Two-photon physics

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Outline

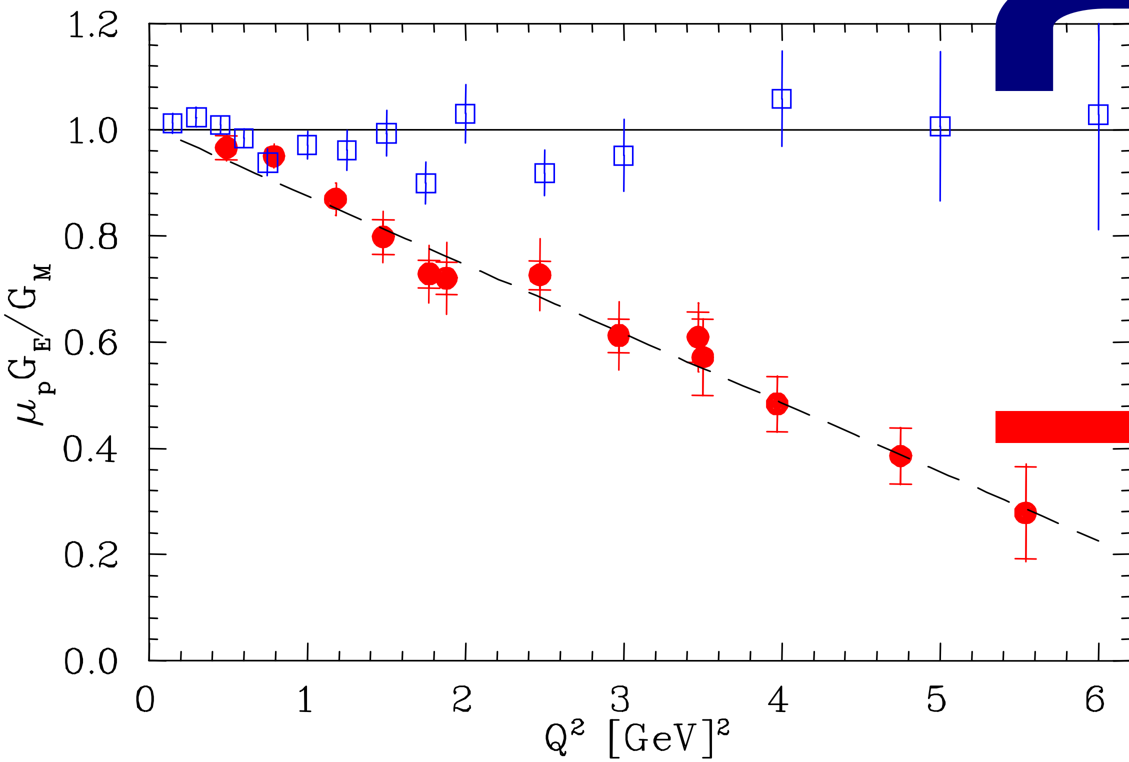
- Introduction : Rosenbluth vs polarization measurements of G_E and G_M of nucleon

  ⟹ puzzle : different results extracted for G_E/G_M
- Elastic electron-nucleon scattering beyond the one-photon exchange approximation
- Partonic calculation of two-photon exchange contribution

  ⟹ generalized parton distributions of nucleon
- Results for cross section and polarization transfer
- SSA in elastic electron-nucleon scattering

P.A.M. Guichon, M.Vdh PRL 91, 142303 (2003)
Y.C. Chen, A.Afanasev, S. Brodsky, C. Carlson, M.Vdh PRL 93 (2004) in press
M. Gorchtein, P.A.M. Guichon, M. Vdh NPA 741 (2004) 234
B. Pasquini, M. Vdh hep-ph/0405303

Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



SLAC

Rosenbluth data

Jlab/Hall A

Polarization data

Jones et al. (2000)

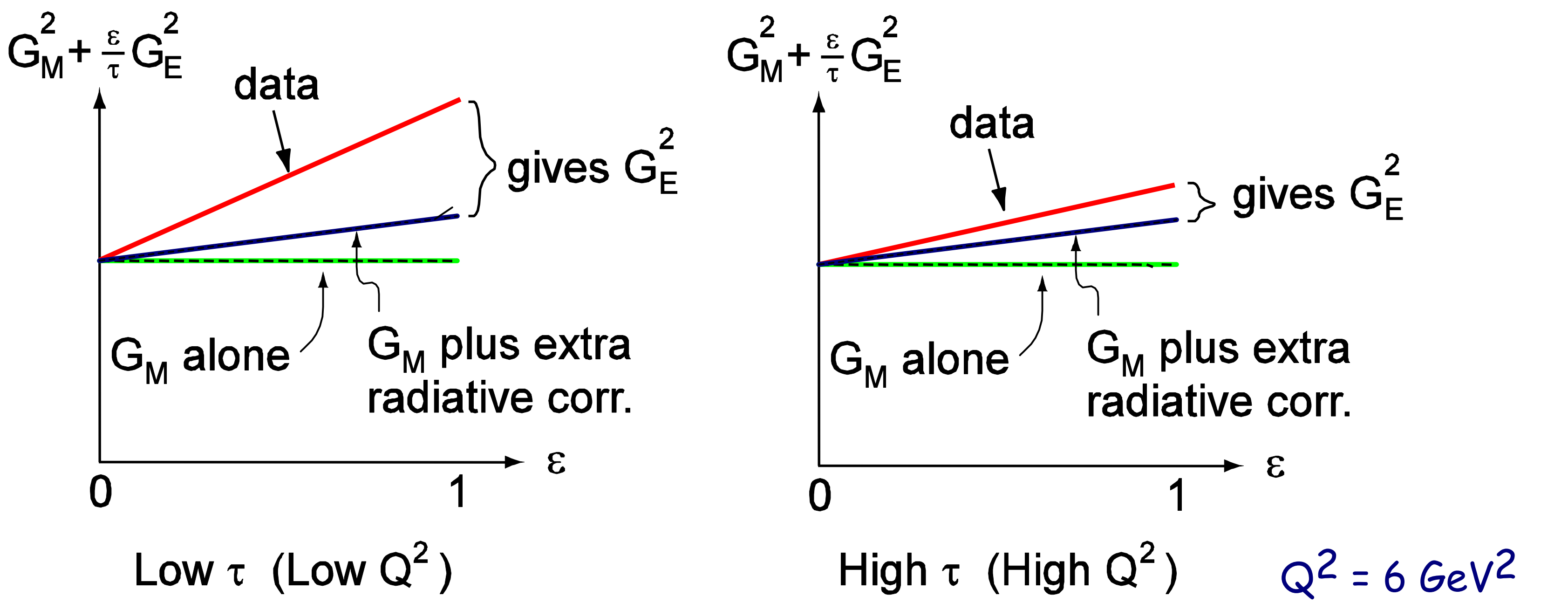
Gayou et al. (2002)

Two methods, two different results !

Speculation : missing radiative corrections

Speculation : there are radiative corrections to Rosenbluth experiments that are important and are not included

missing correction : linear in ε, not strongly Q^2 dependent



$Q^2 = 6\ GeV^2$

G_E term is proportionally smaller at large Q^2

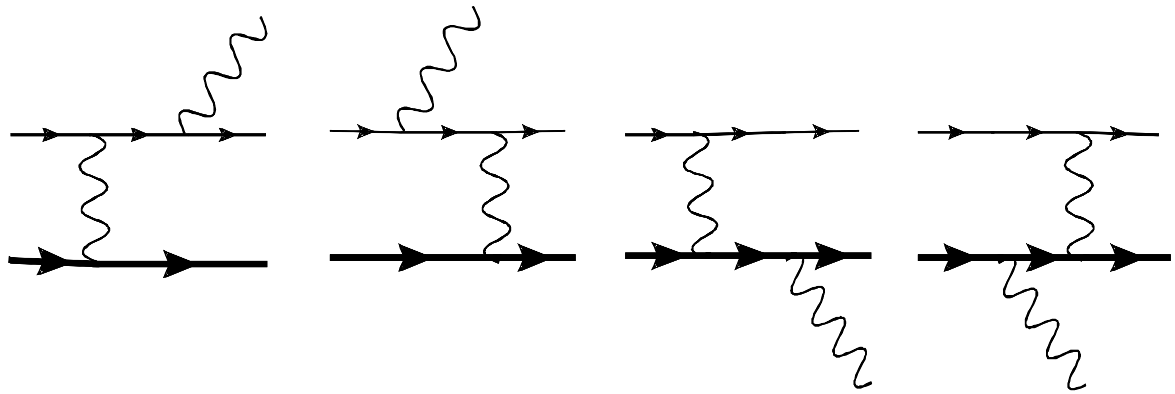
$$\frac{G_E^2}{\tau G_M^2} = \frac{4M^2}{Q^2 \mu_p^2} = 7.5\%$$

effect more visible at large Q^2

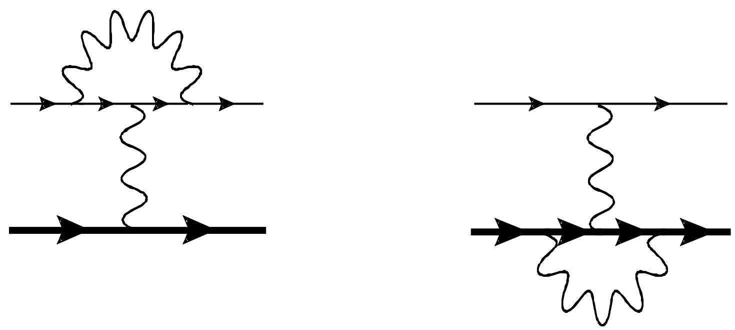
if both FF scale in same way

Radiative correction diagrams

bremsstrahlung

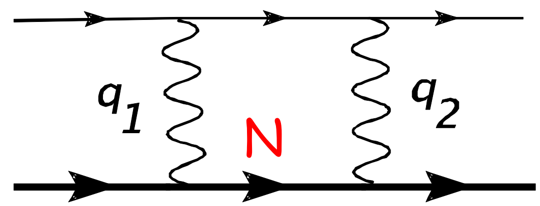


vertex corrections



2 photon exchange box diagrams

Status of radiative corrections



- Tsai (1961), Mo & Tsai (1968)

  box diagram calculated using only nucleon intermediate state and using q_1 ¼ 0 or q_2 ¼ 0 in both numerator and denominator (calculate 3-point function) -> gives correct IR divergent terms

- Maximon & Tjon (2000)

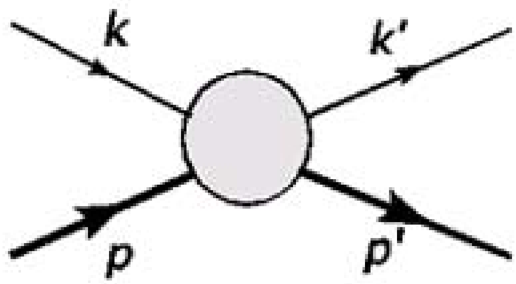
  same as above, but make the above approximation only in numerator (calculate 4-point function)

  + use on-shell nucleon form factors in loop integral

- Blunden, Melnitchouk, Tjon (2003)

  further improvement by keeping the full numerator

Elastic eN scattering beyond one-photon exchange approximation



$$P \equiv \frac{p+p'}{2}, \quad K \equiv \frac{k+k'}{2}$$

Kinematical invariants :

$$\begin{aligned} Q^2 &= -(p-p')^2 \\ \nu &= K \cdot P = (s-u)/4 \end{aligned}$$

$$T^{non-flip}_{h'\lambda'_N, h\lambda_N} = \frac{e^2}{Q^2}\bar{u}(k',h')\gamma_\mu u(k,h)$$

$(m_e = 0)$ $$\times \quad \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \tilde{F}_3\frac{\gamma.K P^\mu}{M^2}\right)u(p,\lambda_N)$$

$$\begin{aligned} \tilde{G}_M(\nu,Q^2) &= G_M(Q^2) + \delta\tilde{G}_M \\ \tilde{F}_2(\nu,Q^2) &= F_2(Q^2) + \delta\tilde{F}_2 \\ \tilde{F}_3(\nu,Q^2) &= 0 + \delta\tilde{F}_3 \end{aligned}$$

equivalently, introduce

$$\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\,\tilde{F}_2$$

$$\tilde{G}_E(\nu,Q^2) = G_E(Q^2) + \delta\tilde{G}_E$$

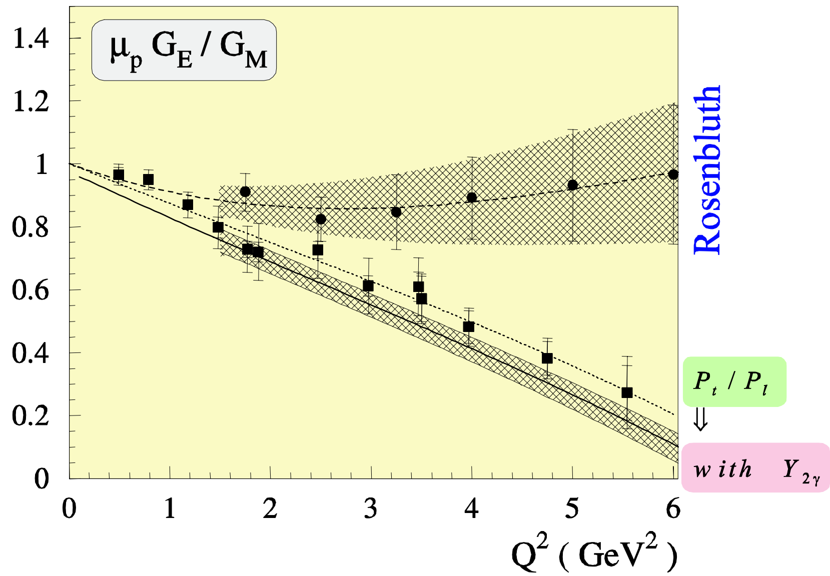
Observables including two-photon exchange

