ref. [4].

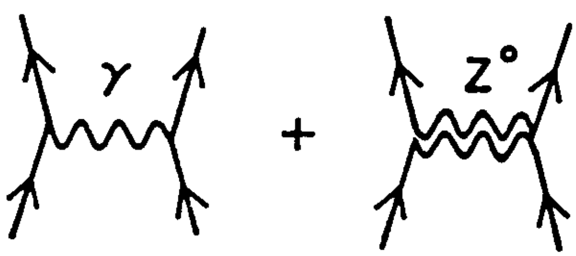


Figure 4. Parity violation in electron scattering (e,e').

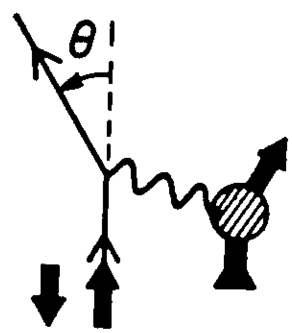


Figure 5. Scattering of right- and left-handed longitudinally polarized electrons. The parity-violating asymmetry depends on the difference of these cross sections.

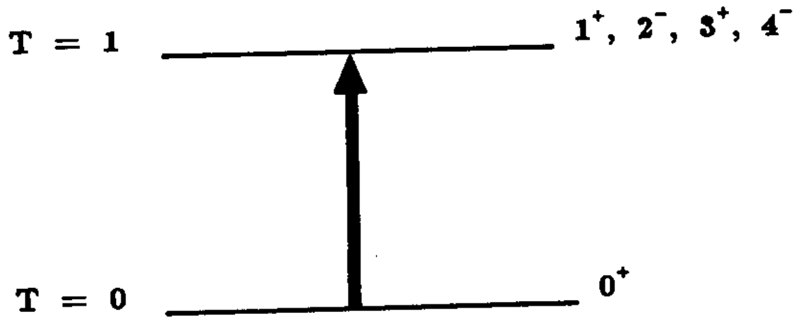


Figure 6. Unnatural parity, isovector nuclear transitions.

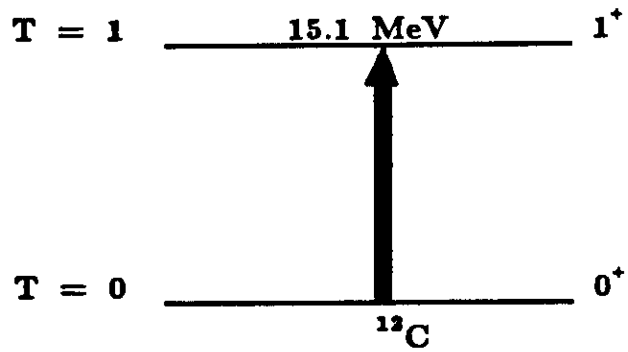


Figure 7. Magnetic dipole, isovector transition in ^{12}C.

