JLAB-ACC-00-01

Measurement and modeling of mirror distortion in a high power FEL

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ABSTRACT

Mirror heating in a high power FEL can alter the optical mode and affect the gain of the laser. This can lead to a large reduction of the laser power from ideal values. Measurements of the power and mode size in the Jefferson Lab IR Demo laser have shown clear evidence of mirror distortion at high average power loading (up to 17 kW incident on the mirrors and over 40 W absorbed per mirror). The measurements and comparisons with modeling will be presented. Both steady state and transient analyses and measurements are considered.

Keywords: accelerator, FEL, infrared, laser, superconducting, resonators

1. INTRODUCTION

The free-electron laser (FEL) is a potentially extremely bright source of coherent light. Part of the reason for this is the high energy density in the gain medium. The average and peak power density in a high brightness electron beam may be hundreds of megawatts per square centimeter and tens of terawatts per square centimeter respectively. The circulating intensity of a FEL can approach these intensities. No optical materials can withstand such high peak or average intensities. The beam must therefore be allowed to expand via diffraction before being reflected off any mirror surface. The magnification of the optical mode *M,* defined as the ratio of the optical mode size at the mirrors to the mode size at the waist, must be on the order of 100 or more to keep the mirrors from being destroyed. Two approaches are available to provide for such a large magnification. The first is to use a negative branch confocal resonator in a ring configuration [1]. Output coupling in this case is accomplished using a scraper mirror. This design is scaleable to very high average power. It allows the use of non-transparent mirror substrates such as copper or silicon that have excellent thermal properties. It is also much less sensitive to steering and radius of curvature errors. Finally, an increase in the radius of curvature actually increases the mode spot size on the mirrors, which leads to a more stable behavior vs. laser power. It has disadvantages as well. The cavity length and mirror steering are coupled, leading to great difficulty in setting up the cavity to be the correct length and aligned with the electron beam at the same time. In addition, diffraction from the output coupler reduces the output coupling efficiency. The output mode can be focussed to a tight spot but it is an annulus and is difficult to transport. Finally, the number of mirror bounces per round trip is at least four, leading to higher absorption and scatter losses. The simpler approach, used in initial experiments at Jefferson Lab, is to use a nearly concentric stable resonator. The nearly concentric configuration is much simpler to align and has excellent output coupling efficiency since diffractive losses can be made negligible. These advantages are offset by a thermal runaway problem. When the mirrors heat up from the circulating laser light absorbed on the mirrors, they bulge out, increasing their radii of curvature. This increases the Rayleigh range and moves the cavity away from concentricity, thus reducing the mode size on the cavity mirrors. The high intensity from this arrangement causes the mirror to distort even more. This positive feedback can lead to very large distortion resulting in a runaway process. The nearly concentric cavity also has an extreme sensitivity to mirror steering and changes in the radii of curvature in general. As an example note that the change in Rayleigh range with changes in the radii of curvature is given by:

$$\frac{dz_R}{z_R} = \frac{M}{2}\frac{dR_c}{R_c} \tag{1}$$

Thus, for a magnification of 100, a 1% change in the radii of curvature results in a 50% change in the Rayleigh range. Due to the simplicity and ease of setup of the nearly concentric resonator configuration, however, we decided to use such a cavity for the kilowatt free-electron laser contructed at Jefferson Lab[2] referred to here as the IR Demo FEL.

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2. THE IR DEMO FEL

Initial performance estimates for the IR Demo FEL concentrated on estimating the power levels which could be maintained for various mirror substrates in steady state[3]. This analysis indicated that the coating absorption was critical and that sapphire was the best substrate for the output coupler, which needs to be highly transparent. The transparency of sapphire is high for wavelengths shorter than 4 µm. For longer wavelengths, ZnSe was found to be the best material. Calcium fluoride was also used due to its excellent optical and mechanical properties even though its thermal properties are a factor of ten worse than those of sapphire. The optical cavity parameters for the IR Demo FEL are given in table 1. The magnification of 101 was chosen as a compromise between mirror heating and resonator stability issues.