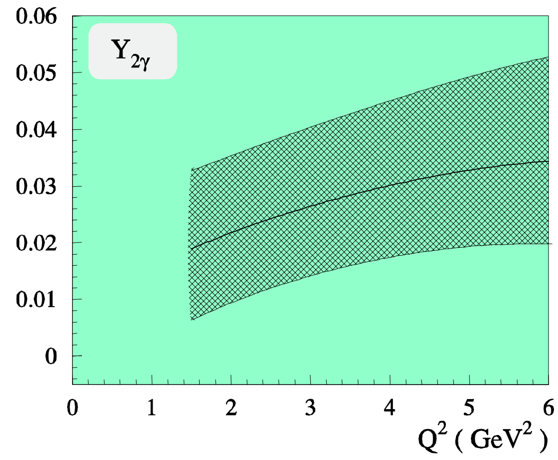
Real parts of two-photon amplitudes

$$\sigma_R = G_M^2 \left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \varepsilon\left\{\frac{1}{\tau}G_E^2\left(1+2\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_E}\right) + 2G_M^2\left(1+\frac{1}{\tau}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\,\varepsilon}{\tau\,(1+\varepsilon)}}\left\{\frac{G_E}{G_M}\left(1-\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right)+\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_M} + \left(1-\frac{2\,\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

Phenomenological analysis

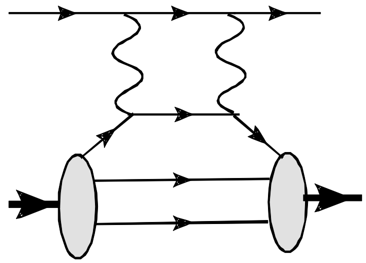




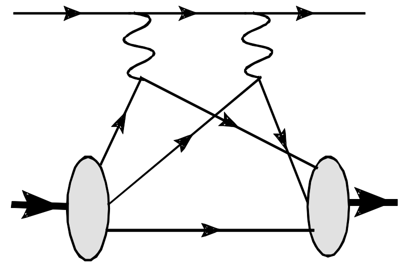
2-photon exchange is a candidate to explain the discrepancy between both experimental methods

Guichon, Vdh (2003)

Partonic calculation of two-photon exchange contribution



"handbag"



"cat's ears"

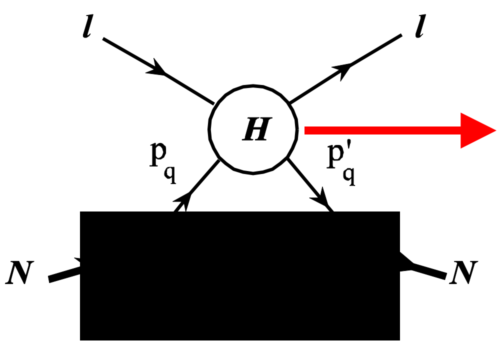
- main contribution at large Q^2 :

  handbag diagrams (one active quark)

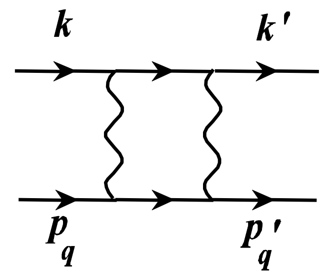
- to reproduce the IR divergent contribution at nucleon correctly (i.e. to satisfy the Low Energy Theorem)

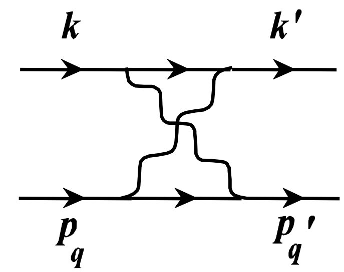
  need cat's ears diagrams (two active quarks)

Calculation of hard scattering amplitude



hard scattering amplitude





$$H_{h,\lambda} = \frac{(e\, e_q)^2}{Q^2}\, \bar{u}(k',h)\gamma_\mu u(k,h) \cdot \bar{u}(p'_q,\lambda)\left(\tilde{f}_1\, \gamma^\mu + \tilde{f}_3\, \gamma.K\, P_q^\mu\right) u(p_q,\lambda)$$

electron helicity quark helicity

$$K \equiv (k+k')/2 \qquad P_q \equiv (p_q + p'_q)/2$$

- Calculation for eμ –> eμ can be found in literature (e.g. van Nieuwenhuizen (1971)), which we verified explicitly
- IR divergences of boxes must disappear or cancel in the end, regularize through photon mass λ

Separation soft-hard parts in electron-quark box

Follow the decomposition of Grammer and Yennie (1973) :

soft part calculated as 3-point function

reproduces Low Energy Theorem

kinematics partonic subprocess : $\hat{s} \equiv (k + p_q)^2, \qquad \hat{u} \equiv (k - p'_q)^2, \qquad \hat{s} + \hat{u} = Q^2$

$$\mathcal{R}\left(\tilde{f}_1^{soft}\right) = \frac{e^2}{4\pi^2}\left\{\ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\hat{u}}}\right)\ln\left|\frac{\hat{s}}{\hat{u}}\right| + \frac{\pi^2}{2}\right\}$$

$$\mathcal{R}\left(\tilde{f}_1^{hard}\right) = \frac{e^2}{4\pi^2}\left\{\frac{1}{2}\ln\left|\frac{\hat{s}}{\hat{u}}\right| + \frac{Q^2}{4}\left[\frac{1}{\hat{u}}\ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{1}{\hat{s}}\ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{1}{\hat{s}}\pi^2\right]\right\}$$

$$\mathcal{R}\left(\tilde{f}_3\right) = \frac{e^2}{4\pi^2}\frac{1}{\hat{s}\,\hat{u}}\left\{\hat{s}\,\ln\left|\frac{\hat{s}}{Q^2}\right| + \hat{u}\,\ln\left|\frac{\hat{u}}{Q^2}\right| + \frac{\hat{s}-\hat{u}}{2}\left[\frac{\hat{s}}{\hat{u}}\ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}}\ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}}\pi^2\right]\right\}$$

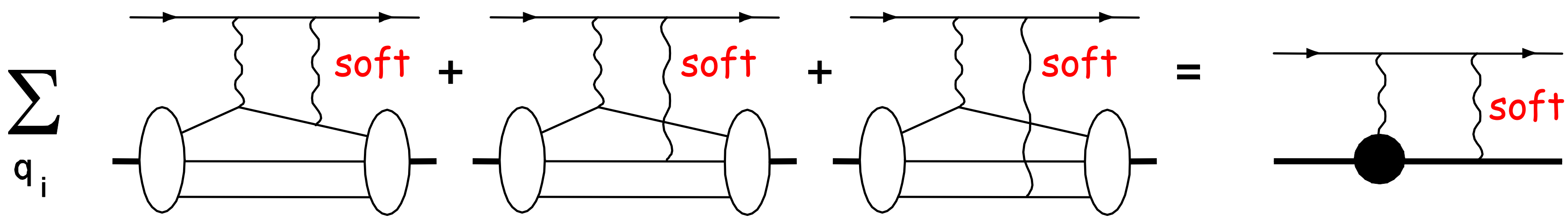
Calculation of soft part at nucleon level

LET : sum of soft contributions from the partonic calculation has to match the soft contributions at nucleonic level

To satisfy the LET, one has to include the

soft-photon contributions from the cats' ears diagrams

Pictorially :



Calculation of bremsstrahlung

$$\sigma_R^{lab} = \sigma_{1\gamma}^{lab}\ (1 + \underbrace{\delta_{2\gamma,\,soft} + \delta_{brems}^{ep}}_{\text{IR finite}})$$

soft part of electron-nucleon box

$$\delta_{2\gamma}^{soft} = \frac{e^2}{2\pi^2}\left\{\ln\left(\frac{\lambda^2}{\sqrt{(s-M^2)|u-M^2|}}\right)\ln\left|\frac{s-M^2}{u-M^2}\right| - L\left(\frac{s-M^2}{s}\right) - \frac{1}{2}\ln^2\left(\frac{s-M^2}{s}\right) + \mathcal{R}\left[L\left(\frac{u-M^2}{u}\right)\right] + \frac{1}{2}\ln^2\left(\frac{u-M^2}{u}\right) + \frac{\pi^2}{2}\right\}$$

bremsstrahlung contribution : Maximon, Tjon (2000)

$$\delta_{brems}^{ep} = \frac{e^2}{2\pi^2}\left\{\ln\left(\frac{4\,(\Delta E)^2\,E_e^2}{\lambda^2\,y\,E_e'^2}\right)\ln\left(\frac{E_e}{E_e'}\right) + L\left(1-\frac{1}{y}\frac{E_e}{E_e'}\right) - L\left(1-\frac{1}{y}\frac{E_e'}{E_e}\right)\right\}$$

Experimentalists do corrections according to Mo-Tsai

relative to Mo-Tsai, the above formula gives

a correction factor $(1 + \pi\,\alpha)$ + terms of size 0.001

$$y \equiv (\sqrt{\tau} + \sqrt{1+\tau})^2$$

$$\Delta E = E_e'^{\,el} - E_e'$$

Convolution with GPDs

result for handbag amplitude (large Q^2)

$$
\begin{aligned}
T^{hard}_{h,\lambda'_N\lambda_N} \;=\; & \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2}\left[H^{hard}_{h,+\frac{1}{2}} + H^{hard}_{h,-\frac{1}{2}}\right] \\
\times\; & \left[H^q\left(x,0,q^2\right)\, \bar{u}(p',\lambda'_N)\,\gamma\cdot n\, u(p,\lambda_N) + E^q\left(x,0,q^2\right)\, \bar{u}(p',\lambda'_N)\, \frac{i\,\sigma^{\mu\nu}\, n_\mu q_\nu}{2M}\, u(p,\lambda_N)\right] \\
+\; & \int_{-1}^{1} \frac{dx}{x} \sum_q \frac{1}{2}\left[H^{hard}_{h,+\frac{1}{2}} - H^{hard}_{h,-\frac{1}{2}}\right] \mathrm{sgn}(x)\, \tilde{H}^q\left(x,0,q^2\right)\, \bar{u}(p',\lambda'_N)\,\gamma\cdot n\,\gamma_5\, u(p,\lambda_N)
\end{aligned}
$$

work in frame $q^+ = 0$, n^μ is a Sudakov vector ($n^2 = 0$, $n \,.\, P = 1$)

handbag amplitude depends on GPD(x, $\xi = 0$, Q^2),

which also appear in other wide angle scattering processes (e.g. WACS)

Hard part to invariant amplitudes for elastic eN scattering

$$
\begin{aligned}
\delta\tilde{G}_M^{hard} &= C \\
\delta\tilde{G}_E^{hard} &= -\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C)+\sqrt{\frac{1+\varepsilon}{2\varepsilon}}\,B \\
\tilde{F}_3 &= \frac{M^2}{\nu}\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C)
\end{aligned}
$$

GPD integrals

$$A \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard}-\hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\left(H^q+E^q\right),$$ "magnetic" GPD

$$B \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard}-\hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\left(H^q-\tau E^q\right)$$ "electric" GPD

$$C \equiv \int_{-1}^{1}\frac{dx}{x}\tilde{f}_1^{hard}\,\mathrm{sgn}(x)\sum_q e_q^2\,\tilde{H}^q$$ "axial" GPD

Observables including two-photon exchange in terms of A, B, C (real parts)

$$
\begin{aligned}
\sigma_R &= G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \\
&+ (1+\varepsilon)\, G_M\, \mathcal{R}\,(A) + \sqrt{2\,\varepsilon\,(1+\varepsilon)}\frac{1}{\tau}\, G_E\, \mathcal{R}\,(B) + (1-\varepsilon)\, G_M\, \mathcal{R}\,(C)
\end{aligned}
$$

$$
\begin{aligned}
P_t &= -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\, \frac{1}{\sigma_R} \left\{ G_E G_M + G_E\, \mathcal{R}\,(C) + G_M \sqrt{\frac{1+\varepsilon}{2\,\varepsilon}}\, \mathcal{R}\,(B) \right. \\
&\qquad \left. + \mathcal{O}(e^4) \right\} \\
P_l &= \sqrt{1-\varepsilon^2}\, \frac{1}{\sigma_R} \left\{ G_M^2 + G_M\, \mathcal{R}\,(A+C) + \mathcal{O}(e^4) \right\}
\end{aligned}
$$

