



Deformed PDFs and SSA

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Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs

- $H(x, 0, -\Delta_{\perp}^2) \longrightarrow q(x, \mathbf{b}_{\perp})$

- $\tilde{H}(x, 0, -\Delta_{\perp}^2) \longrightarrow \Delta q(x, \mathbf{b}_{\perp})$

- $E(x, 0, -\Delta_{\perp}^2)$

↔ \perp deformation of unpol. PDFs in \perp pol. target

- physics: orbital motion of the quarks

↔ intuitive explanation for SSAs

- exclusive SSAs

- Sivers effect

- $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of \perp pol. PDFs in unpol. target

- correlation between quark angular momentum and quark transversity

↔ Boer-Mulders function $h_1^{\perp}(x, \mathbf{k}_{\perp})$

- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t , w.r.t. the average momentum fraction $x = \frac{1}{2} (x_i + x_f)$ of the active quark

$$\begin{aligned} \int dx H_q(x, \xi, t) &= F_1^q(t) & \int dx \tilde{H}_q(x, \xi, t) &= G_A^q(t) \\ \int dx E_q(x, \xi, t) &= F_2^q(t) & \int dx \tilde{E}_q(x, \xi, t) &= G_P^q(t), \end{aligned}$$

- x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
- $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)

Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q} \left(-\frac{x^-}{2} \right) \gamma^+ q \left(\frac{x^-}{2} \right) \right| p \right\rangle = H(x, \xi, \Delta^2) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, \Delta^2) \bar{u}(p') \frac{i\sigma^{+\nu} \Delta_\nu}{2M} u(p)$$

- in the limit of vanishing t and ξ , the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x, 0, 0) = q(x) \quad \tilde{H}_q(x, 0, 0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction** x
- ↪ t dependence of GPDs for fixed x , provides information on the **position space distribution** of quarks carrying a certain momentum fraction x

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x, \xi, t)$?

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$\int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x, 0, t)$	$q(x, \mathbf{b}_\perp)$

$q(x, \mathbf{b}_\perp) = \text{impact parameter dependent PDF}$

Impact parameter dependent PDFs

- define \perp localized state [D.Soper,PRD15, 1141 (1977)]

$$|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |p^+, \mathbf{p}_\perp, \lambda\rangle$$