Table 1. Design parameters for the IR Demo optical resonator.

Parameter	Value
Length	8.0105 m
Mirror radius of curvature	404.5 cm
Rayleigh range	40 cm
Magnification	101
Mirror diameter	5 cm
Mirror tilt tolerance	$1.5\mu\text{rad}\sqrt{\lambda(\mu\text{m})}$
Typical Output coupler reflectivity	90%
HR reflectivity	>99.5%
Coating absorption	<0.1%

The design details of the accelerator have been reported in previous publications[4]. The accelerator can deliver up to 240 kW of continuous electron beam power. The laser efficiency is up to 0.7% at full power and twice that at low power. With a 40 cm Rayleigh range and 10% output coupling, the CW intensity at the cavity center could be as high as 2.8 MW/cm^2. With a magnification of 101 the intensity at the mirrors is only 28 kW/cm^2. The gain is difficult to measure but is of the order of 100% per pass. This is inferred from the fact that the laser efficiency does not drop greatly when the electron beam repetition rate is lowered to one-fourth the round trip frequency of the cavity. The threshold gain in this case is over 52%.

3. STEADY STATE MIRROR DISTORTION

In reference [3] it was shown that the mirror distortion in the case of a Gaussian absorption pattern on a mirror whose edge is held at a constant temperature has the form:

$$\delta z(\chi) = \frac{P_l}{8\pi F}\left[\gamma + \ln\left(\frac{2a^2}{w_m^2}\right) - 1.17R_{20}(\chi) + 0.44R_{40}(\chi) - 0.17R_{60}(\chi)\right] \tag{2}$$

where P_l is the output laser power, a is the mirror radius, w_m is the $1/e^2$ laser mode radius on the mirror, R_{mn} is the Zernike circle polynomial of radial order m and azimuthal order n, and γ is the Euler-Mascheroni constant, equal to 0.57722. The quantities F(the figure of merit for the mirror) and χ are defined by

$$\chi = \frac{r}{2w_m} \quad \text{and} \quad F = \frac{k_{th}}{\left(h\alpha_B + \alpha_s(1+1/t_c)\right)\alpha_e} \tag{3}$$

where k_{th} is the thermal conductivity of the mirror substrate, h is the mirror thickness, t_c is the output coupler transmission, α_B is the bulk absorption coefficient, α_s is the coating absorption (assumed to be the same for all coatings), and α_e is the thermal expansion coefficient of the mirror substrate. The quantity in parantheses in the denominator of the expression for F is the absorption of the mirror. From this solution, it is possible to calculate the change in the Rayleigh range and the aberration for a given set of mirrors, laser wavelength, and power output. In reference [3] it was assumed that only the output coupler is distorted by absorbed power. It was assumed that a better thermal material could be used for the high reflector. Here let us assume that both mirrors have the same distortion. This is done to better agree with the IR Demo experience where the same substrate is used for both mirrors. Note that the coating absorption may be higher for the high reflector but this is partially offset by the lack of absorption in the bulk and AR coating for the high reflector. Nevertheless, this is a possible error in the calculation. With these assumptions the change in the Rayleigh range is given by

$$\frac{\Delta z_R}{z_R} \cong \frac{1.17P_l}{16F\lambda}\frac{M}{\sqrt{M-1}} \tag{4}$$

Note how the change becomes larger for a larger magnification when the power, wavelength, and figure of merit are held constant. For a given magnification, wavelength, and allowable change in the Rayleigh range, the maximum power allowed can be easily calculated. Note that the ultimate change in the Rayleigh range will be larger than the value calculated by this formula when the change is large. This is due to the positive feedback of the mirror heating. For the IR Demo, for instance, a 60% initial change will lead to a 100% change after the full change is calculated self-consistently.

The aberration may be calculated from the higher order Zernike polynomial coefficients. The resulting equation can be rewritten in terms of the change in the Rayleigh range:

$$\frac{\delta z(0)}{\lambda} = 0.332\frac{\sqrt{M-1}}{M}\frac{\Delta z_R}{z_R} \tag{5}$$

Note that, for large magnification and moderate changes in the Rayleigh range, the aberration is much smaller than one wave. For a magnification of 101 and a 100% change in the Rayleigh range, the aberration is only 3% of a wave.