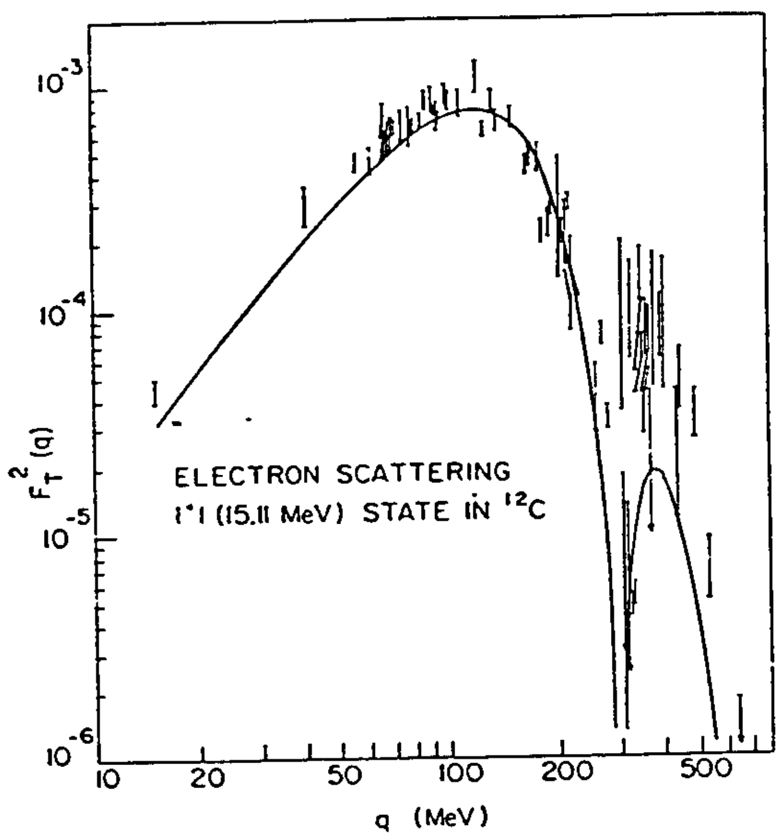


Figure 8. Transverse magnetic dipole from factor squared [Eq. (22)] for the $0+,0 \rightarrow 1+,1$ (15.11 MeV) state in ^{12}C [1].

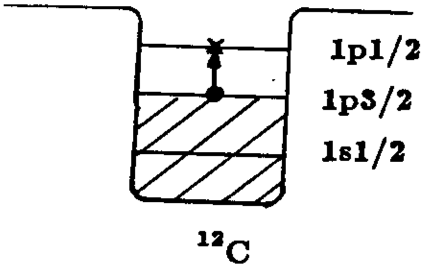
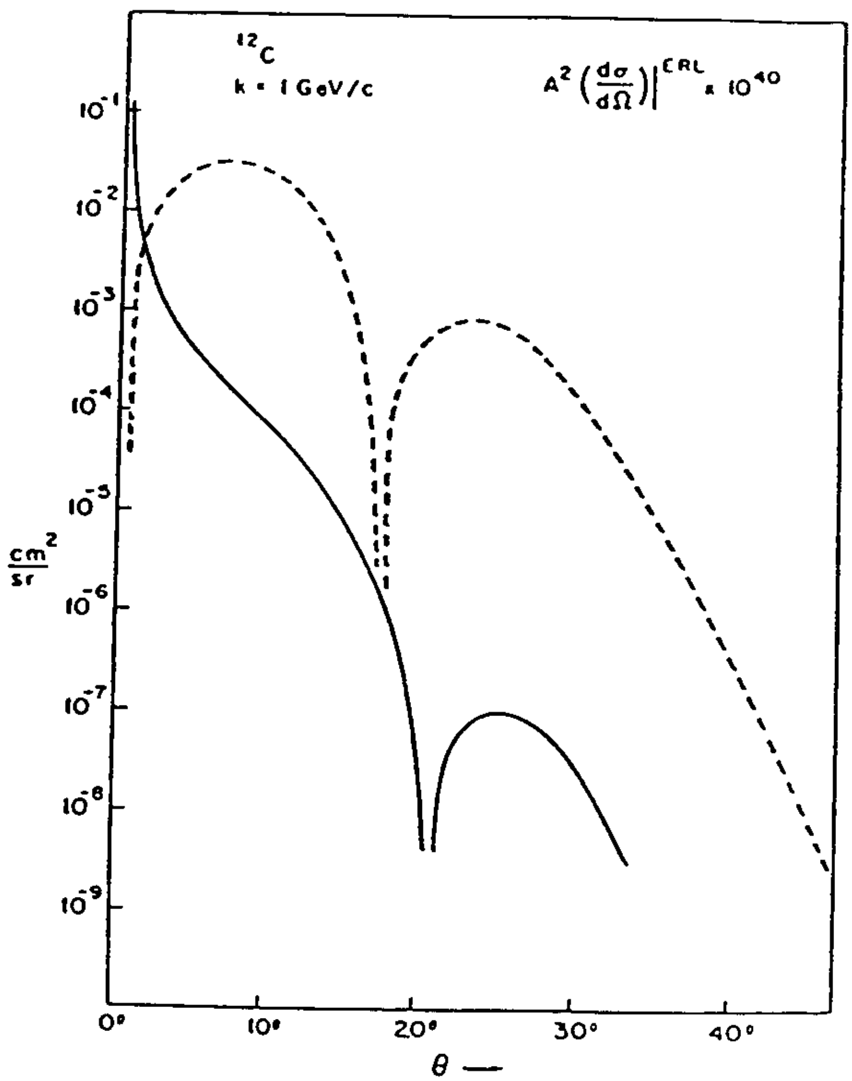


Figure 9. Simple particle-hole picture of the transition in Figure 7.