Note: \perp boosts in IMF form Galilean subgroup \Rightarrow this state has

$$\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$$

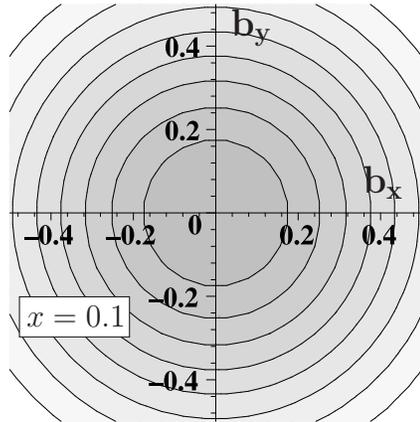
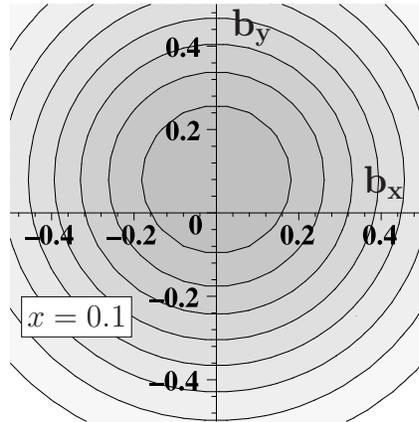
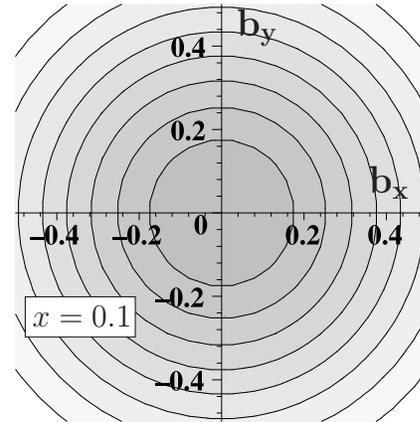
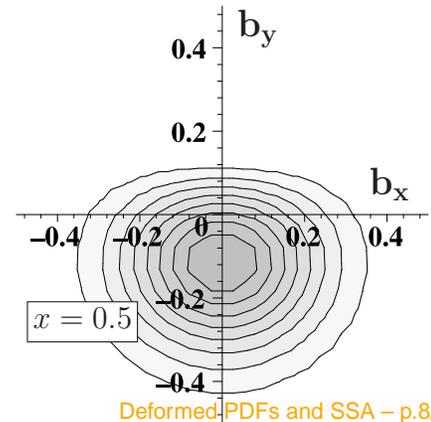
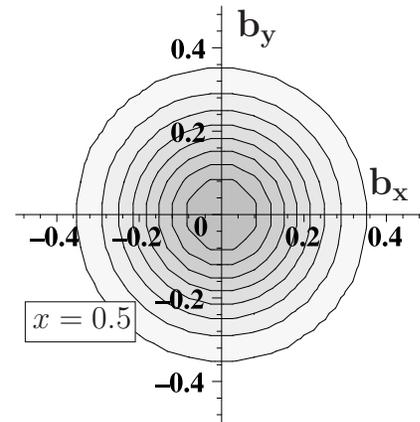
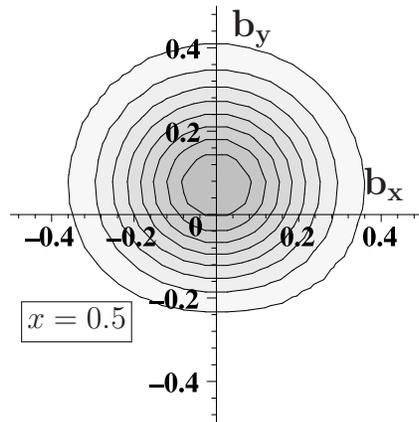
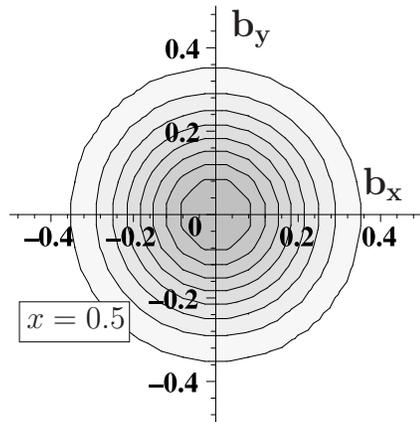
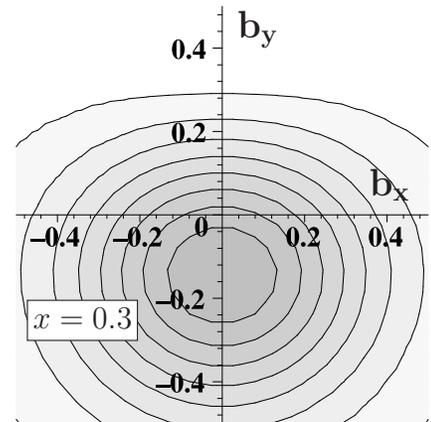
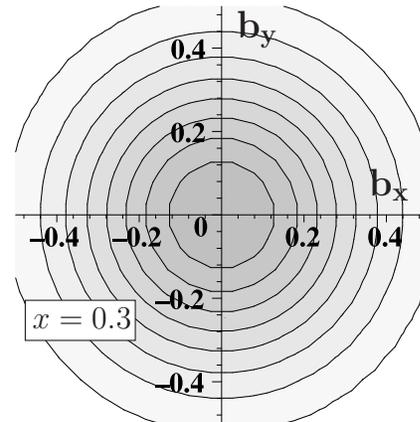
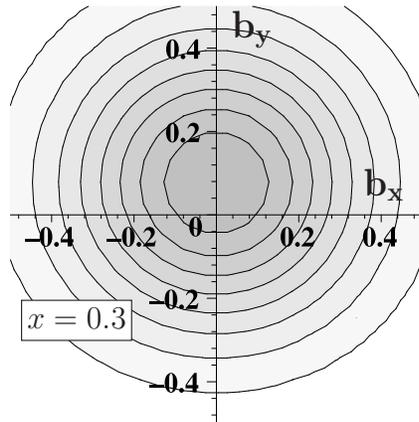
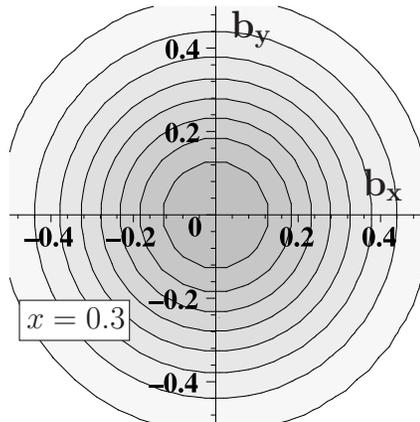
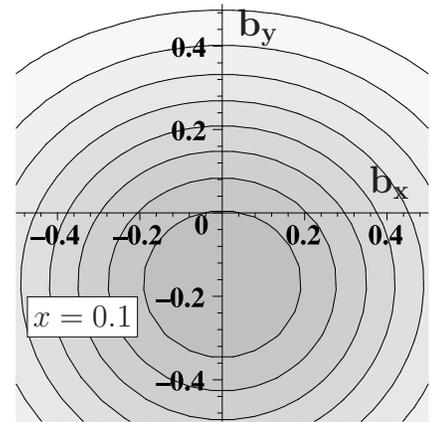
(cf.: working in CM frame in nonrel. physics)

