NPL Polarized Source Group
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The Design of the Electrostatic Bend

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1 Introduction

The Illinois/CEBAF polarized electron source incorporates a novel spin manipulator system first proposed by E. Reichert [1]. The system consists of two 107.7° electrostatic bends separated by four solenoids and followed by four solenoids. This technical note describes the procedures used in the design of the electrostatic bends.

In order to bend our electron beam, it is necessary to employ a radially concentric toroidal condenser (see Figure 1). Due to the absence of a 'nice', fully 3–D, electrostatic-ray-tracing program, we must complete our design analytically. In the ensuing discussion we follow closely the work of Wollnik [2] and Wollnik, Matsuo, and Matsuda [3]; and adopt the conventions of reference [2] whenever possible.

2 Definitions Used

Before proceeding, we pause to define the coordinate system and parameters we use to characterize a toroidal condenser; Figure 1 displays the coordinate system while Figures 2 and 3 display the parameters.

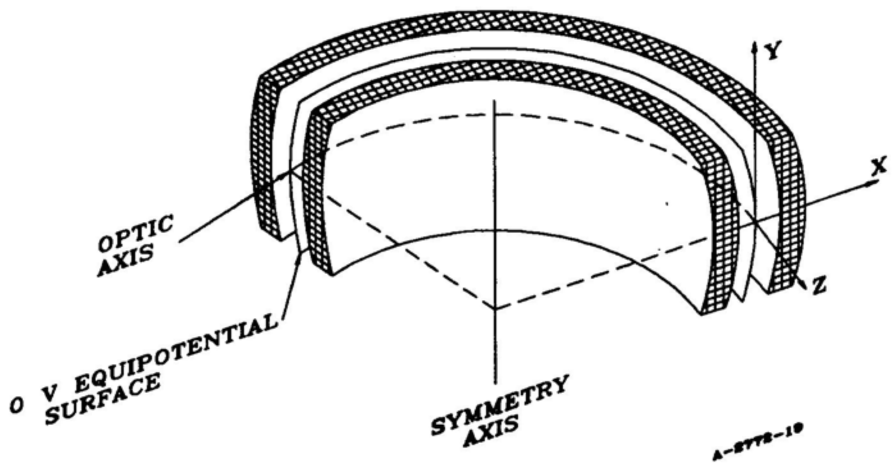

Figure 1: The coordinates used in characterizing a toroidal condenser.

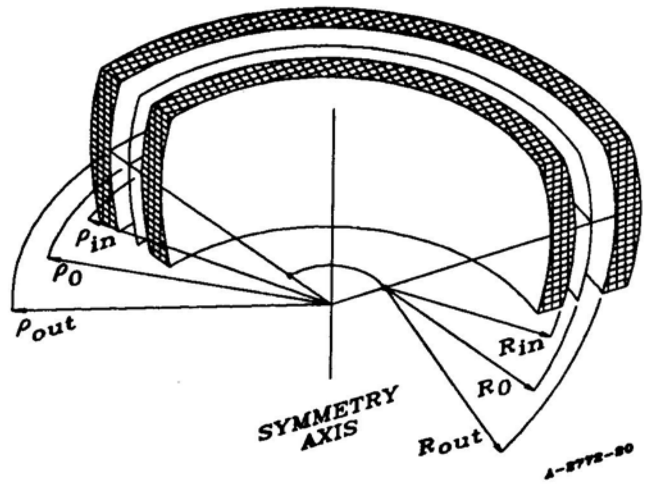

Figure 2: The radii that characterize the electrodes and 0 V equipotential of a toroidal condenser.

2.1 The Coordinate System

First, we assume that the electrodes are radially concentric and that the optic axis lies in the 0 V equipotential surface and has a radius of curvature ρ_0. If we now let x and y represent small deviations from the optic axis in the horizontal and vertical directions (respectively), then by choosing to work the problem in cylindrical coordinates, the radial coordinate becomes $\rho = \rho_0 + x$, the axial coordinate becomes y, and the azimuthal coordinate becomes z.

2.2 The Parameters

A general toroidal equipotential surface is described by two radii: the vertical radius of curvature and the horizontal radius of curvature. We show in Figure 2 the inner electrode, the 0 V equipotential surface, and the outer electrode of a toroidal condenser. We signify the horizontal (vertical) radius of curvature of the inner electrode, the 0 V equipotential surface, and the outer electrode as $\rho_{\rm in}$, ρ_0, and $\rho_{\rm out}$ ($R_{\rm in}$, R_0, and $R_{\rm out}$) respectively.