If we assume that the coating absorption in the coatings used in the IR Demo is 0.05% and use the thermal parameters for the mirrors quoted in the literature we find that a 60% change in the Rayleigh range occurs at 400 W using calcium fluoride at 5 microns, at 1900 W using ZnSe at 6 μm, and at 2650 W using sapphire at 3.1 microns. Note that the actual limit to the power might not be the change in the Rayleigh range. For sapphire, for example, the limit is the allowable power density on the mirrors. High average intensities have been shown to cause mirror damage at intensities exceeding 50 kW/cm^2 so we have defined a design limit of 50 kW/cm^2 for this resonator. This intensity is exceeded at around 2000 W so operation at 2650 W is not advisable. For ZnSe, the aberration due to heating in the output coupler exceeds 1/4 wave for power output higher than 1300 W. If good mode quality is required, this level of aberration should not be exceeded.

4. TRANSIENT MIRROR DISTORTION

When the laser first turns on there is a period during which the laser power has not had a chance to spread laterally. The mirror distortion therefore has the shape of the optical mode instead of the shape given in equation 2. This is the transient regime. The transient regime is present for times short compared to the time necessary for the heat to spread laterally by a mode radius. This time is given by:

$$t_0 = \frac{w_m^{\ 2}}{8\kappa} \tag{6}$$

where w_m is the mode $1/e^2$ waist at the mirror, and κ is the thermal diffusivity of the mirror subtrate. The temperature rise of a half plane on which a Gaussian mode is incident is given by the following equation[6].

$$T(r,z,t) = \frac{2P}{\rho C(4\pi\kappa)^{3/2}}\int_0^t \frac{1}{(t'+t_0)\sqrt{t'}}\exp\left[\frac{-z^2}{4\kappa t'} - \frac{r^2}{4\kappa(t'+t_0)}\right]dt' \tag{7}$$

The distortion versus time may be calculated by integrating this equation along the z coordinate. The integral over t' may then also be carried out. When the resulting equation is multiplied by the expansion coefficient we get

$$\alpha_e\int_0^\infty T(r,z,t)dz = \frac{P_{abs}\alpha_e}{4\pi k_{th}}\left[Ei\left(\frac{r^2}{4\kappa(t_0+t)}\right) - Ei\left(\frac{r^2}{4\kappa t_0}\right)\right] \tag{8}$$

where Ei is the tabulated exponential integral[5]. For times small compared with t_0 the expression in brackets can be approximated by the Gaussian mode on the mirror. For large times the term in brackets resembles the shape of equation [2]. The distortion at the mode center versus time is given by

$$\delta z(t) = \frac{P_{abs}\alpha_e}{4\pi k_{th}}\ln\left(1+\frac{t}{t_0}\right) \tag{9}$$

For small times, the Zernike coefficients will be different from those in equation (2). If the Gaussian mode is expanded in the same Zernike polymonials, the coeffciencts are

$$c_1 = -0.2831,\quad c_2 = 0.2747,\ c_3 = -0.1856,\text{ and } c_4 = 0.1103$$

If these are compared with those of equation 2, it is found that the aberration amplitude is about the same as for the steady state case. The curvature coefficient is smaller by a factor of 4. This means that the relative importance of radius of curvature effects and aberrations may be reversed for transient versus steady state operation. It is likely that aberrations are always the limiting factor for transient distortion while they are only a factor in steady state for nearly concentric cavities with small magnification.