Model for GPDs at large Q^2

gaussian-valence model : Radyushkin (1998), Diehl et al. (1999)

$$
\begin{aligned}
H^q(x,0,q^2) &= q_v(x)\,\exp\left(-\frac{(1-x)\,Q^2}{4\,x\,\sigma}\right) \\
\tilde{H}^q(x,0,q^2) &= \Delta q_v(x)\,\exp\left(-\frac{(1-x)\,Q^2}{4\,x\,\sigma}\right) \\
E^q(x,0,q^2) &= \frac{\kappa^q}{N^q}(1-x)^2 q_v(x)\exp\left(-\frac{(1-x)Q^2}{4\,x\,\sigma}\right)
\end{aligned}
$$

$\sigma = 0.8$ GeV^2

Forward parton distributions at μ^2 = 1 GeV^2

MRST2002 NNLO

$$
\begin{aligned}
u_v &= 0.262\,x^{-0.69}(1-x)^{3.50}\,(1+3.83\,x^{0.5}+37.65\,x) \\
d_v &= 0.061\,x^{-0.65}(1-x)^{4.03}\,(1+49.05\,x^{0.5}+8.65\,x)
\end{aligned}
$$

Leader, Sidorov, Stamenov (2002)

$$
\begin{aligned}
\Delta u_v &= 0.505x^{-0.33}(1-x)^{3.428}\,(1+2.179x^{0.5}+14.57x) \\
\Delta d_v &= -0.0185x^{-0.73}(1-x)^{3.864}\,(1+35.47x^{0.5}+28.97x)
\end{aligned}
$$

Model for GPDs at large Q^2

modified Regge model : Guidal, Polyakov, Radyushkin, Vdh (2004)

$$H^q(x,0,q^2) \;=\; q_v(x)\; x^{\alpha_1'\,(1-x)\,Q^2}$$
$$E^q(x,0,q^2) \;=\; \frac{\kappa_q}{N_q}\,(1-x)^{\eta_q}\; q_v(x)\; x^{\alpha_2'\,(1-x)\,Q^2}$$

- forward parton distributions at $\mu^2 = 1\ \mathrm{GeV}^2$ (MRST2002 NNLO)
- regge slopes : α_1' , α_2' determined from rms radii

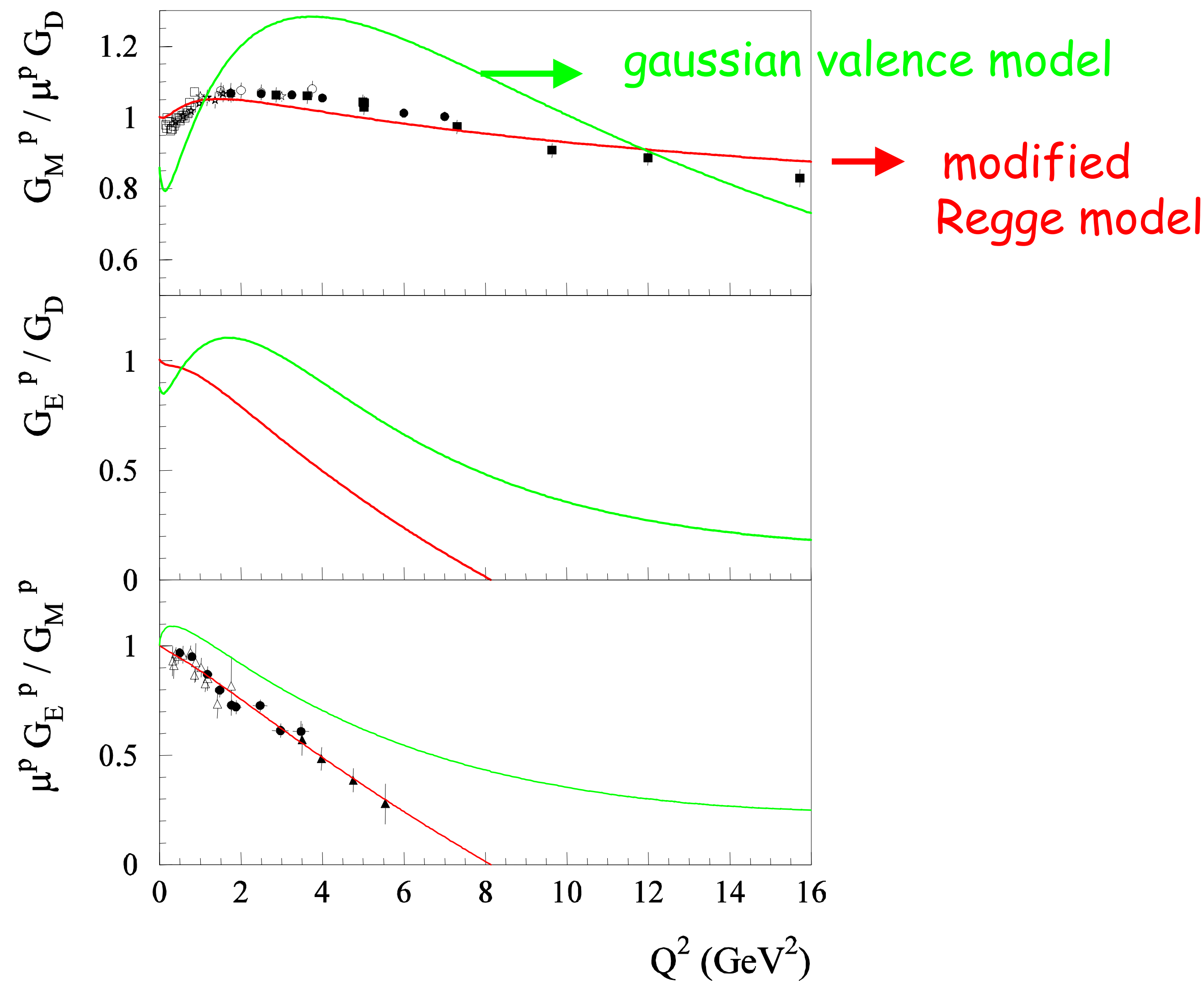
$$r_{1,p}^2 \;=\; -6\,\alpha_1' \int_0^1 dx \left\{ e_u\, u_v(x) + e_d\, d_v(x) \right\} \ln x$$
$$r_{1,n}^2 \;=\; -6\,\alpha_1' \int_0^1 dx \left\{ e_u\, d_v(x) + e_d\, u_v(x) \right\} \ln x$$

$$\alpha_1' = 1.098\ \mathrm{GeV}^{-2}$$
$$\alpha_2' = 1.158\ \mathrm{GeV}^{-2}$$

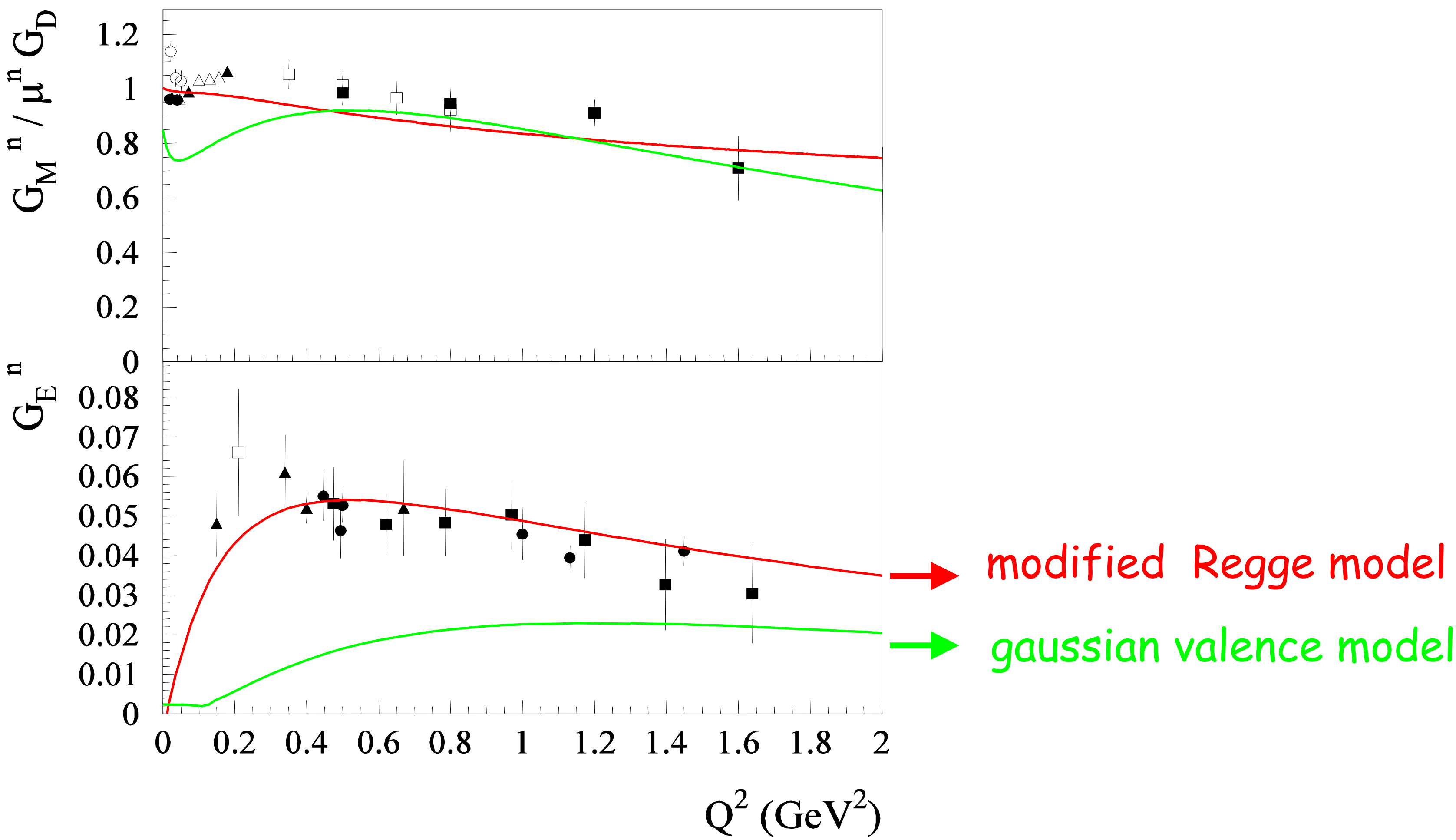
- η_u , η_d determined from F_2 / F_1 at large Q^2

$$\eta_u = 1.52$$
$$\eta_d = 0.31$$

Test of GPD models : proton form factors

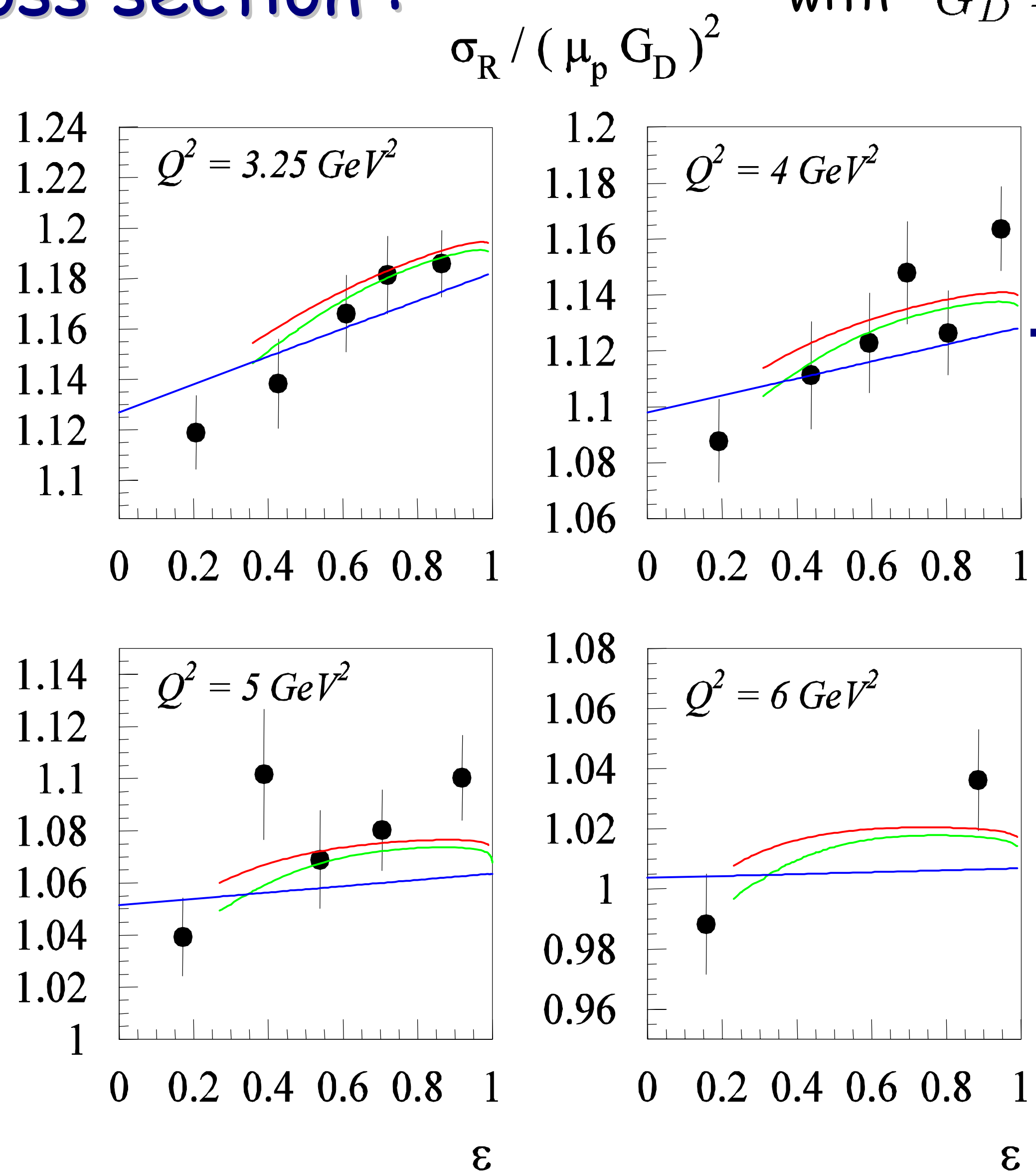


Test of GPD models : neutron form factors



Cross section :