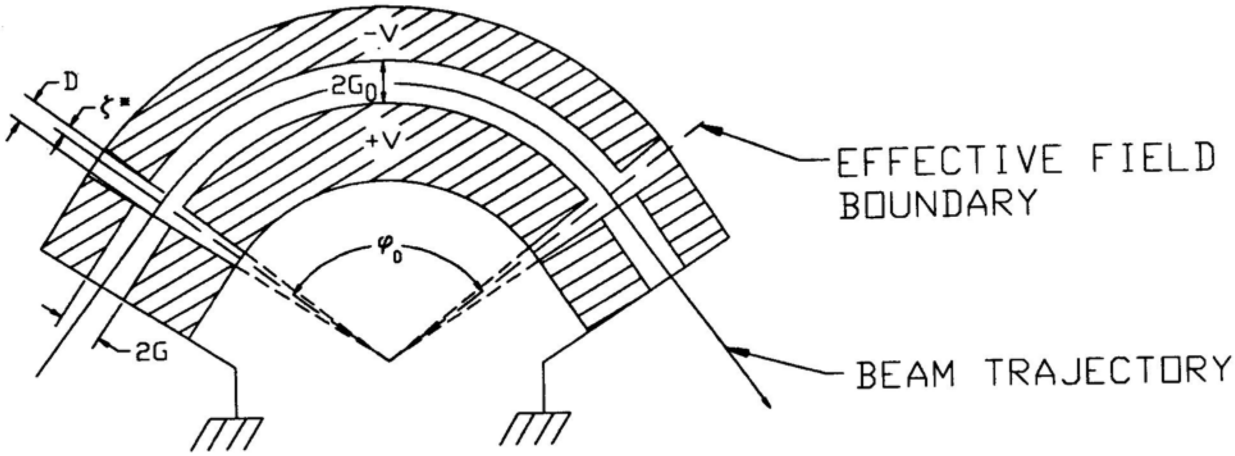

Figure 3: A cross sectional view of a complete toroidal bend in the $y = 0$ plane.

In Figure 3 we show a cross sectional view of a complete toroidal bend (including both the electrodes and the fringing field shunts) in the $y = 0$ plane. We use this figure to assist us in the definition of five more parameters:

$2G_0$	Electrode gap
$2G$	Shunt gap
D	Electrode/shunt gap
ζ^*	Electrode/effective-field-edge spacing
φ_0	Angle enclosed by the effective field boundaries (the bend angle)

3 Design Criteria

In order to define our bend mechanically, we must specify φ_0, ρ_0, ρ_{in}, ρ_{out}, R_{in}, R_{out}, ζ^*, the vertical radius of curvature of the fringing field shunts, and the radial deflection suffered by the effective electron trajectory at the field boundaries ($\Delta\xi$). As it behooves us to maintain the radial inhomogeneity of the bend's electric field, we equate the vertical radius of curvature of the grounded shunts to R_0.

To allow us to arrive at meaningful values for the nine parameters above, we observe four criteria in the design of the bend.

1. We wish to bend our 100 kV electron beam through an angle of 107.7° (which corresponds to a 90.0° spin precession at this energy).

2. Because we want to use relatively inexpensive, commercial power supplies, we limit the electrode potentials to ≤ 15 kV. We therefore choose a design potential of +14 kV for the inner electrode and −14 kV for the outer electrode.

3. We desire that the optical properties of the bend in the x direction be as similar as possible to the optical properties of the bend in the y direction (where x and y are defined by Figure 1).

4. Due to emittance and energy spread considerations, we require our electron beam to traverse the bend along the 0 V equipotential.

4 Theory

4.1 Transfer Matrices

In order to be able to compare the optical properties of the bend in the x and y directions, we must know the first order equations of motion of an electron beam as it traverses the bend. To write the equations of motion in the customary matrix form, we introduce the 4×4 matrices $\mathcal{F}$ and $\mathcal{S}$. $\mathcal{F}$ represents the defocussing of the beam in the x direction as it crosses the bend's effective-field boundaries:

$$\mathcal{F} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mathcal{F}_{21} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

where

$$\mathcal{F}_{21} = \frac{-2\,G_0}{\pi\,{\rho_0}^2}\left[2\,\ln\frac{2\,G_0}{\sqrt{D^2+(G_0+G)^2}} - \frac{G_0}{D}\cos^{-1}\left(1-\frac{2\,D^2}{D^2+(G_0+G)^2}\right)\right]. \tag{2}$$