Though we can calculate the change in the aberrations and Rayleigh range from a given power absorption on a mirror, it is as difficult to predict how this distortion affects the laser. The most definitive way to determine the effect is to do a full two dimensional simulation that includes the mirror distortion and cavity phase advances and calculates the saturated power. This can be done by a simulation code such as FELIX, developed at Los Alamos. Modeling a deep UV system using FELIX[7] found that the power drops rapidly with increasing wavefront distortion for a small mirror surface distortion per unit intensity. Let us define the mirror surface distortion per unit incident intensity as Δ. The quantity Δ is a measure of the mirror coating absorption or the expansion coefficient of the mirror substrate. For large values of Δ, the circulating power falls off sufficiently fast to keep the total wavefront distortion of the circulating mode constant with respect to Δ. This is shown in figure 1. It can also be a function of the repetition rate if one assumes that the right axis is the power per micropulse instead of the total power. Finally, the power vs. time will be similar to the power vs. Δ as long as the distortion is linearly dependent on time. The latter assumption is true for times small compared with the characteristic time of equation 6 (this can be seen by expanding equation 9 for small t/t_0).

One can also do a simple Fox and Li type analysis of the cavity using the commercial package PARAXIA with a phase error added to the cavity. When this is done for a cavity with parameters of the IR Demo FEL operating at 3 μm, the mode quality factor M^2 and the waist spot size increase as shown in figure 2. Clearly a distortion in the mirror of 1/10 of a wave is very deleterious to mode quality and should strongly affect the laser gain. On the other hand, a distortion of less than 1/20 wave should have quite a small effect on the laser. Note that the fact that the cavity is nearly concentric makes the total aberration worse. The aberrations from each of the two mirrors add linearly due to the degeneracy of the cavity. For a smaller magnification, the mirror distortion induced aberration will be larger (see equation 5) but the aberration from the two mirrors will not necessarily add linearly because the phase advance may alter or even reverse the wavefront distortion after propagating to the other mirror.

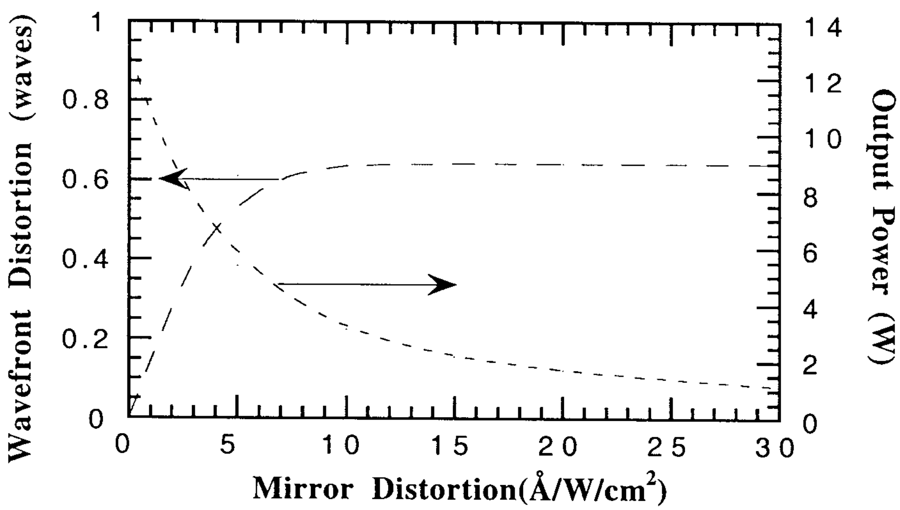

Figure 1. Wavefront distortion and output power versus the mirror distortion per unit intensity for a UV unstable ring resonator with scraper output coupling. The shape of the distortion was assumed to follow the mode shape.

Another question to answer is how much the Rayleigh range can change before the gain is greatly reduced. For a system like the IR demo the answer is that the Rayleigh range can change a great deal without strongly affecting the gain. This is shown in figure 3 in which the small signal gain is plotted as a function of Rayleigh range. The Rayleigh range must increase by a factor of four to decrease the gain by a factor of two. Note that, for this large a change in the Rayleigh range, the aberration will not necessarily be negligible. For a smaller magnification or in the transient regime the aberration will almost certainly dominate the reduction in laser gain.