with $G_D = 1/(1+Q^2/0.71)^2$



1 γ

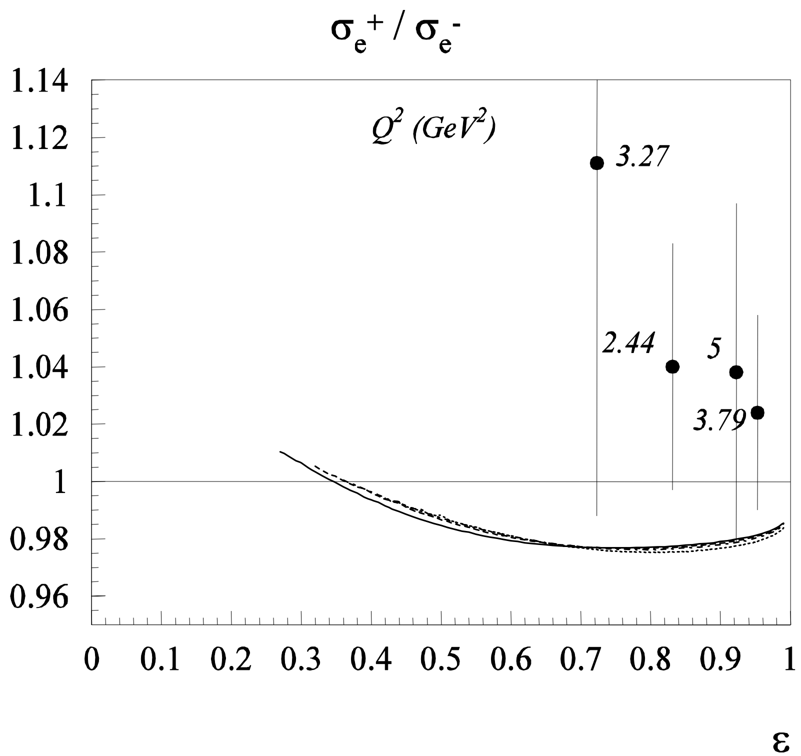
1γ + 2γ (modified Regge model)

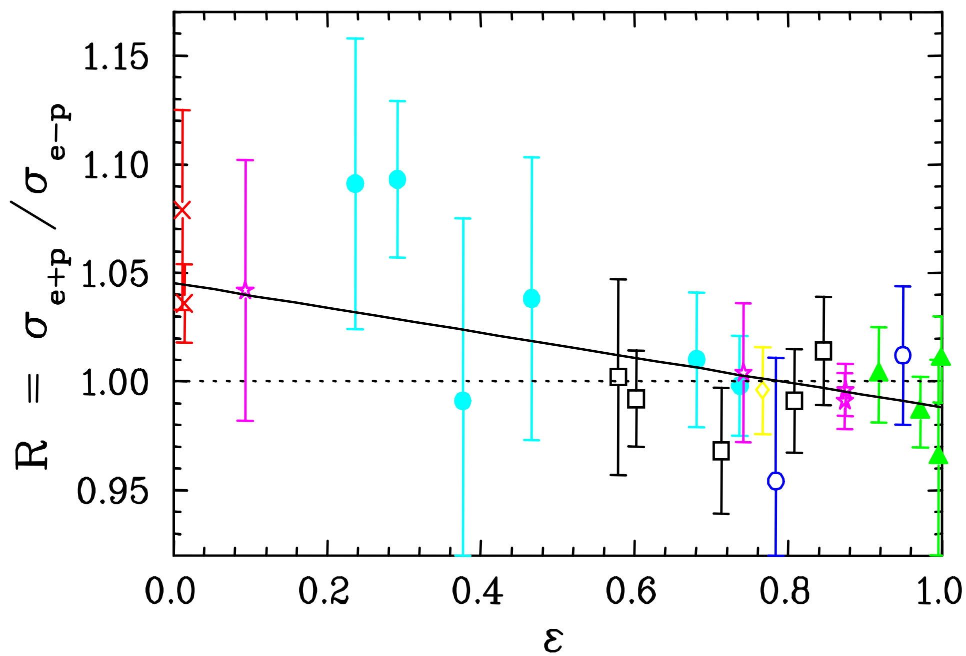
1γ + 2γ (gaussian valence model)

form factors from polarization transfer method

e+ / e- ratio

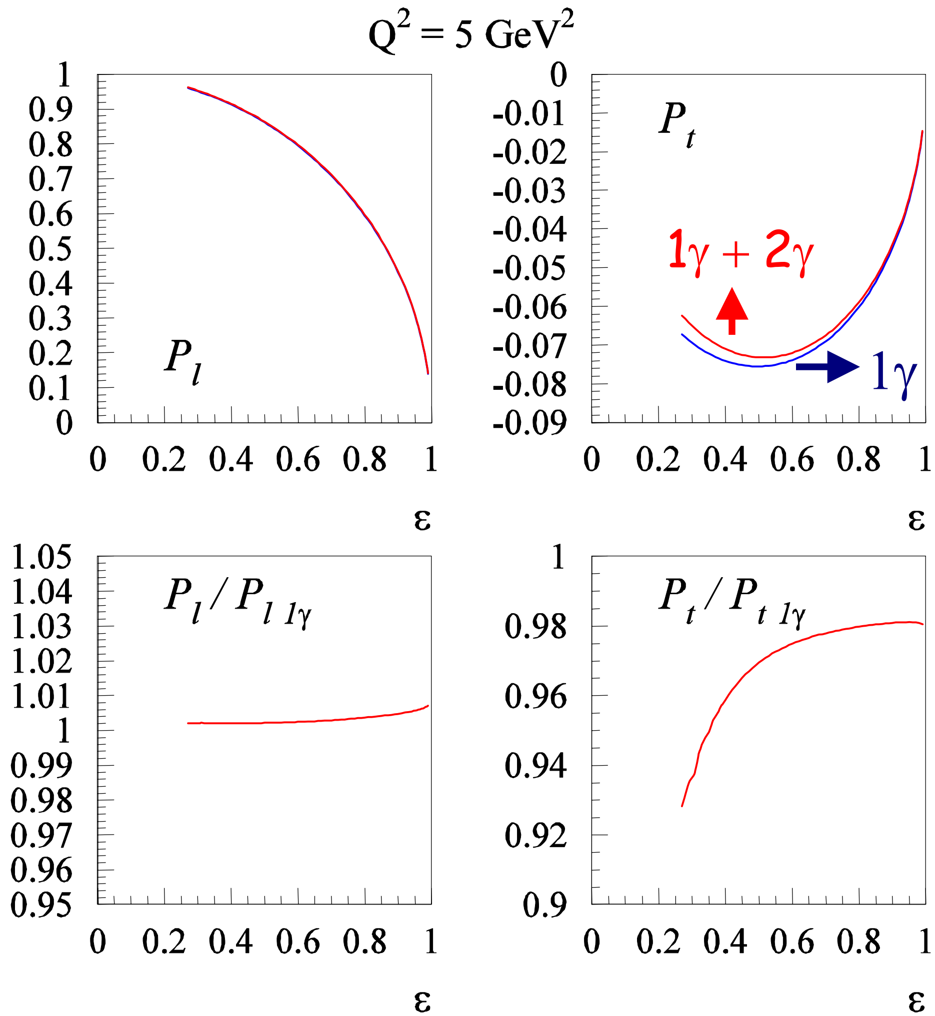
Direct test of real part of 2γ amplitude





data figure from
Arrington (2003)

Polarization transfer observables



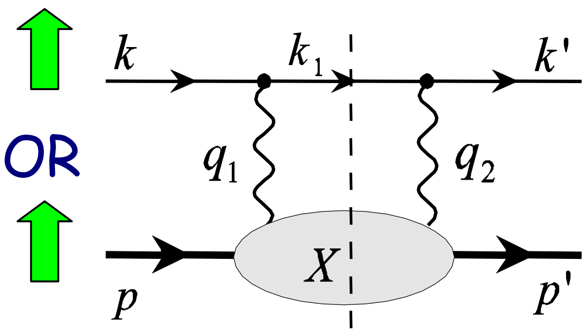
2 γ correction on P_l is small

2 γ correction on P_t can be tested at small ε !

Normal spin asymmetries in elastic eN scattering

- directly proportional to the imaginary part of 2-photon exchange amplitudes

spin of beam OR target NORMAL to scattering plane



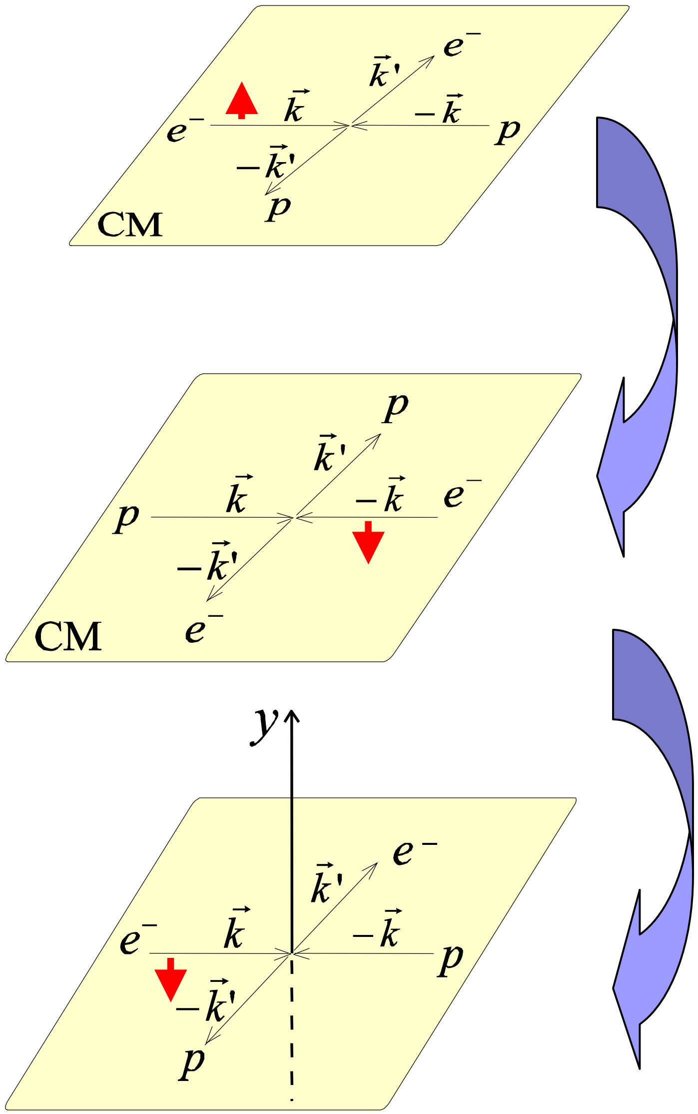
on-shell intermediate state

- order of magnitude estimates :

target : $A_n \sim \alpha_{em} \sim 10^{-2}$

beam : $B_n \sim \alpha_{em} \cdot m_e \sim 10^{-6} - 10^{-5}$

SSA in elastic eN scattering



$$\boxed{\uparrow \;\equiv\; (\hat{k} \times \hat{k}')}$$

$$T_{f\,i} \;\equiv\; T_{\uparrow}\,(\vec{k}, \vec{k}')$$

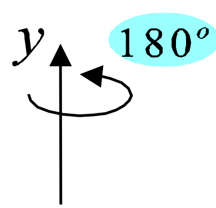
$$|T_{f\,i}|^2 \sim \sigma_{\uparrow}$$

time reversed states

$$i \to \tilde{i}$$
$$f \to \tilde{f}$$

$$T_{\tilde{f}\,\tilde{i}} \;\equiv\; T_{\downarrow}\,(-\vec{k}, -\vec{k}')$$

momenta and spins reversed



$$T_{\downarrow}\,(-\vec{k}, -\vec{k}') = \underset{\text{phase}}{\eta}\, T_{\downarrow}\,(\vec{k}, \vec{k}')$$

$$|T_{\tilde{f}\,\tilde{i}}|^2 \;=\; |T_{\downarrow}\,(\vec{k}, \vec{k}')|^2 \sim \sigma_{\downarrow}$$