- define **impact parameter dependent PDF**

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+ x^-}$$

\hookrightarrow

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\Delta_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\Delta_\perp^2), \end{aligned}$$

$u(x, \mathbf{b}_\perp)$  $u_X(x, \mathbf{b}_\perp)$  $d(x, \mathbf{b}_\perp)$  $d_X(x, \mathbf{b}_\perp)$ 

Transversely Deformed Distributions and $E(x, 0, -\Delta_{\perp}^2)$

M.B., Int.J.Mod.Phys.A18, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta_{\perp}^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, 0, -\Delta_{\perp}^2).$$

- Consider nucleon polarized in x direction (in IMF)

$$|X\rangle \equiv |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \uparrow\rangle + |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \downarrow\rangle.$$

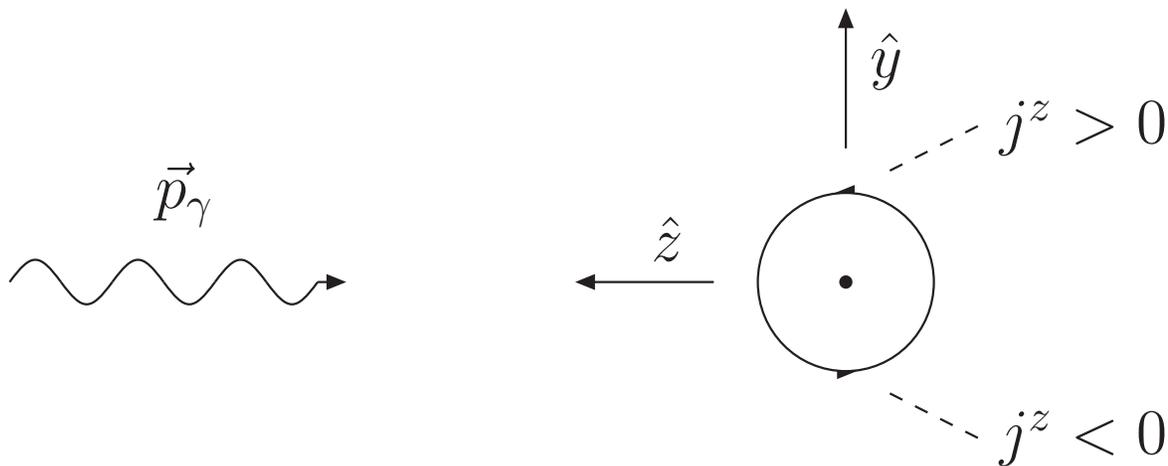
- ↪ unpolarized quark distribution for this state:

$$q(x, \mathbf{b}_{\perp}) = \mathcal{H}(x, \mathbf{b}_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} E(x, 0, -\Delta_{\perp}^2) e^{-i\mathbf{b}_{\perp} \cdot \Delta_{\perp}}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
[X.Ji, PRL 91, 062001 (2003)]

Intuitive connection with \vec{L}_q

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — \hat{z} -axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the \hat{x} -direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



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- If up-quarks have positive orbital angular momentum in the \hat{x} -direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side
- ↪ j^+ is deformed not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) “see” the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side.
- \perp deformation described by $E_q(x, 0, -\Delta_{\perp}^2)$
- ↪ not surprising to find that $E_q(x, 0, -\Delta_{\perp}^2)$ enters the Ji relation

$$\langle J_q^i \rangle = S^i \int dx [H_q(x, 0, 0) + E_q(x, 0, 0)] x.$$

Transversely Deformed Distributions and $E(x, 0, -\Delta_{\perp}^2)$

- $q(x, \mathbf{b}_{\perp})$ in \perp polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean \perp deformation of flavor q (\perp flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2 \mathbf{b}_{\perp} q_X(x, \mathbf{b}_{\perp}) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1 - 2) \Rightarrow d_y^q = \mathcal{O}(0.2 fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

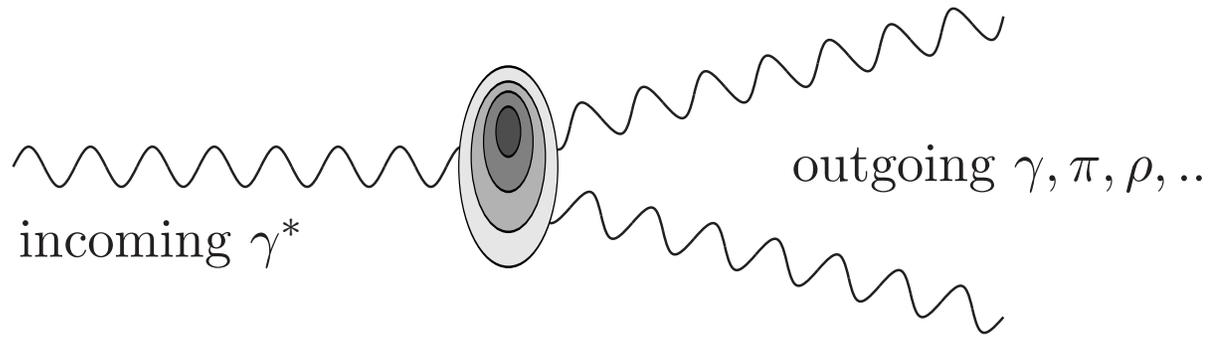
$$E_u(x, 0, -\Delta_{\perp}^2) = \frac{\kappa_u^p}{2} H_u(x, 0, -\Delta_{\perp}^2)$$

$$E_d(x, 0, -\Delta_{\perp}^2) = \kappa_d^p H_d(x, 0, -\Delta_{\perp}^2)$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$ $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$.