Finally, one must remember that it is the saturated gain that is important in determining the output power. The dependence of the saturated gain on the small signal gain is a complicated function of the cavity length detuning and the gain to loss ratio. If one assumes the cold cavity parameters of the IR Demo and uses Dattoli's approximation for the saturated gain versus the gain to loss ratio[8], one finds the curves in figure 4 for the output power versus the ratio of the small-signal-gain to the saturated gain (equal to the inverse of one minus the losses). One can see that, for the cavity length detuning which produces the highest gain the output power is quite sensitive to the gain ratio. For more detuning, the output power is much less sensitive to the small signal gain.

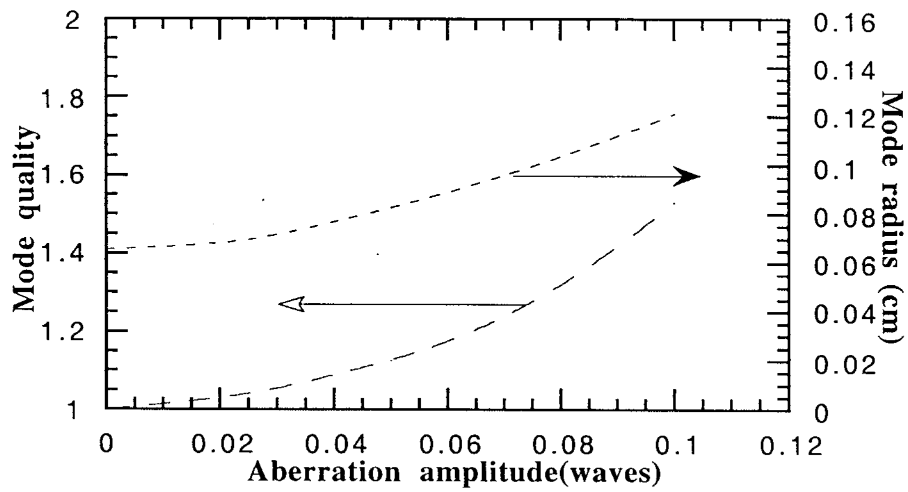

Figure 2. Mode quality M^2 and waist spot radius as a function of Gaussian aberration added to both mirrors. The total wavefront distortion will be four times this value.

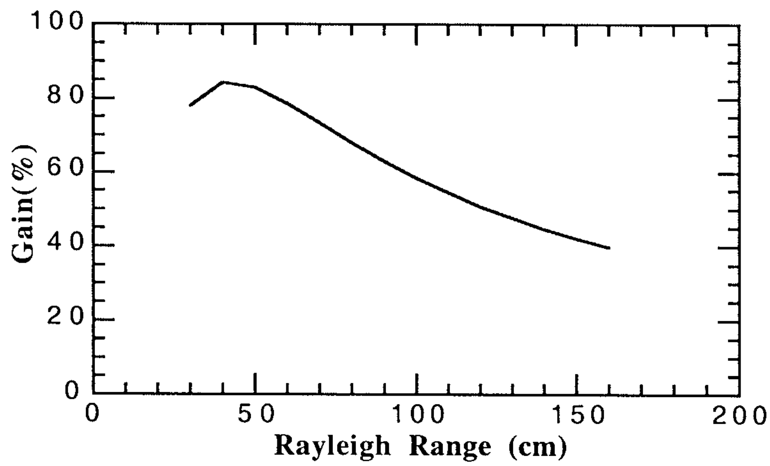

Figure 3. Small signal gain versus Rayeliegh range for the IR Demo parameters at 3 microns. The design Rayleigh range is 40 cm. The Rayleigh range must increase to 160 cm before the gain is halved.