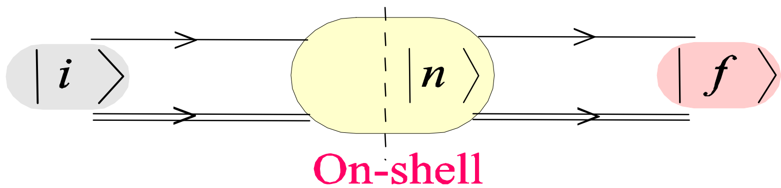
rotation over 180°
around axis ? to plane

$$\boxed{A = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = \frac{|T_{f\,i}|^2 - |T_{\tilde{f}\,\tilde{i}}|^2}{|T_{f\,i}|^2 + |T_{\tilde{f}\,\tilde{i}}|^2}}$$

- Unitarity

$$S\,S^\dagger = S^\dagger\,S = \mathcal{I} \qquad \text{with} \qquad S_{f\,i} = \delta_{f\,i} - iT_{f\,i}$$

$$\Rightarrow \quad i(T_{f\,i} - T^\dagger_{f\,i}) = \Sigma_n T^\dagger_{f\,n} T_{n\,i} \equiv \mathrm{Abs}\, T_{f\,i}$$



- Time reversal invariance : $\;|\,T_{f\,i}\,|^2 \equiv |\,T_{\tilde{i}\,\tilde{f}}\,|^2$

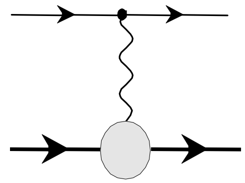
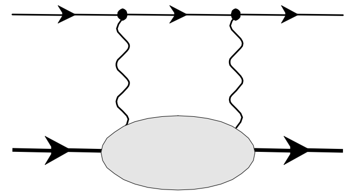
$$\begin{cases} |\mathrm{Abs}\, T_{f\,i}\,|^2 = |T_{f\,i}|^2 + |T_{\tilde{f}\,\tilde{i}}|^2 - 2\,\mathrm{Re}(T_{f\,i}\,T_{i\,f}) \\[2ex] 2\,\mathrm{Im}\,(T^*_{f\,i}\mathrm{Abs}\,T_{f\,i}) = 2\,\mathrm{Re}\,(|T_{f\,i}|^2 - T^*_{f\,i}T^*_{i\,f}) \end{cases}$$

$$\boxed{2\,\mathrm{Im}\,(T^*_{f\,i}\mathrm{Abs}\,T_{f\,i}) - |\mathrm{Abs}\,T_{f\,i}\,|^2 = |T_{f\,i}|^2 - |T_{\tilde{f}\,\tilde{i}}|^2 \sim A}$$

$$A \sim 2\,\mathrm{Im}\,(T^*_{f\,i}\mathrm{Abs}\,T_{f\,i}) - |\mathrm{Abs}\,T_{f\,i}|^2 = |T_{f\,i}|^2 - |T_{\tilde{f}\,\tilde{i}}|^2$$

- Perturbation theory in α_{em}

$T^{1\gamma}_{f\,i}$ $\qquad$ $T^{2\gamma}_{f\,i}$

$T_{f\,i} =$  $+$  $+ \;\cdots$

$\mathcal{O}(\alpha_{em})$ $\qquad$ $\mathcal{O}(\alpha_{em}^2)$

➡ to $\mathcal{O}(\alpha_{em}^2)$ $\quad |T^{1\,\gamma}_{f\,i}|^2 - |T^{1\,\gamma}_{\tilde{f}\,\tilde{i}}|^2 = 0$

1 γ exchange gives no contribution to spin asymmetries

➡ to $\mathcal{O}(\alpha_{em}^3)$

$$|T_{f\,i}|^2 - |T_{\tilde{f}\,\tilde{i}}|^2 = 2\,\mathrm{Im}\,(T^{*\,1\gamma}_{f\,i}\ \mathrm{Abs}\,T^{2\,\gamma}_{f\,i})$$

spin asymmetries arise from interference between
1γ exchange and absorptive part of 2γ exchange

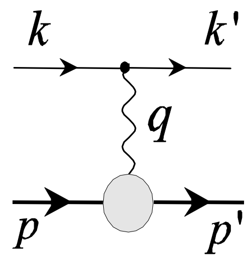
→ to $\mathcal{O}(\alpha_{em})$

$$\boxed{A = \frac{\mathrm{Im}\,(T^{*\,1\gamma}_{f\,i}\,\mathrm{Abs}\,T^{2\gamma}_{f\,i})}{|T^{1\gamma}_{f\,i}|^2}}$$

De Rujula et al. (1971)

→ 1γ exchange

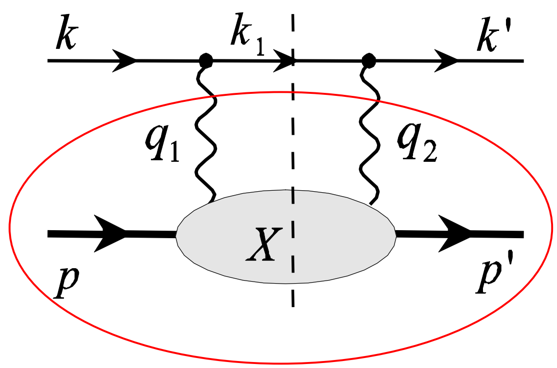
$$s = (k+p)^2$$



$$\Sigma_{\rm spin}\,|T^{1\gamma}_{f\,i}|^2 = \frac{e^4}{Q^4}\,D(s,Q^2)$$

function of elastic nucleon form factors

→ 2γ exchange

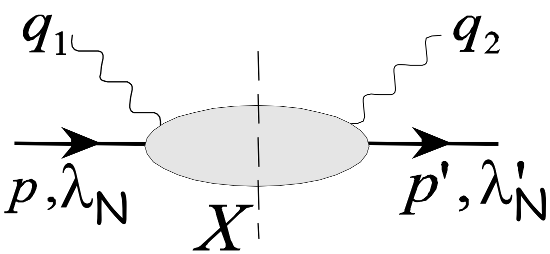


absorptive part of double virtual Compton scattering

$$\mathrm{Abs}\,T^{2\gamma} = e^4 \int \frac{d^3\vec{k}_1}{(2\pi)^3 2E_{k_1}}\,\bar{u}(k',h')\gamma_\mu(\gamma\cdot k_1 + m_e)\gamma_\nu u(k,h) \times \frac{1}{Q_1^2 Q_2^2}\, W^{\mu\nu}(p',\lambda'_N;p,\lambda_N)$$

Hadronic tensor: absorptive part of Double Virtual Compton tensor

$$W^{\mu\nu} = \sum_X (2\pi)^4 \delta^4(p + q_1 - p_X) \, \langle p' \lambda'_N | J^\mu(0) | X \rangle \, \langle X | J^\nu(0) | p \lambda_N \rangle$$



on-shell intermediate state ($M_X^2 = W^2$)

Normal spin asymmetry

$$\boxed{A_n = -\frac{1}{(2\pi)^3} \frac{e^2 (1-\varepsilon)}{8\, Q^2} \int_{M^2}^{s} dW^2 \frac{|\vec{k}_1|^2}{4\sqrt{s}} \int d\Omega_{k1} \frac{1}{Q_1^2\, Q_2^2} \mathcal{I}\left(L_{\alpha\mu\nu} H^{\alpha\mu\nu}\right)}$$

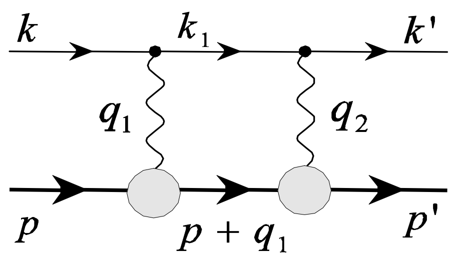
lepton $\quad L_{\alpha\mu\nu} = \bar{u}(k', h') \gamma_\mu (\gamma \cdot k_1 + m_e) \gamma_\nu u(k, h) \cdot [\bar{u}(k', h') \gamma_\alpha u(k, h)]^*$

hadron $\quad H^{\alpha\mu\nu} = W^{\mu\nu} \cdot \left[\bar{u}(p', \lambda'_N) \left(G_M \gamma^\alpha - F_2 \frac{P^\alpha}{M} \right) u(p, \lambda_N) \right]^*$

spin of beam OR target NORMAL to scattering plane

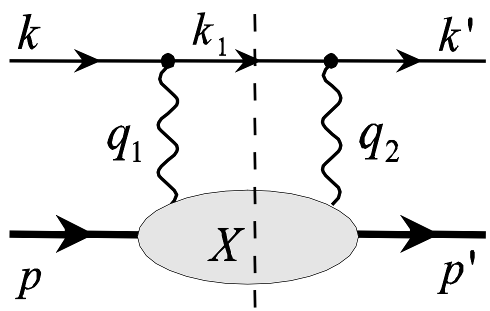
and sum over spins of unpolarized particles

- elastic contribution



on-shell nucleon intermediate nucleon

- inelastic contribution



X= π N

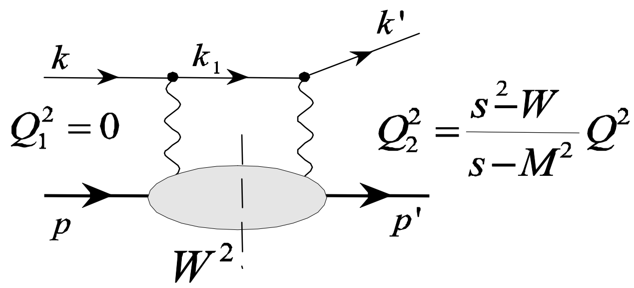
resonant and non-resonant π N intermediate states
calculated with MAID2000