$\mathcal{S}$ represents the sector-field focussing of the condenser:

$$\mathcal{S} = \begin{pmatrix} \mathcal{S}_{11} & \mathcal{S}_{12} & 0 & 0 \\ \mathcal{S}_{21} & \mathcal{S}_{22} & 0 & 0 \\ 0 & 0 & \mathcal{S}_{33} & \mathcal{S}_{34} \\ 0 & 0 & \mathcal{S}_{43} & \mathcal{S}_{44} \end{pmatrix} \tag{3}$$

where

$$\begin{aligned} \mathcal{S}_{11} = \mathcal{S}_{22} &= \cos\left(k_x\,\rho_0\,\varphi_0\right), & \mathcal{S}_{33} = \mathcal{S}_{44} &= \cos\left(k_y\,\rho_0\,\varphi_0\right), \\ \mathcal{S}_{12} &= \frac{\sin\left(k_x\,\rho_0\,\varphi_0\right)}{k_x}, & \mathcal{S}_{21} &= -k_x \sin\left(k_x\,\rho_0\,\varphi_0\right), \\ \mathcal{S}_{34} &= \frac{\sin\left(k_y\,\rho_0\,\varphi_0\right)}{k_y}, & \mathcal{S}_{43} &= -k_y \sin\left(k_y\,\rho_0\,\varphi_0\right), \end{aligned} \tag{4}$$

with

$$k_x = \frac{\sqrt{1-n_1+\gamma^{-2}}}{\rho_0}, \quad k_y = \frac{\sqrt{n_1}}{\rho_0}, \quad n_1 = \frac{\rho_0}{R_0}, \quad \gamma = \frac{E_{\text{beam}}}{m_e}. \tag{5}$$

If we take $x' = dx/dz$ and $y' = dy/dz$, and if we let a subscript i (f) refer to conditions at the entrance (exit) of the bend, then we can finally write the equations of motion as

$$\begin{pmatrix} x_f \\ x'_f \\ y_f \\ y'_f \end{pmatrix} = \mathcal{F}\,\mathcal{S}\,\mathcal{F} \begin{pmatrix} x_i \\ x'_i \\ y_i \\ y'_i \end{pmatrix},$$

or rather

$$\begin{pmatrix} x_f \\ x'_f \\ y_f \\ y'_f \end{pmatrix} = \begin{pmatrix} \mathcal{S}_{11}+\mathcal{S}_{12}\mathcal{F}_{21} & \mathcal{S}_{12} & 0 & 0 \\ \mathcal{S}_{21}+2\mathcal{S}_{11}\mathcal{F}_{21}+\mathcal{S}_{12}\mathcal{F}_{21}^2 & \mathcal{S}_{11}+\mathcal{S}_{12}\mathcal{F}_{21} & 0 & 0 \\ 0 & 0 & \mathcal{S}_{33} & \mathcal{S}_{34} \\ 0 & 0 & \mathcal{S}_{43} & \mathcal{S}_{44} \end{pmatrix} \begin{pmatrix} x_i \\ x'_i \\ y_i \\ y'_i \end{pmatrix}. \tag{6}$$

4.2 Electrostatic Potential

We can express the electrostatic potential (ϕ) between the condenser's electrodes as a power series in x and y:

$$\phi(x,y) = -E_0\,\rho_0 \sum_{i,j>0} \frac{a_{ij}}{i!\,j!} \left(\frac{x}{\rho_0}\right)^i \left(\frac{y}{\rho_0}\right)^j, \tag{7}$$

where we assume that the equilibrium point about which the potential is expanded ($x = y = 0$) lies in the 0 V equipotential surface. Note that E_0 is the electric field strength at the optic axis ($x = y = 0$) which will cause the beam to be bent along a curve of radius ρ_0.

By requiring a solution to Laplace's equation, it is possible to generate a recursion relation for all the nonzero a_{ij} in equation 7 in terms of the a_{i0}:

$$0 = (i+1)\,a_{i,j+2} + a_{i+1,j+2} + (i+2)\,a_{i+2,j} + a_{i+3,j}\,. \tag{8}$$

The first six a_{i0} are given by

$$\begin{aligned}
a_{10} &= 1\,, \\
a_{20} &= -1 - n_1\,, \\
a_{30} &= 2 + 2\,n_1 + n_1^2 - n_2\,, \\
a_{40} &= -6 - 5\,n_1 - 5\,n_1^2 + 2\,n_2 + 6\,n_1\,n_2\,, \\
a_{50} &= 24 + 19\,n_1 + 13\,n_1^2 + 15\,n_1^3 - 6\,n_1^4 \\
&\quad - 7\,n_2 - 18\,n_1\,n_2 - 24\,n_1^2\,n_2 + 6\,n_2^2\,, \\
a_{60} &= -120 - 93\,n_1 - 60\,n_1^2 - 30\,n_1^3 - 57\,n_1^4 + 30\,n_1^5 \\
&\quad + 33\,n_2 + 72\,n_1\,n_2 + 117\,n_1^2\,n_2 + 60\,n_1^3 - 18\,n_2^2 - 90\,n_1\,n_2^2\,,
\end{aligned} \tag{9}$$