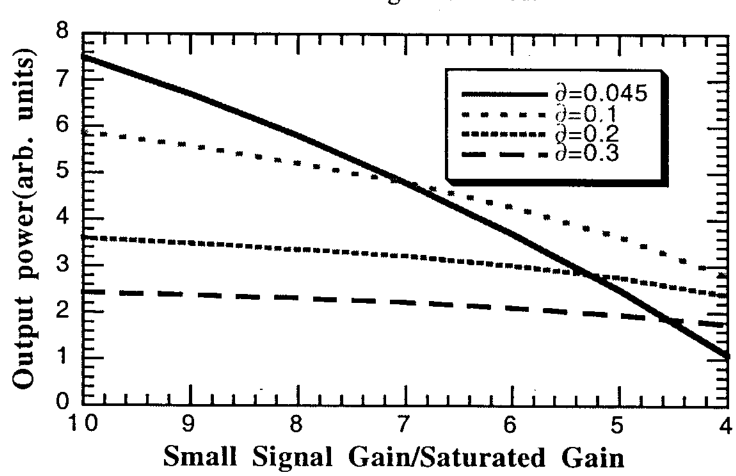

Figure 4. Output power versus the ratio of the small signal gain to the saturated gain for four different cavity length detunings. A detuning of 0.045 corresponds to the maximum in the detuning curve for a gain ratio of 10. Though the power is much higher for small detuning and high gain, it is not as high when the gain drops.

5. EXPERIMENTAL RESULTS

The IR demo has indeed shown mirror limited performance in both the steady state and transient regimes. Using a pair of mirrors with calcium fluoride substrates to achieve 5 μm lasing, we achieved 550 W of output power. The power was achieved with ~3 mA of beam current. Increasing the current above this value did not increase the power. This saturation behavior is similar to that shown in figure 1. Once a distortion is reached, the efficiency drops as the current increases such that the power is essentially constant. Note that the drop in efficiency was only seen at the peak of the detuning curve. When the efficiency versus current is plotted for a point one third of the way out the detuning curve, it agrees with the cold cavity theory as shown in figure 5. For the highest currents run at a that detuning, however, the power is still less than that at the peak of the detuning curve. The figure of 550 W is consistent with the mirror power limit of 400 W quoted above. Unfortunately no data was taken on the Rayleigh range and waist position versus power for this case. The inferred mirror coating loss is 0.04%. This value has a large uncertainty since more detailed simulations must be carried out before knowing where the power actually saturates.

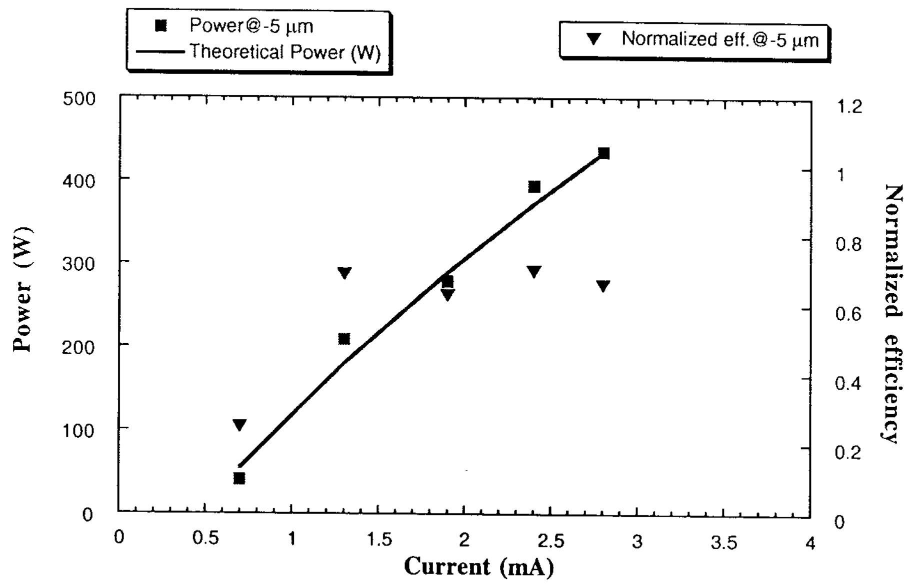

Figure 5. Power and efficiency versus electron beam current for a point 5 microns away from the peak of the cavity length detuning curve. The efficiency is normalized to one over four times the number of wiggler periods. A normalized efficiency of unity corresponds to 0.61% absolute efficiency.