(near) collinear singularities

- $Q_1^2 = 0,\ Q_2^2 \neq 0$

  $\downarrow$

  $\Theta_1 = 0$

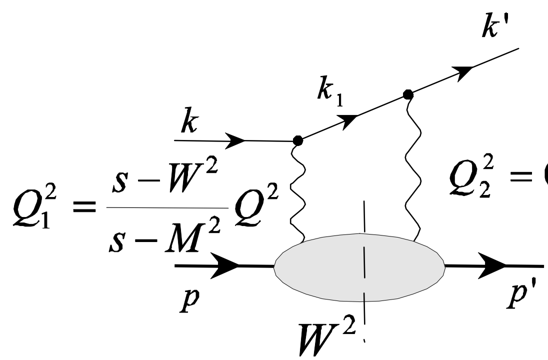


VCS

- $Q_1^2 \neq 0,\ Q_2^2 = 0$

  $\downarrow$

  $\Theta_2 = 0$

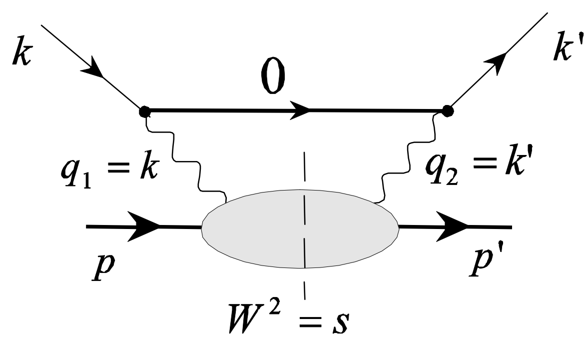


VCS

- $Q_1^2 = 0,\ Q_2^2 = 0$

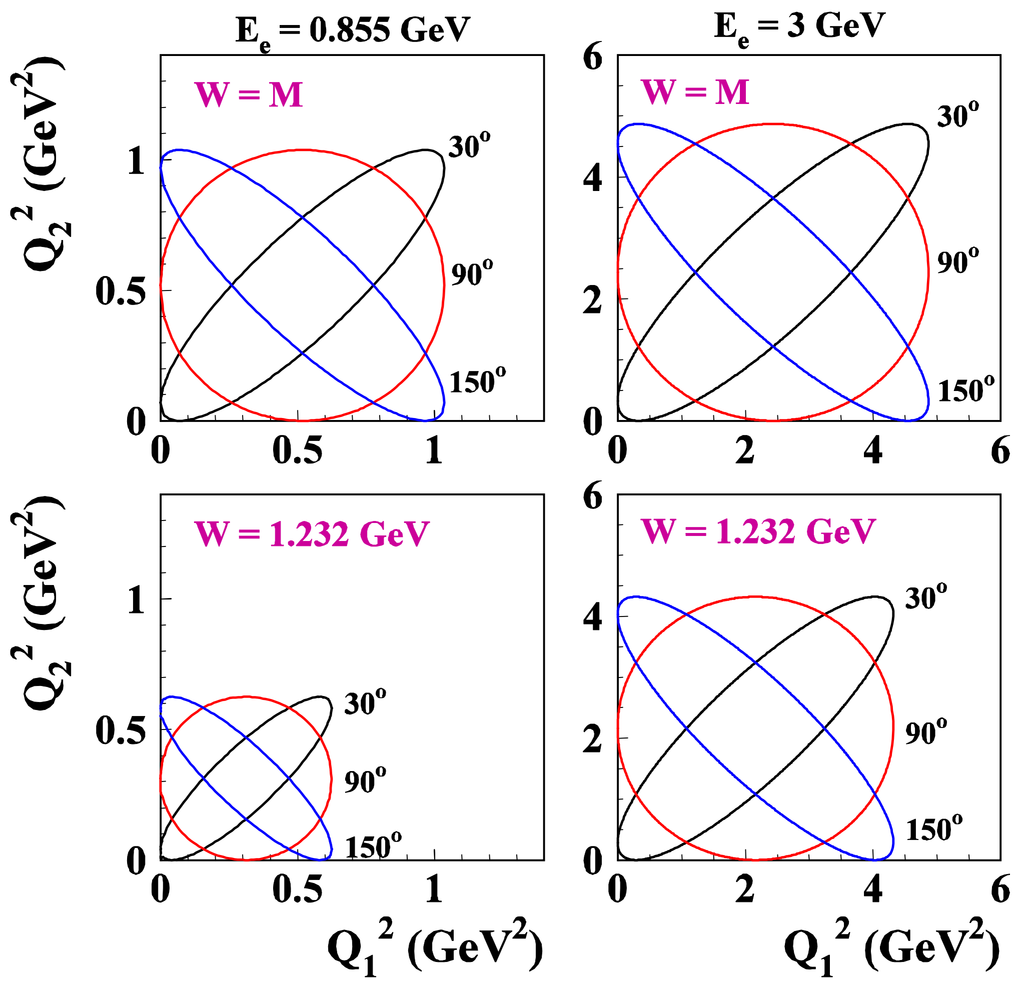
  $\downarrow$

  $k_1 = 0,\ W^2 = s$



Quasi - RCS

Kinematical bounds for Q_1^2 and Q_2^2

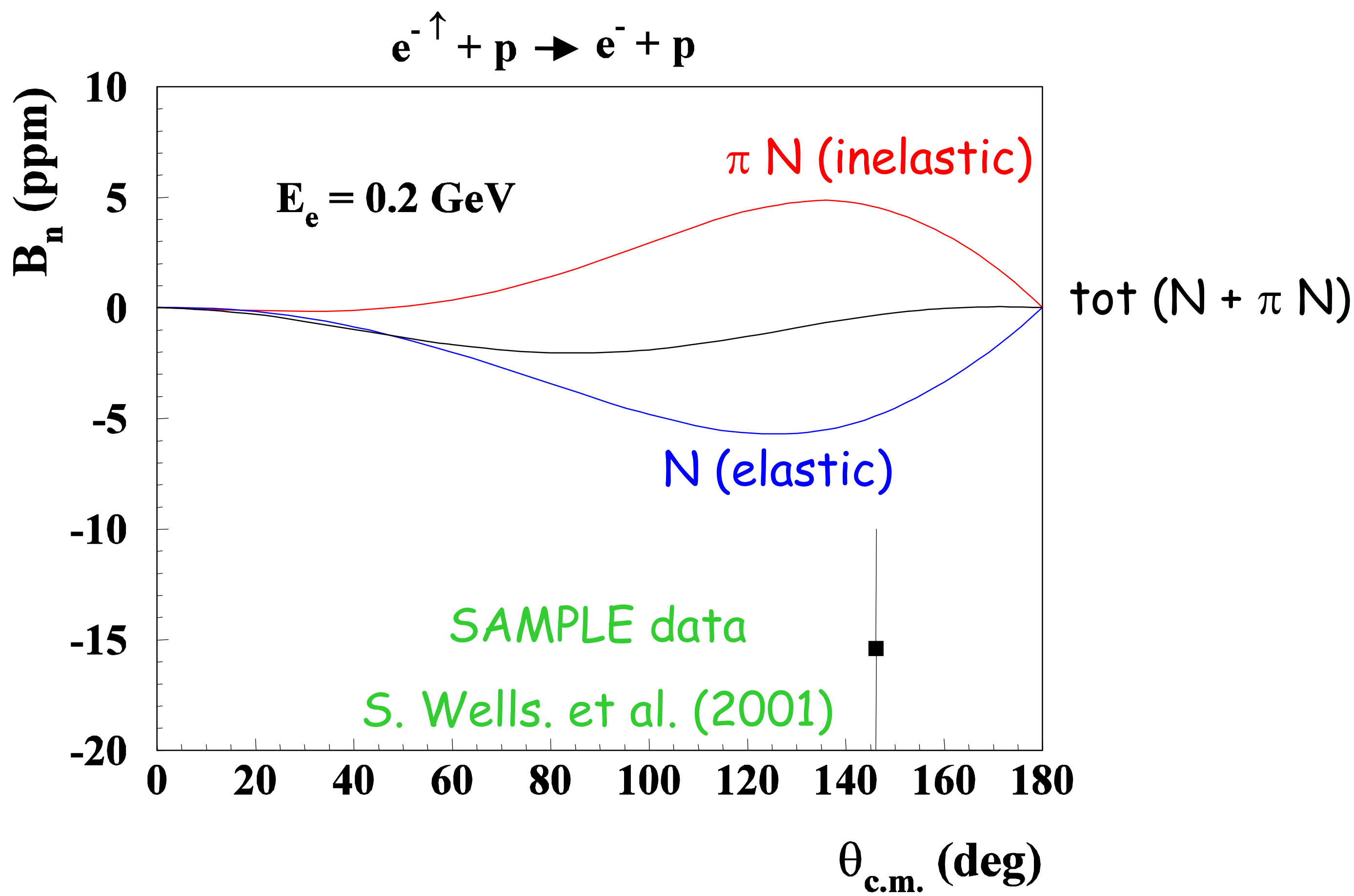


Elastic contribution

Inelastic contribution

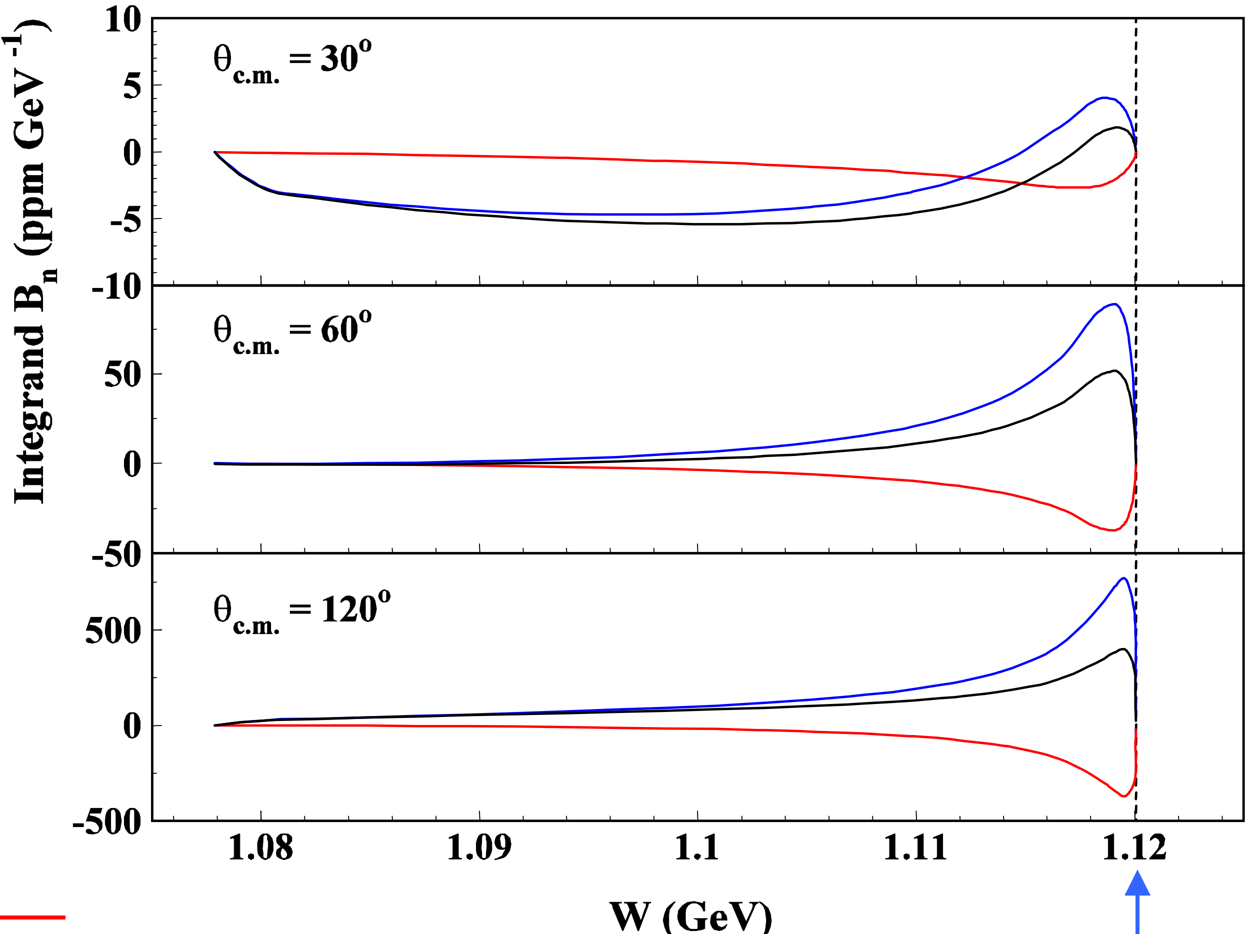
Beam normal spin asymmetry

E_e = 0.2 GeV



Integrand : beam normal spin asymmetry

E_e = 0.2 GeV



π^0 p

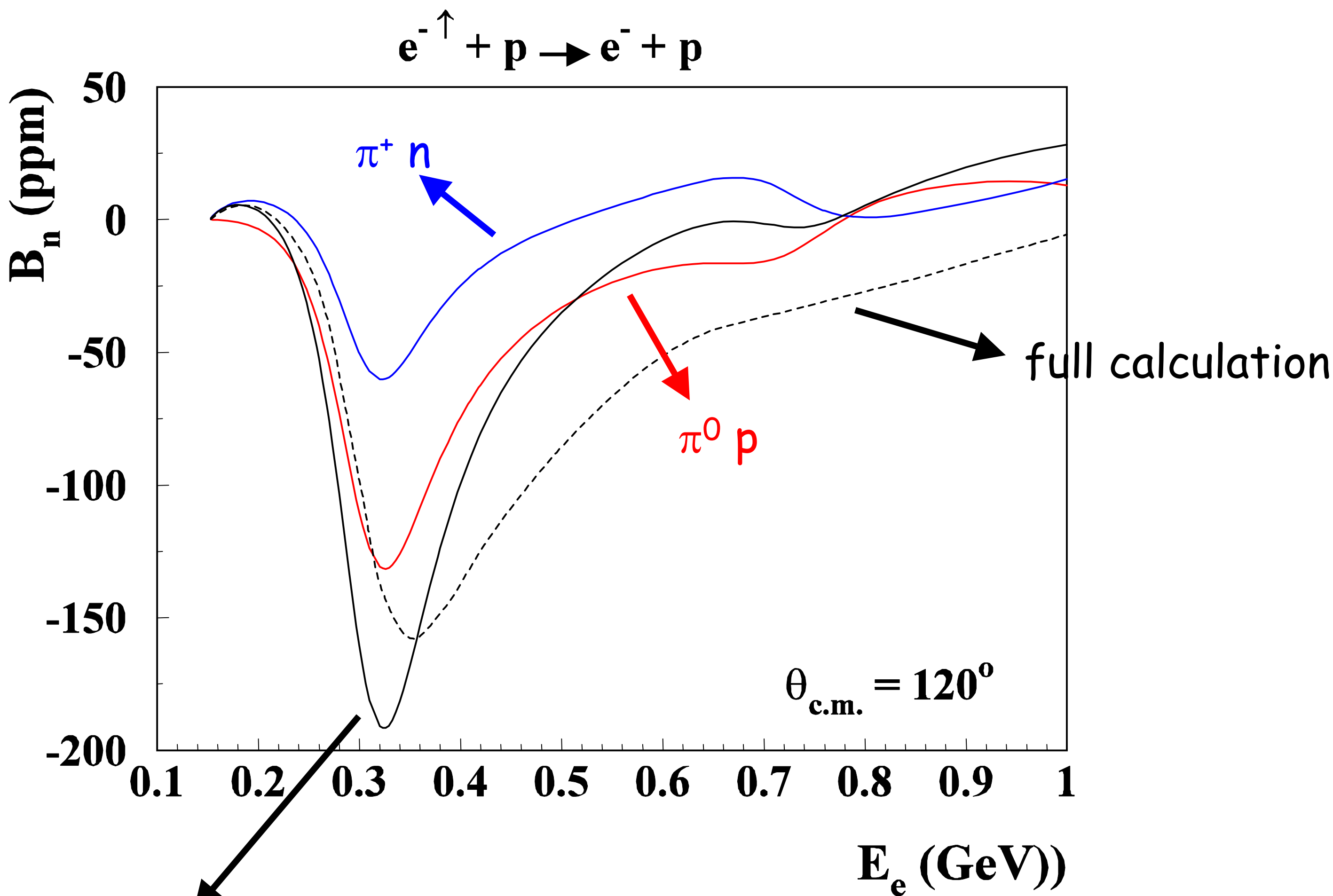
π^+ n

tot

$\sqrt{s}$

Quasi-RCS peak

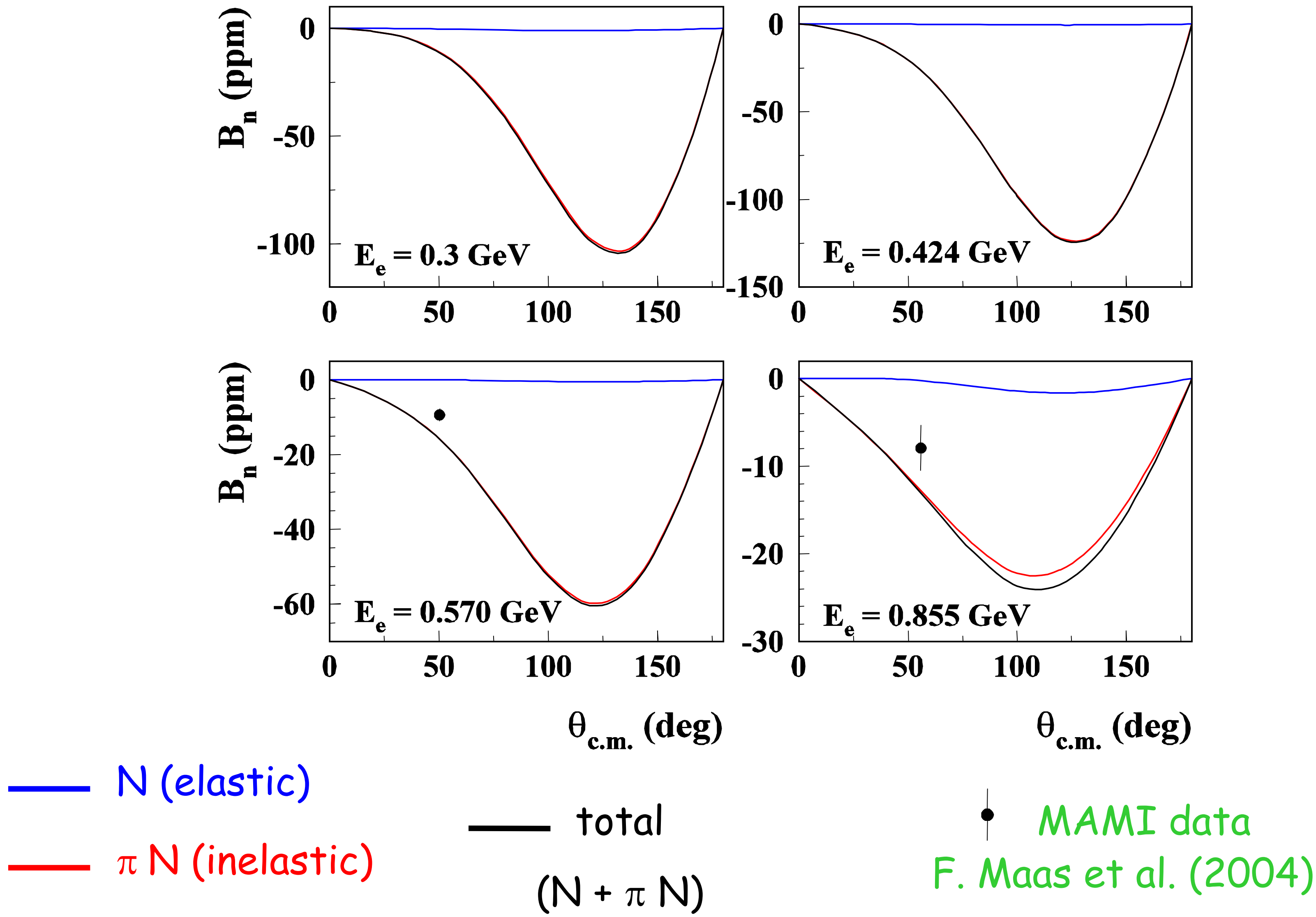
Beam normal spin asymmetry : energy dependence at fixed $\theta_{cm} = 120^o$