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!

Exclusive SSAs



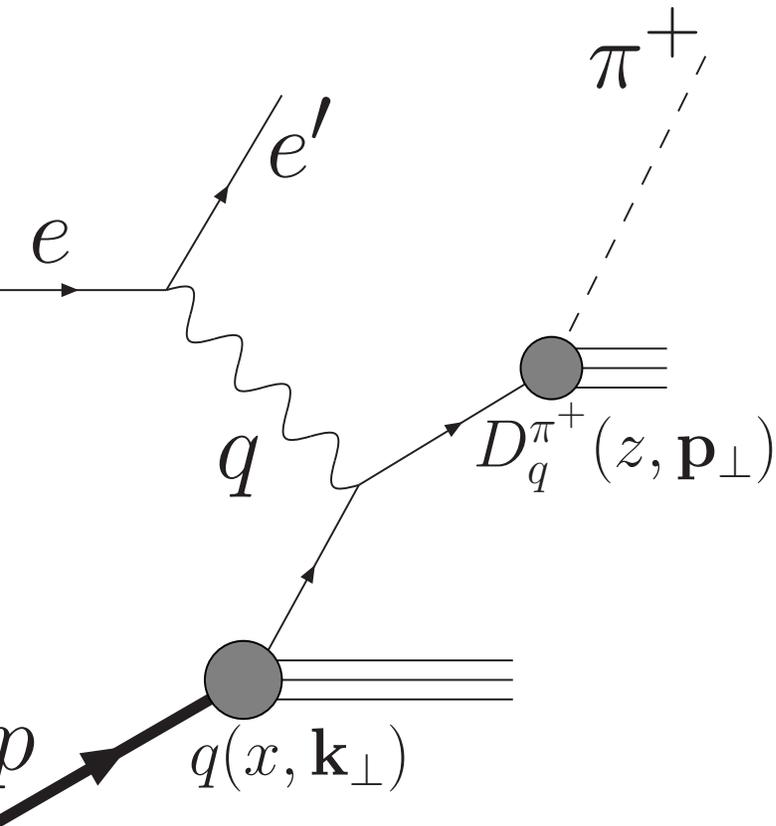
- For simplicity, only \perp momentum transfer

$$\mathcal{A} \propto \int d^2\mathbf{b}_\perp q(\mathbf{b}_\perp, x) T_q e^{i\Delta_\perp \cdot \mathbf{b}_\perp}$$

where T_q is the parton-level “Compton”-amplitude

- ↪ left-right asymmetry of quark distribution translates directly into left-right asymmetry of the scattering amplitude
- ↪ SSA probe GPDs which describe this deformation

SSAs in SIDIS ($\gamma + p \uparrow \longrightarrow \pi^+ + X$)



● use factorization (high energies) to express momentum distribution of outgoing π^+ as **convolution** of

- momentum distribution of quarks in nucleon
- ↪ **unintegrated parton density** $f_{q/p}(x, \mathbf{k}_\perp)$
- momentum distribution of π^+ in jet created by leading quark q
- ↪ **fragmentation function** $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average \perp momentum of pions obtained as sum of
 - average \mathbf{k}_\perp of quarks in nucleon (Sivers effect)
 - average \mathbf{p}_\perp of pions in quark-jet (Collins effect)

GPD \longleftrightarrow SSA (Sivers)