When operated at 6 microns with ZnSe mirrors or at 3.1 microns with sapphire mirrors the laser reached >1000 W and 1720 W respectively. The efficiency at the peak of the detuning curve was constant as a function of current for these cases, showing that the power was limited by the electron beam and not the mirrors, in agreement with the predictions in section 3. In figure 6 we show the efficiency as a function of cavity length detuning and current for 3.1 micron operation with sapphire substrates. No reduction in efficiency with power is evident.

Transient behavior was also seen. In figure 7 we show the laser power versus time for two long single pulses of 200 msec. The laser was operated at the peak of the detuning curve at 3.1 microns with sapphire mirrors. The sapphire mirrors had rather lossy coatings (absorption estimated at 0.4%). Two pulses are shown. The first is for 18.7 MHz operation and the second is with all the same parameters but with 37.4 MHz operation. In the absence of mirror distortion, the power should have been independent of time for the two curves and equal to the initial power level. Instead, the power for both cases falls rapidly and is the same after 50 msec. This implies that the laser efficiency allowed at 37.4 MHz is exactly half that allowed at 18.7 MHz. Note that the characteristic time for these mirrors from equation 6 is 0.33 second, so the behavior seen is definitely occurring in the transient regime. Also note that the behavior is qualitatively consistent with the behavior seen in figure 1. As noted above, the data in figure 1 can be regarded as the power per micropulse train as a function of the repetition rate. One expects the power per micropulse to drop by a factor of two when the repetition rate is doubled. This means that the power should be independent of the repetition rate when the distortion is high enough. Also note that the initial slope of the curve is proportional to the repetition rate. This implies that the effect leading to the power reduction is linear in the initial power.

The speed with which the power drops is surprising. When the response seen in figure 7 is analyzed using the equations in section 4 one finds that the absorption required for the aberration amplitude to reach 1/10 of a wave in 30 msec. (the time it takes for the experimental power to drop by a factor of two) is 2.8%. The reflectivity of the high reflector is 99.2% so absorption this high is not possible.

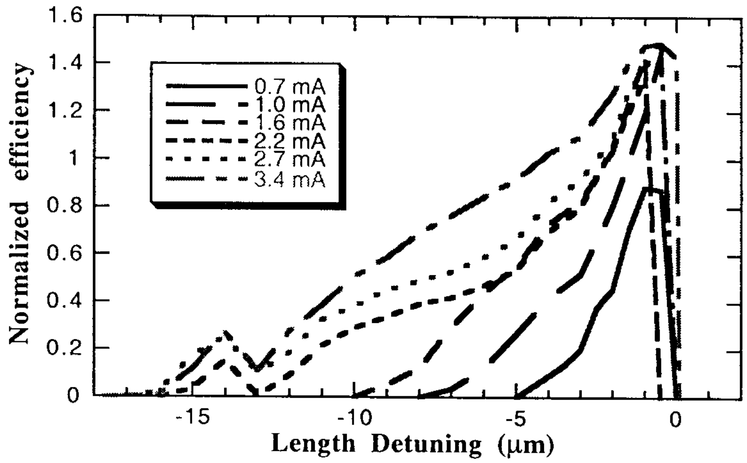

Figure 6. Normalized efficiency vs. cavity length for several different electron beam currents. Sapphire mirror substrates with low loss coatings were used for the resonator optics for this data. The highest efficiency is for the highest current. The cavity length detuning curve shortens for low current operation due to reduced gain.