where

$$n_1 = \frac{\rho_0}{R_0}, \qquad n_2 = \rho_0 \frac{d}{dx}\left(\frac{\rho_0}{R(x)}\right)\bigg|_{x=0}, \tag{10}$$

with $R(x)$ being the vertical radius of curvature of any equipotential surface that intersects the $y = 0$ plane at x.

5 Applying Design Criteria

From Section 3, we know that we have to assign values to φ_0, ρ_0, ρ_{in}, ρ_{out}, R_0, R_{in}, R_{out}, ζ^*, and $\Delta\xi$ in order to define our mechanical design. If we concentrate on determining values for the set of variables G_0, G, D, φ_0, n_1, n_2, $E_0\,\rho_0$, and ρ_0, which are easier to manipulate, then values for the mechanical parameters become transparent. It turns out that common sense plus the four design criteria listed in Section 3 provide sufficient constraints to allow us to complete our design.

5.1 Criterion 1

Application of criterion 1 tells us two things. First, we see that

$$\varphi_0 = 107.7^\circ\,. \tag{11}$$

Second, by equating the electrostatic force on the electrons with the centripetal force they experience as they traverse the bend, we see that

$$e\,E_0 = \frac{\gamma\, m_e\, v^2}{\rho_0}.$$

By utilizing the facts that $m_e = 511.1$ keV, that $\gamma = 1.196$, and that $v^2 = 1 - \gamma^{-2}$, we can obtain a value for the product of E_0 and ρ_0:

$$E_0\,\rho_0 = 1.839 \times 10^5 \text{ V}. \tag{12}$$

5.2 Common Sense and Criterion 2

At this point in the design process, we consult our common sense to help us narrow the range of acceptable values for our parameters. We don't have enough design criteria to fully specify every parameter.

First, for the sake of simplicity, we allow

$$R(x) = R_0 + x\,. \tag{13}$$

Then equation 10 becomes

$$
\begin{aligned}
n_2 &= \rho_0 \frac{d}{dx}\left(\frac{\rho_0}{R_0 + x}\right)\bigg|_{x=0} \\
&= -\frac{\rho_0^2}{R_0^2} \\
n_2 &= -n_1^2 . \qquad (14)
\end{aligned}
$$

Second, we consider ρ_0. We would like to make ρ_0 as large as possible so that the bend would not focus "too hard." The upper limit of ρ_0 is determined by the capabilities of the lathe available in our machine shop, and is estimated to be $\sim$ 10–12 cm. So we shall take

$$\rho_0 \sim 11 \text{ cm} . \qquad (15)$$

Finally, in the context of equation 15 and of criterion 2 (which tells us that the potential difference between the two electrodes is $\Delta V = 28$ kV), we consider G_0, G, and D. As $2G_0$ goes roughly as $\rho_0 \, \Delta V / E_0 \, \rho_0$, we can use equation 12 to see that

$$2\, G_0 \sim 1.7 \text{ cm} . \qquad (16)$$

As it is simple to keep the shunt gap equal to the electrode gap, we choose

$$2\, G \sim 1.7 \text{ cm} \qquad (17)$$

also. Finally, in picking D we shall choose a nice round number such that the average electric field along the ends of the electrodes facing the shunts (~ 14 kV$/D$) is $\lesssim 20$ kV/cm. We choose

$$D = 1.0 \text{ cm} . \qquad (18)$$

5.3 Criteria 2 and 3

Co-application of criteria 2 and 3 allows us to exactly determine the quantities G_0, G, D, ρ_0, and n_1. These five parameters, together with the already known values of φ_0, n_2, and $E_0\,\rho_0$, allow us to determine all of the mechanical parameters. We now detail the steps used to arrive at our values for G_0, G, D, ρ_0, and n_1.

1. From Section 5.2 we have initial guesses for the values of ρ_0, D, G, and G_0.
2. Application of criterion 3 to equation 6 causes us to equate $\mathcal{S}_{11} + \mathcal{S}_{12}\mathcal{F}_{21}$ with $\mathcal{S}_{33}$. Using numerical methods to solve

$$0 = \mathcal{S}_{11} + \mathcal{S}_{12}\mathcal{F}_{21} - \mathcal{S}_{33} , \qquad (19)$$

it is possible to arrive at a value for n_1 which equalizes as much as possible the optical properties of the bend in the x and y directions. We include at the end of this note a listing of the program, PARAMN, used in this step.