- **Sivers**: distribution of unpol. quarks in \perp pol. proton

$$f_{q/p\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S}{M}$$

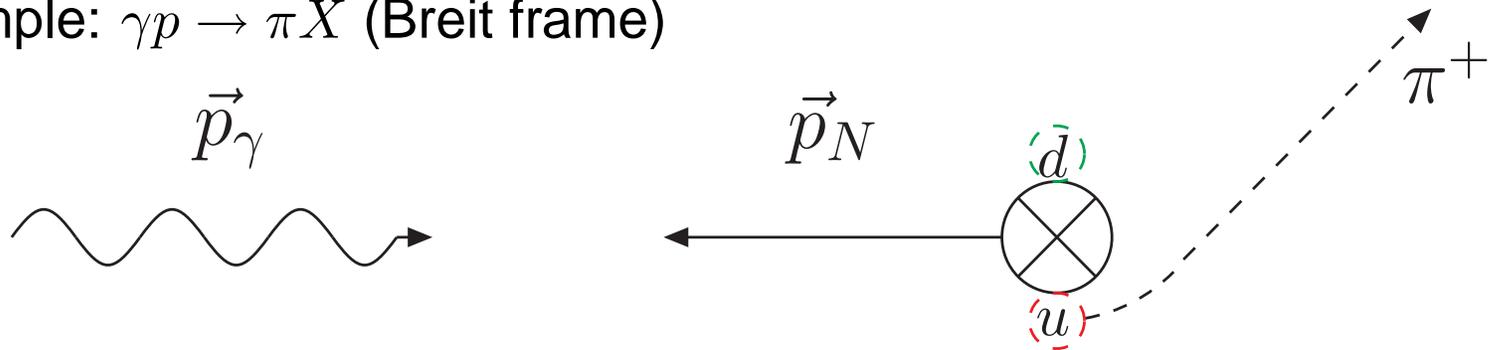
- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) = 0$
 - with FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$ (Brodsky, Hwang, Schmidt)
 - FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $f_{q/p}(x, \mathbf{k}_\perp)$
- \hookrightarrow Qiu, Sterman; Collins; Ji; Boer et al.;...

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P, S \left| \bar{q}(0) \gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P, S \right\rangle$$

- $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is the \perp impulse that the active quark acquires as it moves through color field of “spectators”
- What should we expect for Sivers effect in QCD ?

GPD \longleftrightarrow SSA (Sivers)

- example: $\gamma p \rightarrow \pi X$ (Breit frame)



- u, d distributions in \perp polarized proton have left-right asymmetry in \perp position space (T-even!); sign “determined” by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum
- \hookrightarrow FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$ -direction into momentum asymmetry that favors $-\hat{y}$ direction
- \hookrightarrow correlation between sign of κ_q and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q$
- $f_{1T}^{\perp q} \sim -\kappa_q$ consistent with HERMES results