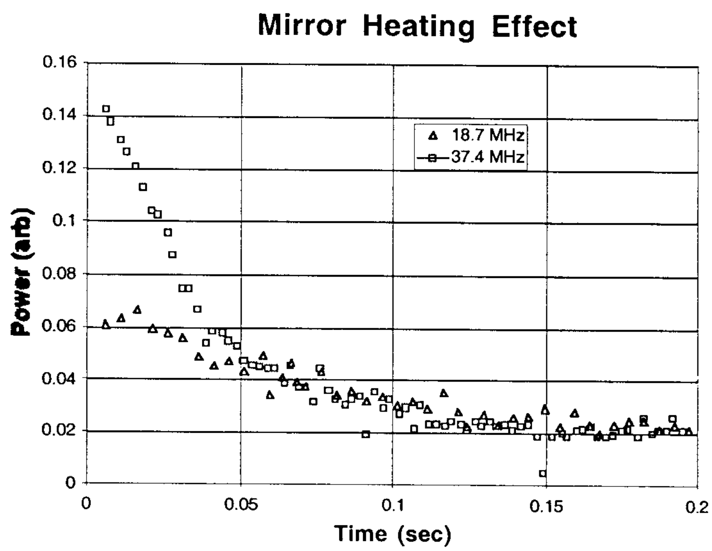

Figure 7. Power vs. time for long single pulse operation. The two curves are for the same laser and accelerator settings except that the repetition rate of the two differs by a factor of two.

Beam quality measurements were made using the calcium fluoride mirror substrates for three different power levels. The mode quality M^2, measured at the end of an optical transport system with 15 mirror bounces and three windows, was independent of power but was equal to 1.9, much higher than expected. It is not clear why the mode quality, even for rather low power should be this high. The most likely explanation is an accumulated wavefront distortion in the optical transport. If the aberration is systematic, the number of mirror bounces is consistent with the M^2 seen. We also need to look at the mode quality with ZnSe and sapphire mirror substrates. Measurements taken with burn paper and scanning blades at 3.1 microns imply an M^2 of 2 with a large uncertainty. This was done at low average power, supporting the idea of an aberration introduced by the optical transport system.

6. CONCLUSIONS

We have seen good qualitative agreement with the laser behavior vs. the theoretical predictions. The power limit for calcium fluoride mirror substrates is certainly good to a factor of two, which is within the error of the predictions. The mode quality does not depend on the power level for low powers, also in agreement with predictions.
Though the transient response also agrees qualitatively with the experimental results, the quantitative agreement is not as good. It is not clear why this discrepancy exists. One possible explanation is that this FEL configuration is quite sensitive to aberrations. Another is that the coating expands more than the substrate and leads to an enhancement in the aberration. Clearly, more data is needed to get good quantitative agreement between mirror distortion calculations and FEL performance. Also a better model of the affect of laser performance versus aberration is needed. Future experiments will look at the mode quality as a function of power and mirror substrate at the laser exit and will study the transient power characteristics as a function of cavity length detuning. In addition, mirror coating absorption will be studied using extracavity measurements to obtain an independent measure of this critical parameter.

ACKNOWLEDGEMENTS

This work was supported by the U.S. DOE Contract No. DE-AC05-84-40150, the Office of Naval Research, the Commonwealth of Virginia, and the Laser Processing Consortium. Many thanks to Matt Poelker for reviewing the paper and suggesting many excellent improvements.

REFERENCES

1. C. C. Shih and S. M. Shih, "Advanced Free-Electron Laser Resonator", Nucl. Inst. and Meth. **A 304**, 788(1991).
2. G. R. Neil. et al., "First Operation of an FEL in Same-cell Energy Recovery Mode", To appear in Phys. Rev. Lett.
3. G. R. Neil, S. V. Benson, M. D. Shinn, P. C. Davidson, and P. K. Kloeppel,"Optical Modeling of the Jefferson Lab IR Demo FEL", **SPIE 2989** 160–171(1997).
4. C. L. Bohn et al., "Performance of the Accelerator Driver of Jefferson Laboratory's Free-Electron Laser", Proceedings of the 1999 Particle Accelerator Conference, New York, 1999.
5. G. Arfken, "Mathematical Methods for Physicists", Section 5.11, Academic Press, New York, N.Y., 1970.
6. N. N. Rykalin and Y. L. Krasulin, Soviet Phys. Doklady **163** (1966) 87, English transl. **10** (1966) 659.
7. J. C. Goldstein and B. D. McVey, "Thermal distortion limits on the performance of XUV Free-electron Lasers configured with a multifacet ring resonator", **SPIE 1739**, *High Heat Flux Engineering*, (1992).
8. G. Dattoli and P. L. Ottaviani, "Logistic Equation, FEL Dynamics and Self Induced Harmonic Generation", presented at the 20th International Free-electron Laser Conference, Williamsburg VA, Aug. 1998.