quasi-RCS approximation $W = \sqrt{s}$

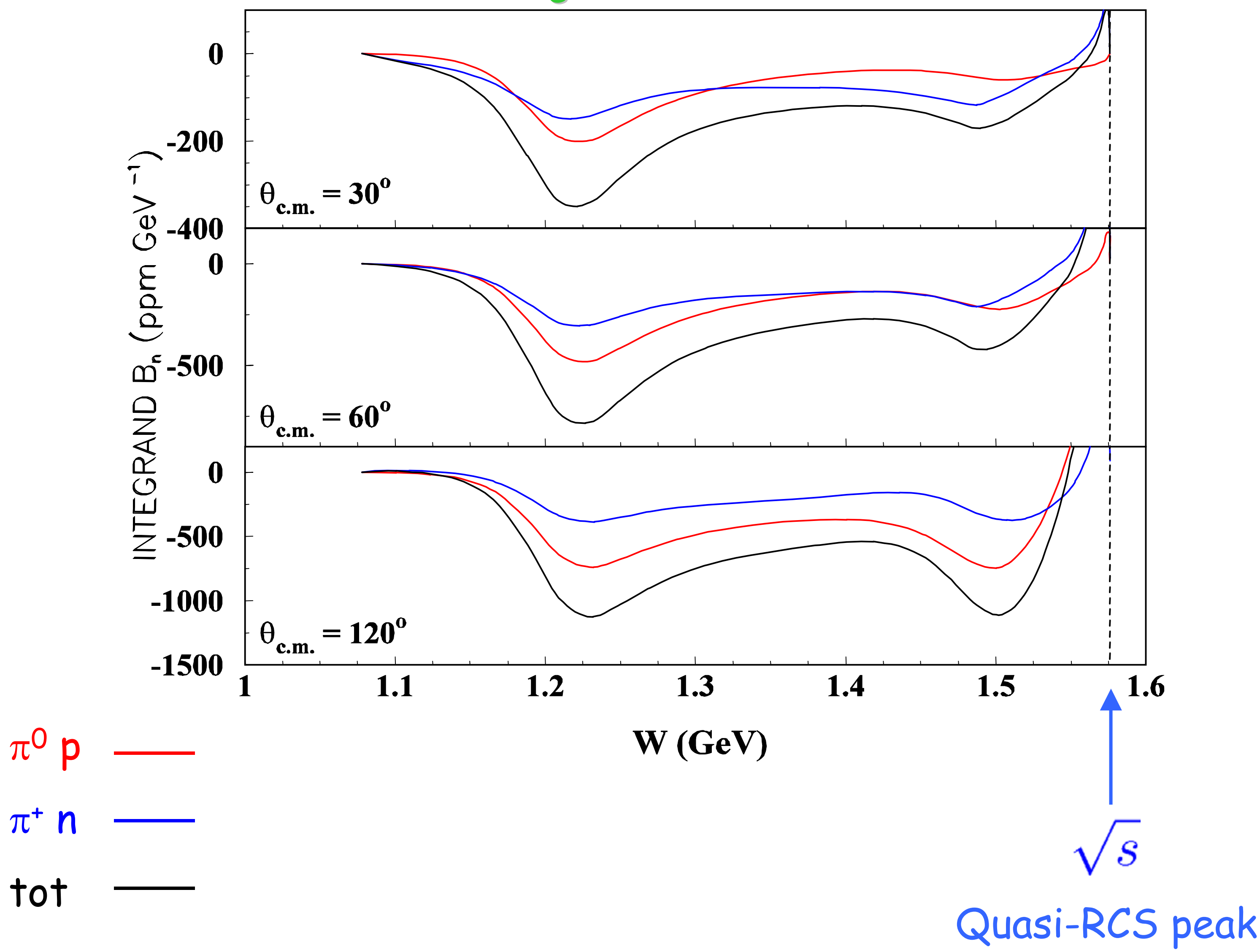
Beam normal spin asymmetry

$e^{-}\uparrow + p \rightarrow e^{-} + p$



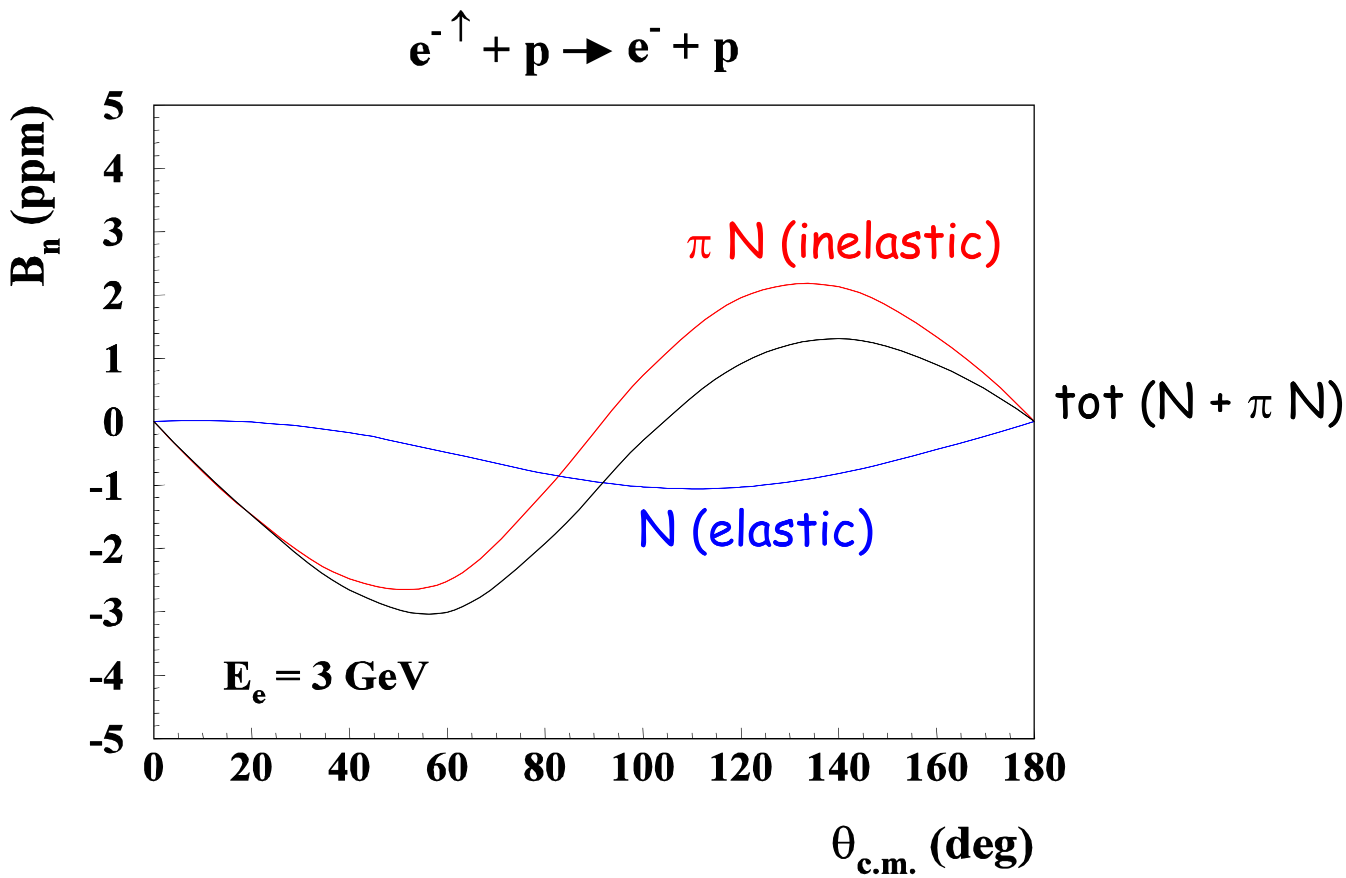
Integrand : beam normal spin asymmetry

E_e = 0.855 GeV



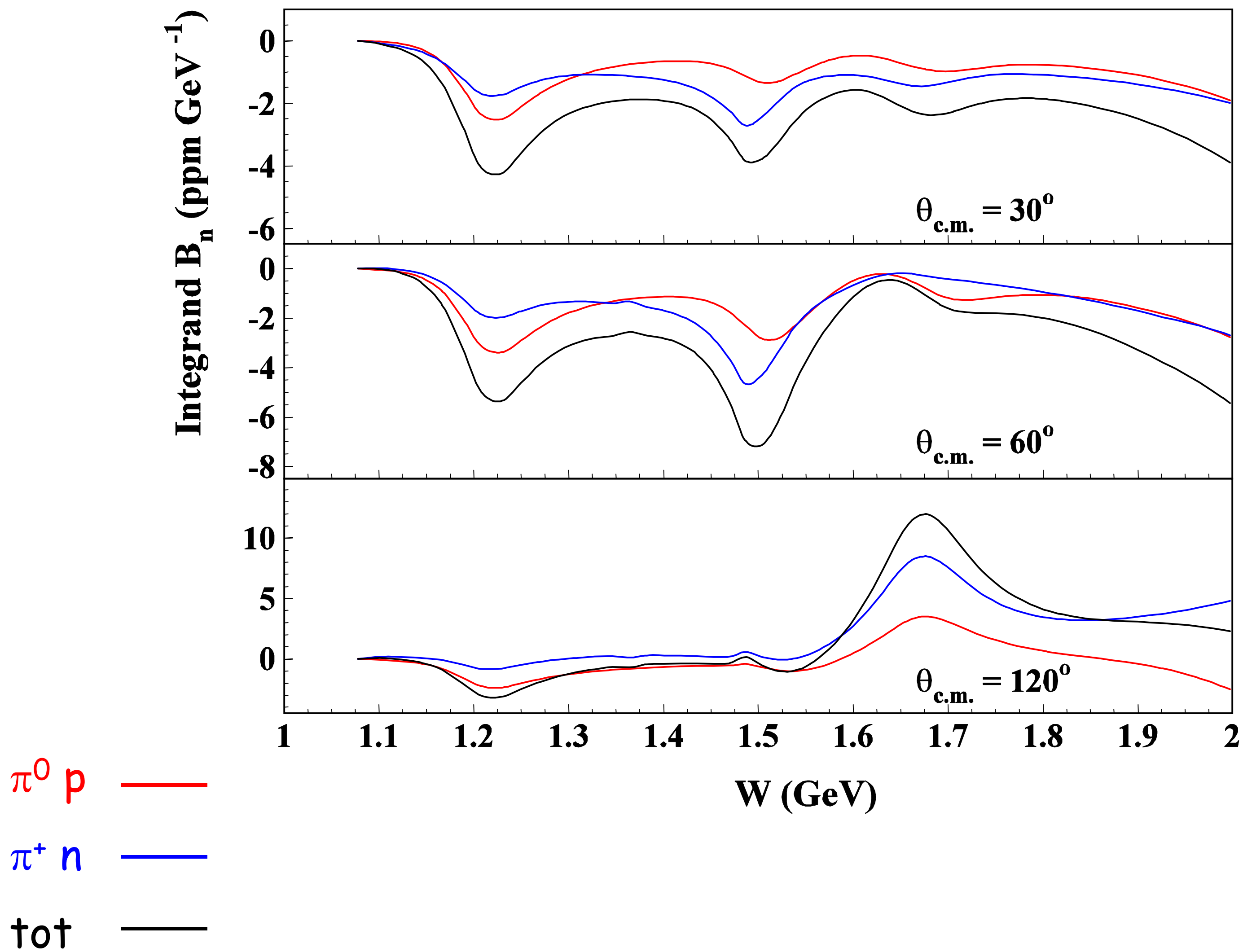
Beam normal spin asymmetry