3. Aplication of criterion 2 to equation 7 and utilization of equations 9, 10, 12, and 14 allow us to write two equations in the $y = 0$ plane: one for the outer electrode

$$0.07613 = b_1 \mathcal{X} + b_2 \mathcal{X}^2 + b_3 \mathcal{X}^3 + b_4 \mathcal{X}^4 + b_5 \mathcal{X}^5 + b_6 \mathcal{X}^6 \tag{20}$$

and one for the inner electrode

$$-0.07613 = b_1 \mathcal{X} + b_2 \mathcal{X}^2 + b_3 \mathcal{X}^3 + b_4 \mathcal{X}^4 + b_5 \mathcal{X}^5 + b_6 \mathcal{X}^6 , \tag{21}$$

where we have truncated the series in equation 7 after six terms, and where

$$\begin{aligned} \mathcal{X} &= x/\rho_0 , \\ b_1 &= 1 , \\ b_2 &= (-1 - n_1)/2 , \\ b_3 &= (2 + 2 n_1 + 2 n_1^2)/6 , \\ b_4 &= (-6 - 5 n_1 - 7 n_1^2 - 6 n_1^3)/24 , \\ b_5 &= (24 + 19 n_1 + 20 n_1^2 + 33 n_1^3 + 24 n_1^4)/120 , \\ b_6 &= (-120 - 93 n_1 - 93 n_1^2 - 102 n_1^3 - 192 n_1^4 - 120 n_1^5)/720 . \end{aligned} \tag{22}$$

Using numerical methods to solve equations 20 and 21, it is possible to arrive at values for ρ_{in} and ρ_{out}. We include at the end of this note a listing of the program, ELEC, used in this step.

4. We set $2G = 2G_0 = \rho_{out} - \rho_{in}$.

5. We iterate through steps 2–4 until the process converges.

5.4 Criterion 4

In order to arrive at analytical solutions for the effects of the bend's fringing fields on electron trajectories, it is necessary to consider an *effective* beam trajectory. As is shown in Figure 4, the effective trajectory is straight outside the effective field boundary, circular inside the effective field boundary (coincident with the optic axis), and normal to the effective field edge. The fringing field effects are accounted for by a small radial displacement ($\Delta\xi$) of the effective trajectory at the field boundary. We say that as the electrons enter (exit) through the effective field edge, they suffer a radial kick of $-|\Delta\xi|$ ($|\Delta\xi|$), where

$$\Delta\xi \approx -\frac{G_0^2}{\rho_0} \left[0.15 + 0.04 \left(\frac{D}{G_0} \right)^2 + 0.04 \left(\frac{G}{G_0} \right) + 0.05 \left(\frac{G}{G_0} \right)^2 \right] . \tag{23}$$

Application of criterion 4 suggests letting $\rho_0 \rightarrow \rho_0 - |\Delta\xi|$. Doing so and recalculating n_1, ρ_{in}, and ρ_{out} as per steps 2 and 3 of Section 5.3 finally gives us self consistent values for all of the working parameters.