Chirally Odd GPDs

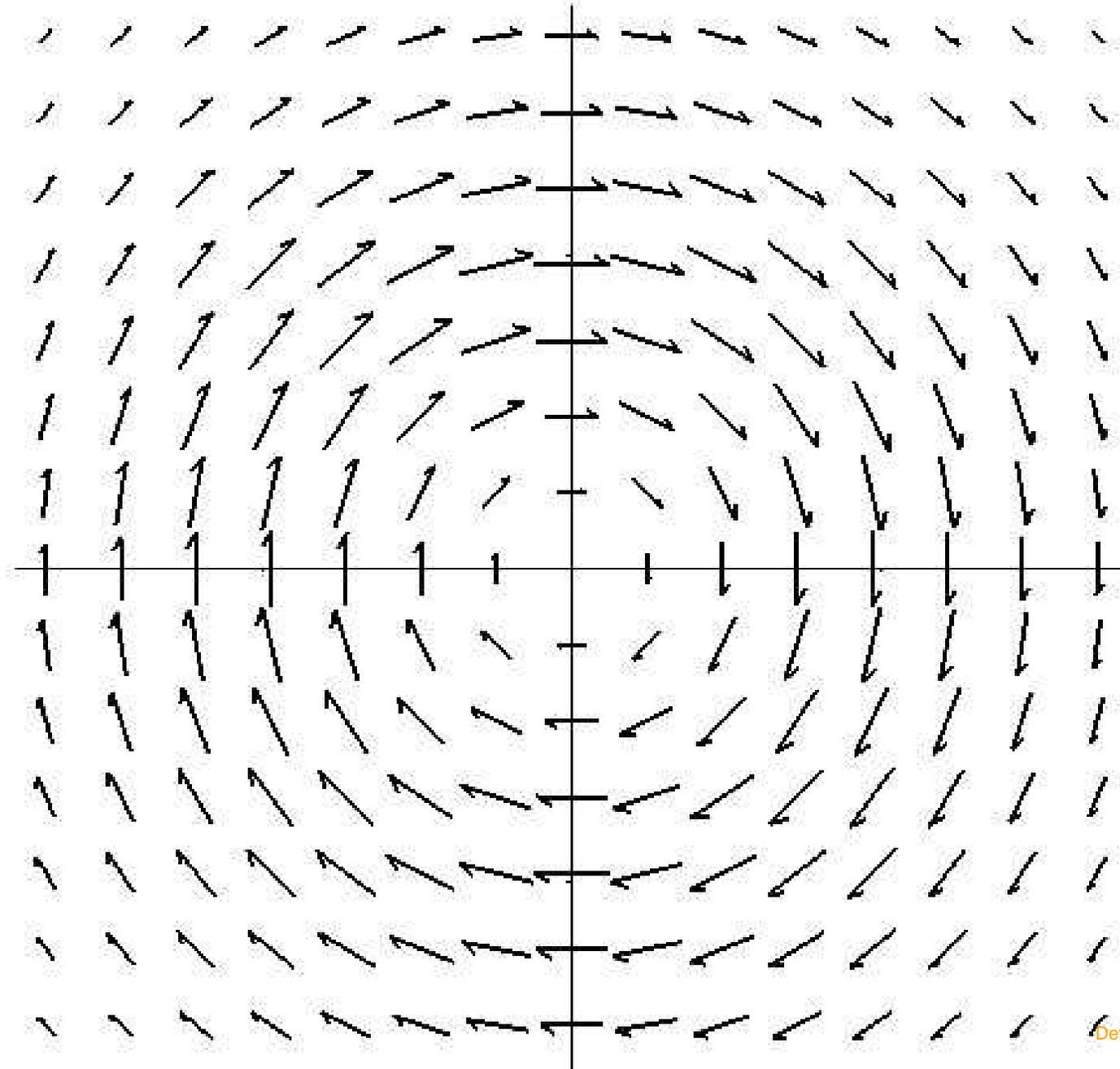
$$\int \frac{dx^-}{2\pi} e^{ixp^+ x^-} \left\langle p' \left| \bar{q} \left(-\frac{x^-}{2} \right) \sigma^{+j} \gamma_5 q \left(\frac{x^-}{2} \right) \right| p \right\rangle = H_T \bar{u} \sigma^{+j} \gamma_5 u + \tilde{H}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha P_\beta}{M^2} u + E_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $\bar{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in \perp plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \Delta_\perp} \bar{E}_T^q(x, 0, -\Delta_\perp^2)$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Chirally Odd GPDs

- $J^i = \frac{1}{2} \varepsilon^{ijk} \int d^3x [T^{0j} x^k - T^{0k} x^j]$

- J_q^x diagonal in transversity, projected with $\frac{1}{2}(1 \pm \gamma^x \gamma_5)$, i.e. one can decompose

$$J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$$

where $J_{q,\pm\hat{x}}^x$ is the contribution (to J_q^x) from quarks with positive (negative) transversity

↪ derive analog to Ji's sum rule (unpol. target), e.g.

$$\langle J_{q,+\hat{y}}^y \rangle = \frac{1}{4} \int dx [H_T^q(x, 0, 0) + \bar{E}_T^q(x, 0, 0)] x$$

↪ \bar{E}_T^q provides quantitative information about the correlation between quark transversity and quark angular momentum)