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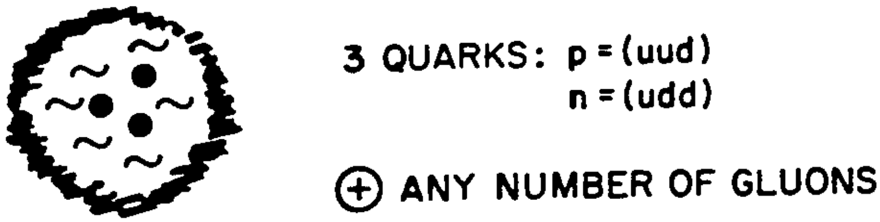
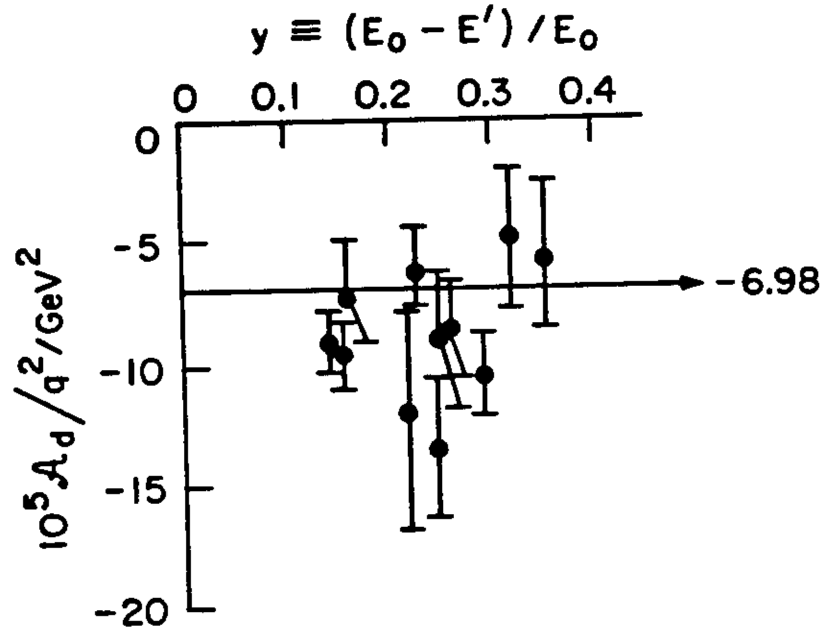


Figure 11. Model of the nucleon.



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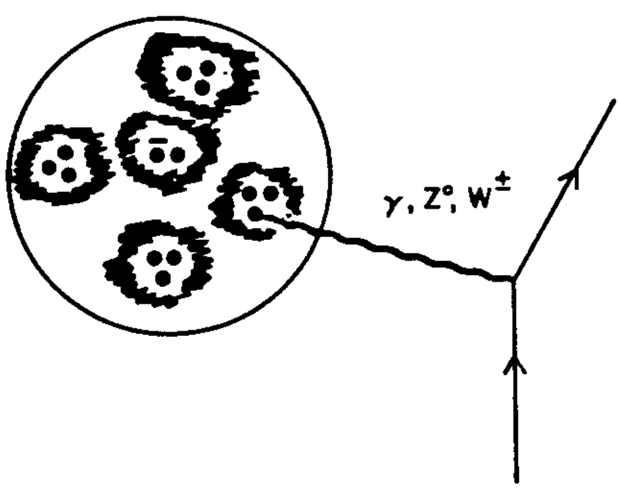

Figure 13. Picture of the nucleus in the Standard Model.