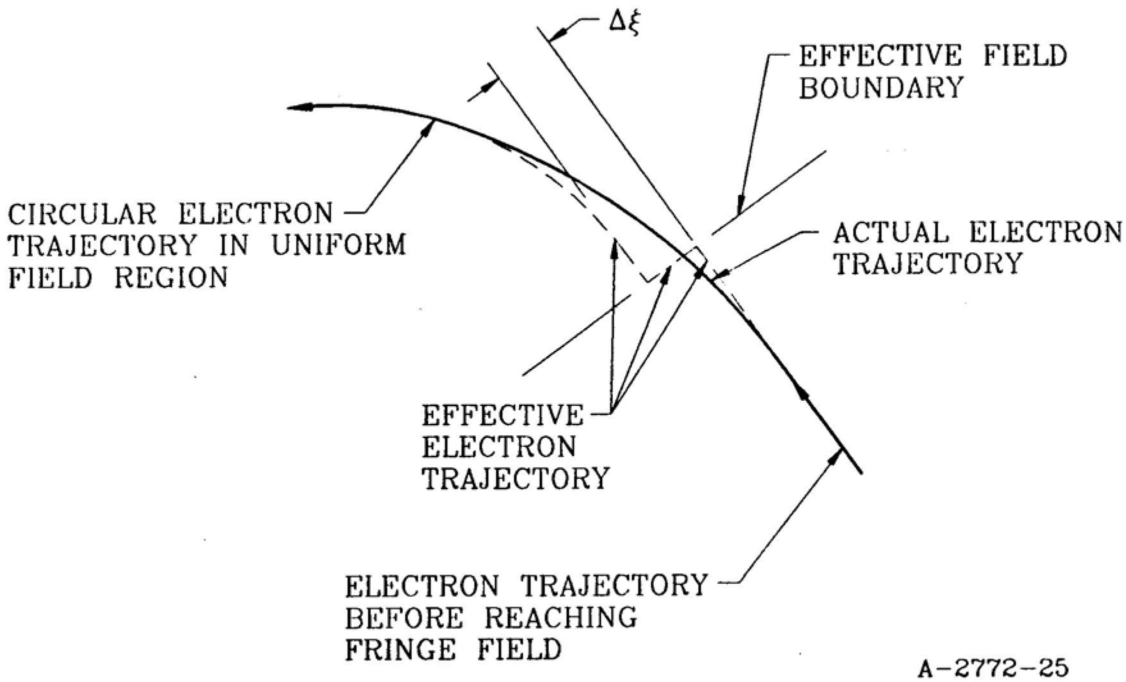

Figure 4: A schematic diagram to demonstrate the concept of the effective beam trajectory.

6 Results

6.1 Working Parameters

The results of applying our design criteria towards the working parameters are summarized in Table 1.

6.2 Mechanical Parameters

We now derive the necessary mechanical parameters from the working parameters listed in Section 6.1. We already know that

$$\rho_0 = 11.081 \text{ cm} \quad \text{and} \quad \varphi_0 = 107.7^\circ . \tag{24}$$

Using the values for ρ_0 and n_1 in equations 20, 21, and 22, we find that

$$\rho_{\text{in}} = 10.292 \text{ cm} \quad \text{and} \quad \rho_{\text{out}} = 11.987 \text{ cm} . \tag{25}$$

As $R_{\text{out}} = R_0 + (\rho_{\text{out}} - \rho_0)$ and as $R_{\text{in}} = R_0 - (\rho_0 - \rho_{\text{in}})$, we see that

$$R_{\text{in}} = 12.739 \text{ cm} \quad \text{and} \quad R_{\text{out}} = 14.434 \text{ cm} . \tag{26}$$

By substituting the values for G, G_0, D, and ρ_0 into equation 23, we can determine that

$$\Delta\xi = 0.019 \text{ cm} . \tag{27}$$

Table 1: Design Parameters for the Electrostatic Bend

Quantity	Design Value
G_0	0.8475 cm
G	0.8475 cm
D	1.0 cm
ρ_0	11.081 cm
n_1	0.8191
n_2	-0.6709
φ_0	107.7°
$E_0\,\rho_0$	183.9 kV

Finally, by using the formula

$$\begin{aligned}\zeta^* &= \frac{G_0}{\pi}\left[\ln\left(\frac{4\,G_0^2}{\sqrt{D^2+(G_0+G)^2}\sqrt{D^2+(G_0-G)^2}}\right) - \frac{G}{2\,G_0}\ln\left(\frac{D^2+(G_0+G)^2}{D^2+(G_0-G)^2}\right)\right. \\ &\quad \left. - \frac{D}{G_0}\cos^{-1}\left(\frac{D^2+G^2-G_0^2}{\sqrt{D^2+(G_0+G)^2}\sqrt{D^2+(G_0-G)^2}}\right)\right] \qquad (28)\end{aligned}$$

with our values for G, G_0, and D we can show that

$$\zeta^* = 0.411\ \text{cm}. \qquad (29)$$

6.3 Interpretation

Finally, in this section we summarize all of the pertinent mechanical parameters of the bend design. In doing so, we try to interpret the design data along the mechanical-engineering lines required by our draftsman and machinist.

The electrostatic bend can be broken down into two basic parts: the condenser electrodes and the fringing-field shunts. The schematic drawings in Figures 3 and 4 may help interpret the data and aid in the understanding of the relationship between the actual trajectory of the beam and the geometric parameters of the bend.