Boer-Mulders function

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry
 - ↪ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction
 - ↪ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD \bar{E}_T that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E .
- **Boer-Mulders**: distribution of \perp pol. quarks in unpol. proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2} \left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M} \right]$$

Transversity Distribution in Unpolarized Target

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 - ↪ semi-quantitative predictions for $h_1^\perp(x, \mathbf{k}_\perp)$
 - sign of h_1^\perp opposite to sign of \bar{E}_T
 - “ $\frac{h_1^\perp}{\bar{E}_T} \approx \frac{f_{1T}^\perp}{E}$ ”
- use measurement of h_1^\perp to learn about spin-orbit correlation
- lattice (→ P.Hägler et al.):
 - $\bar{E}_T^q > 0 \Rightarrow$ expect $h_1^{\perp q}$ same sign as $f_{1T}^{\perp, u}$
 - $\bar{E}_T^q > E^u \Rightarrow \left| h_1^{\perp q} \right| > \left| f_{1T}^{\perp u} \right|$

Transversity Distribution in Unpolarized Target

- $\bar{E}_T > E$ (lattice) means that there is a stronger correlation between the quark orbital motion and the quark spin than between the quark orbital motion and the nucleon spin.
- same sign for \bar{E}_T^u and \bar{E}_T^d (lattice) means that up and down quarks have similar spin-orbit correlation
- ↪ both are e.g. consistent with a mean-field-type picture of the nucleon (e.g. bag model, or large N_C) where all quarks have, up to j_z the same “wave function” and hence spin-orbit correlations
- models calcs: \bar{E}_T in π also same sign as \bar{E}_T
- ↪ seems there is something “universal” about the sign of spin orbit correlations in (ground state) hadrons

Summary

- GPDs \xrightarrow{FT} PDFs in impact parameter space
- $E(x, 0, -\Delta_{\perp}^2) \longrightarrow \perp$ deformation of PDFs for \perp polarized target
- origin for deformation: orbital motion of the quarks
[decomposition of quark spin (J_i relation) w.r.t. quark transversity]
- dipole deformation of the target \Rightarrow interpretation for SSAs in exclusive reactions
- simple mechanism (attractive FSI) to predict sign of f_{1T}^q
- distribution of \perp polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + \tilde{E}_T^q$
- \hookrightarrow origin: correlation between orbital motion and spin of the quarks
- \hookrightarrow simple mechanism (attractive FSI) to predict sign of h_1^{\perp}
- \hookrightarrow or, use lensing mechanism to qualitatively “extract” \bar{E}_T^q from measurement of h_1^{\perp}

⊥ Single Spin Asymmetry (Sivers)

- Naive definition of unintegrated parton density

$$f(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{q}(0) \gamma^+ q(\xi) | P, S \rangle \Big|_{\xi^+ = 0}.$$

- Time-reversal invariance $\Rightarrow f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp)$

↪ Asymmetry $\int d^2\mathbf{k}_\perp f(x, \mathbf{k}_\perp) \mathbf{k}_\perp = 0$

- Same conclusion for gauge invariant definition with straight Wilson line $U_{[0, \xi]} = P \exp \left(ig \int_0^1 ds \xi_\mu A^\mu(s\xi) \right)$

⊥ Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) $f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp)$
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

$$f(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{q}(0) U_{[0, \infty]} \gamma^+ U_{[\infty, \xi]} q(\xi) | P, S \rangle \Big|_{\xi^+ = 0}$$

$$\text{with } U_{[0, \infty]} = P \exp \left(ig \int_0^\infty d\eta^- A^+(\eta) \right)$$

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp \left(ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1$$

- ↪ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!
- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$f(x, \mathbf{k}_\perp) = \int \frac{dy^- d^2 \mathbf{y}_\perp}{16\pi^3} e^{-ixp^+ y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\ \times \langle p, s | \bar{q}(y) \gamma^+ U_{[y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]} U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} U_{[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]} q(0) | p, s \rangle$$

back