We first consider the electrodes. The inner electrode should have a vertical radius of curvature of 12.739 cm and a horizontal radius of curvature of 10.292 cm. The outer electrode should have a vertical radius of curvature of 14.434 cm and a horizontal radius

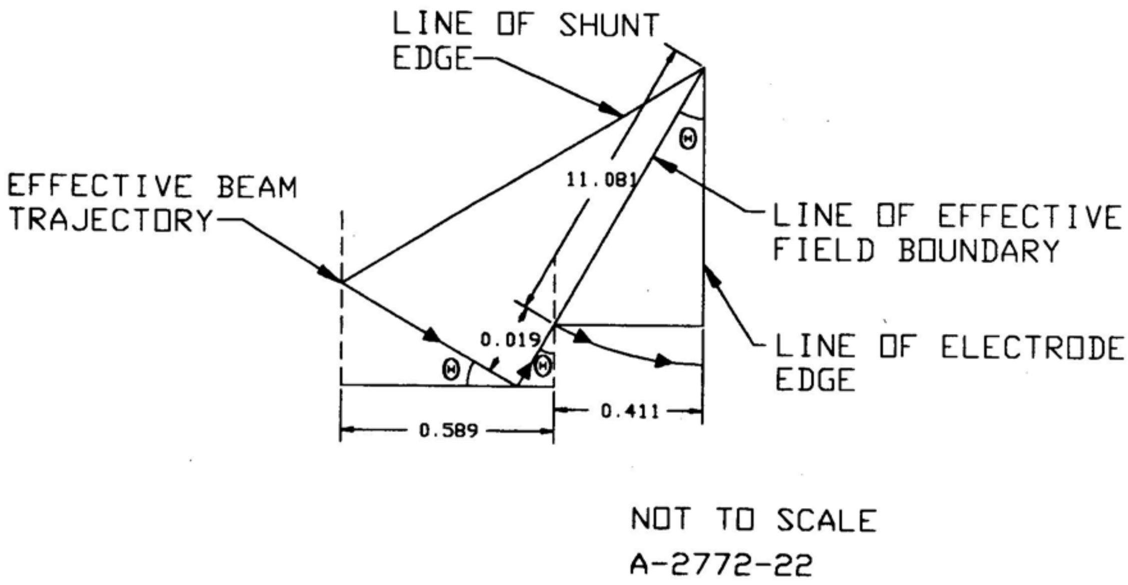

Figure 5: Schematic representation of the effective trajectory and the related numerical values for our electrostatic bend.

of curvature of 11.987 cm. Also, both electrodes should subtend an angle of

$$103.4^\circ = 107.7^\circ - 2\sin^{-1}\left(\frac{0.411}{11.081}\right) = 107.7^\circ - 2\,\Theta\,.$$

Furthermore, the electrodes should be assembled so that the radius at which the effective beam trajectory intersects the effective field boundary is 0.019 cm greater than the radius of the optic axis at the effective field boundary.

We now consider the shunts. The shunts should have a vertical radius of curvature of 13.528 cm and an opening gap of 1.695 cm. Also, the shunt face closer to the electrodes (the inward shunt face) should be aligned along a radius such that the angle subtended by the inner shunt faces is

$$113.8^\circ = 107.7^\circ + 2\tan^{-1}\left\{\frac{0.589 - 0.019(0.411/11.081)}{\cos[\sin^{-1}(0.411/11.081)]}\,\frac{1}{11.1}\right\}.$$

Furthermore, the shunts should be assembled so that the effective beam trajectory passes through the center of the shunt gap at the inward faces at a radius of

$$11.116\ \text{cm} = \sqrt{11.100^2 + \left\{\frac{0.589 - 0.019(0.411/11.081)}{\cos[\sin^{-1}(0.411/11.081)]}\right\}^2}\,.$$

We note that the mechanical realization of the bend and its associated vacuum chamber is documented in the 2771-series of drawings.

Appendix A: The Program ELEC

We provide here a listing of the program ELEC, which calculates the radii of the two electrodes from a number of the design parameters.

```
      program elec
c
c This program reads in the radius of curvature p0, n, a(-.1), b(-.01). It then
c spits out the radii of the two electrodes of a toroidal bend.
c
      common a,b
c
      c1=0.0
      c2=0.0
      c3=0.0
      c4=0.0
      c5=0.0
      c6=0.0
      type*
      type*, 'This program finds toroidal electrode positions.'
      type*
           type*, 'Please enter c, tol, p0, a, and b (where a and b
     + are negative).'
      type*
      read*, c,tol,p0,a,b
c
      c1=1.0
      c2=(-1.0-c)/2.0
      call zeroi(tol,c1,c2,c3,c4,c5,c6,xi)
      call zeroo(tol,c1,c2,c3,c4,c5,c6,xo)
      ri=p0*xi+p0
      ro=p0*xo+p0
      write(6,5) ri,ro
5     format (1x,'SECOND ORDER: ri= ',f8.5,'   ro= ',f8.5)
      type*
      c3=(2.0+2.0*c+2.0*c**2)/6.0
      call zeroi(tol,c1,c2,c3,c4,c5,c6,xi)
      call zeroo(tol,c1,c2,c3,c4,c5,c6,xo)
      ri=p0*xi+p0
      ro=p0*xo+p0
      write(6,10) ri,ro
10    format (1x,'THIRD ORDER: ri= ',f8.5,'   ro= ',f8.5)
      type*
```

```
      c4=(-6.0-5.0*c-7.0*c**2-6.0*c**3)/24.0
      call zeroi(tol,c1,c2,c3,c4,c5,c6,xi)
      call zeroo(tol,c1,c2,c3,c4,c5,c6,xo)
      ri=p0*xi+p0
      ro=p0*xo+p0
      write(6,15) ri,ro
15    format (1x,'FOURTH ORDER: ri= ',f8.5,'   ro= ',f8.5)
      type*
      c5=(24.0+19.0*c+20.0*c**2+33.0*c**3+24.0*c**4)/120.0
      call zeroi(tol,c1,c2,c3,c4,c5,c6,xi)
      call zeroo(tol,c1,c2,c3,c4,c5,c6,xo)
      ri=p0*xi+p0
      ro=p0*xo+p0
      write(6,20) ri,ro
20    format (1x,'FIFTH ORDER: ri= ',f8.5,'   ro= ',f8.5)
      type*
      c6=(-120.0-93.0*c-93.0*c**2-102.0*c**3-192.0*c**4-120.0*c**5)/720.0
      call zeroi(tol,c1,c2,c3,c4,c5,c6,xi)
      call zeroo(tol,c1,c2,c3,c4,c5,c6,xo)
      ri=p0*xi+p0
      ro=p0*xo+p0
      write(6,25) ri,ro
25    format (1x,'SIXTH ORDER: ri= ',f8.5,'   ro= ',f8.5)
      type*
      end
c
      subroutine zeroi (tol,c1,c2,c3,c4,c5,c6,x)
c
      common a,b
c
      f(x)=c1*x+c2*x**2+c3*x**3+c4*x**4+c5*x**5+c6*x**6+0.07613
c
      y=a
      z=b
10         x = y+(z-y)/2
           if (f(x).eq.0.0.or.(z-y)/2.lt.tol) goto 20
c
c
           if (f(y)*f(x).gt.0.0) then
                y=x
           else
                z=x
           endif
      goto 10
c
```

```
20    return
      end
c
      subroutine zeroo (tol,c1,c2,c3,c4,c5,c6,x)
c
      common a,b
c
      f(x)=c1*x+c2*x**2+c3*x**3+c4*x**4+c5*x**5+c6*x**6-0.07613
c
      y=-b
      z=-a
10       x = y+(z-y)/2
         if (f(x).eq.0.0.or.(z-y)/2.lt.tol) goto 20
c
c
         if (f(y)*f(x).gt.0.0) then
            y=x
         else
            z=x
         endif
      goto 10
c
20    return
      end
```

Appendix B: The Program PARAMN

We provide here a listing of the program PARAMN which takes a desired radius of curvature ρ_0 as input and the R_{21} matrix element of the fringing field and calculates the value of n_1 that will match the R_{11} and R_{33} matrix elements for the entire bend. This condition maintains the symmetry of the beam.

```
      program paramn
c
c This program reads in the radius of curvature p0 and the fringing field
c matrix element FR21 and determines the value of n1 that equates R11 with
c R33.
c
      real n
c
      f(n)=cosd(107.7*sqrt(1-n+1.196**-2))+p0*fr*
     +        sind(107.7*sqrt(1-n+1.196**-2))/sqrt(1-n+1.196**-2)
     +        -cosd(107.7*sqrt(n))
c
5     type*
      type*, 'Enter a,b'
      read*, a,b
      type*
      type*, 'Enter p0 and FR21, both in Dekameters.'
      read*, p0,fr
      type*
c
      type*, 'Input tolerance'
      read*, tol
      type*
c
10         n = a+(b-a)/2
           if (f(n).eq.0.0.or.(b-a)/2.lt.tol) then
                type*, 'n =',n
                type*
                type*, 'Want to use again (1,0)?'
                read*, nswer
                if (nswer.eq.1) goto 5
                stop
           endif
c
c
           if (f(a)*f(n).gt.0.0) then
                a=n
```

```
          else
               b=n
          endif
     goto 10
c
     end
```

Bibliography

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