E_e = 3 GeV

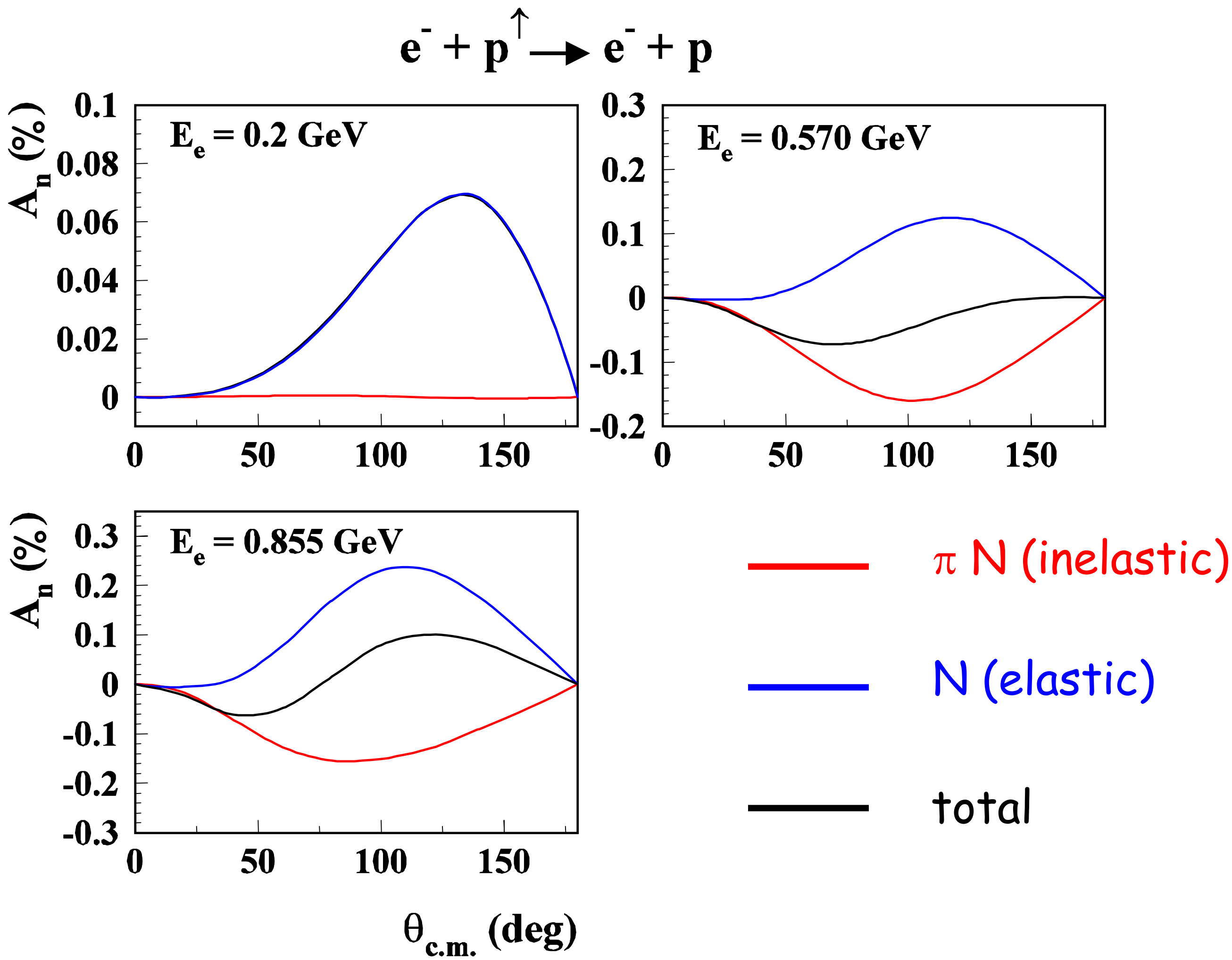


Integrand : beam normal spin asymmetry

E_e = 3 GeV

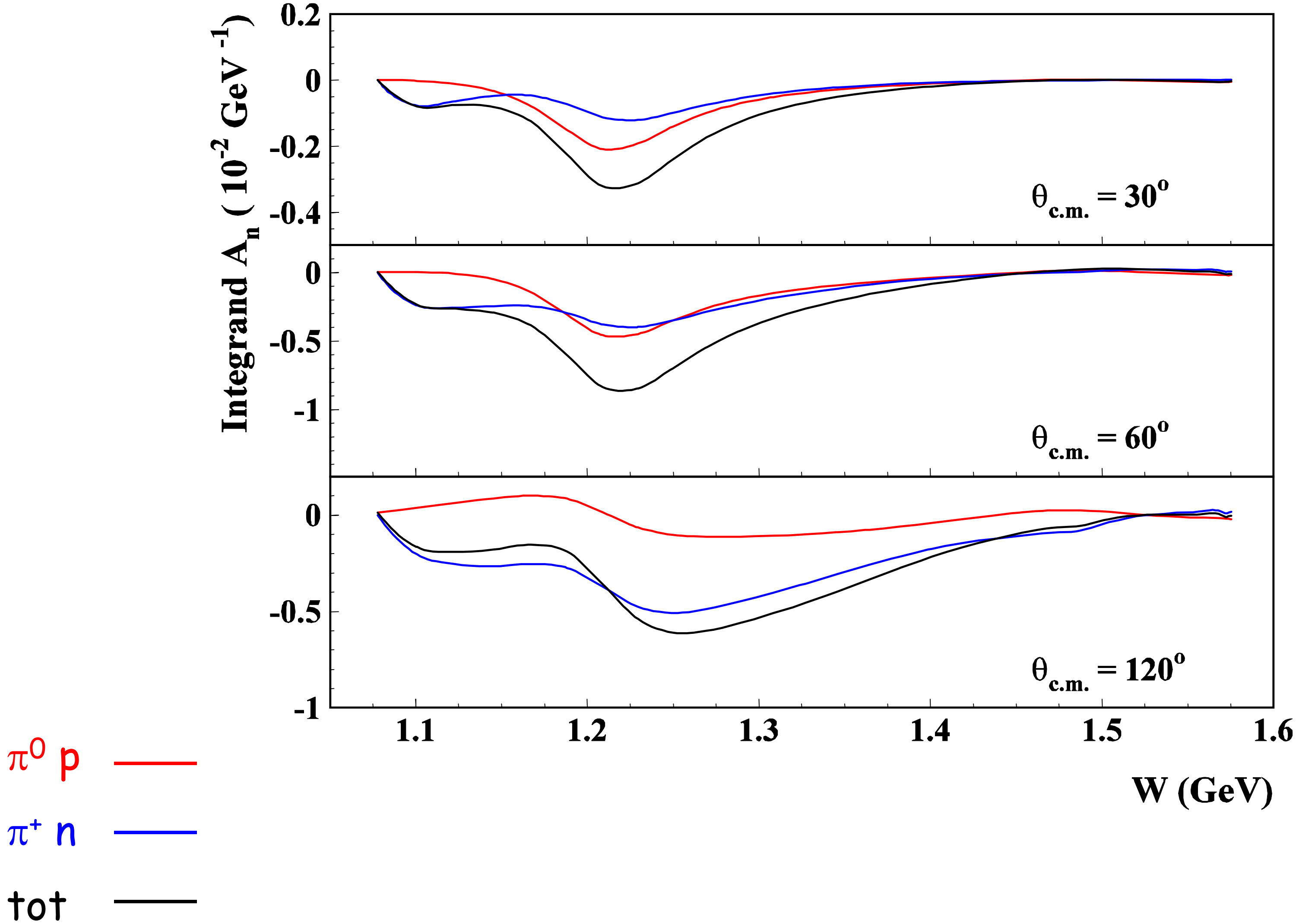


Target normal spin asymmetry



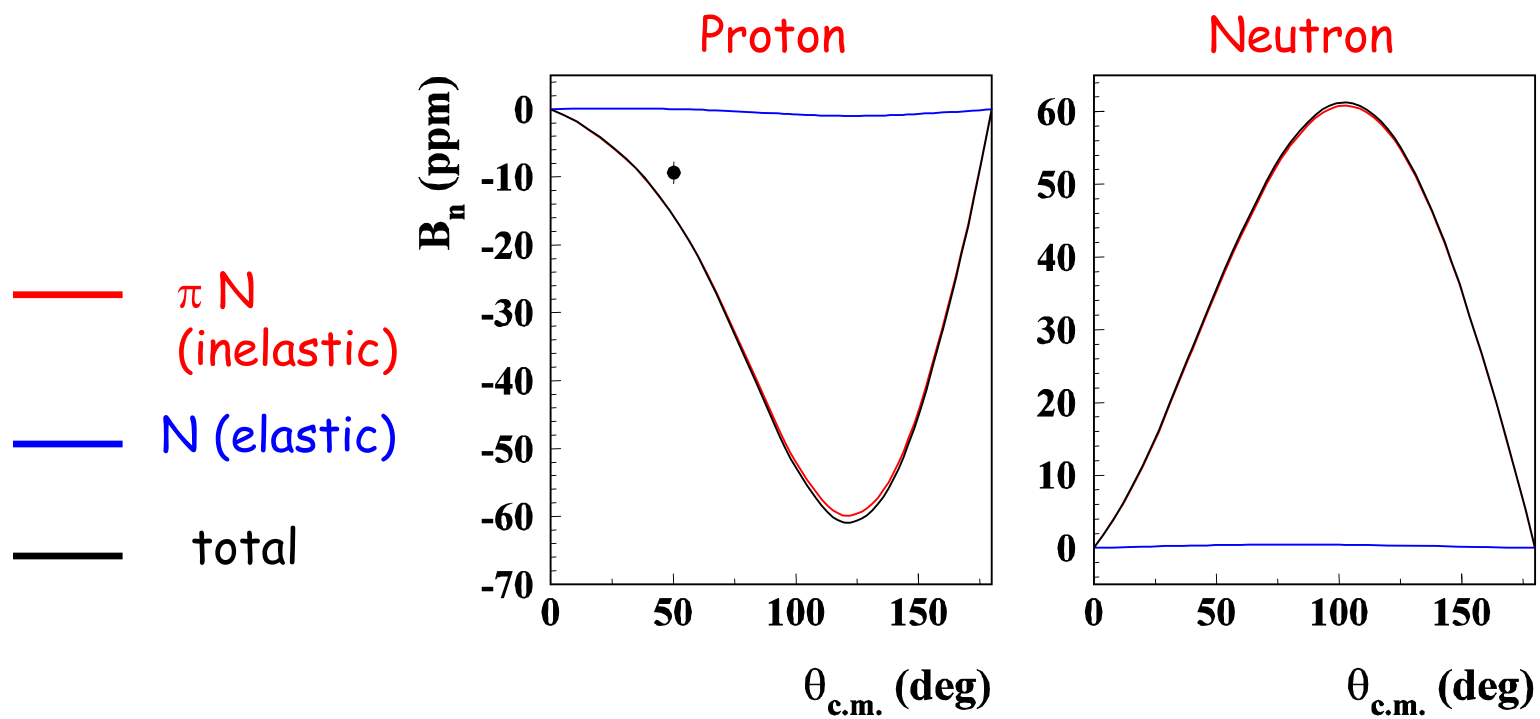
Integrand : target normal spin asymmetry

$E_e = 0.855$ GeV



Beam normal spin asymmetry

E_e = 0.570 GeV

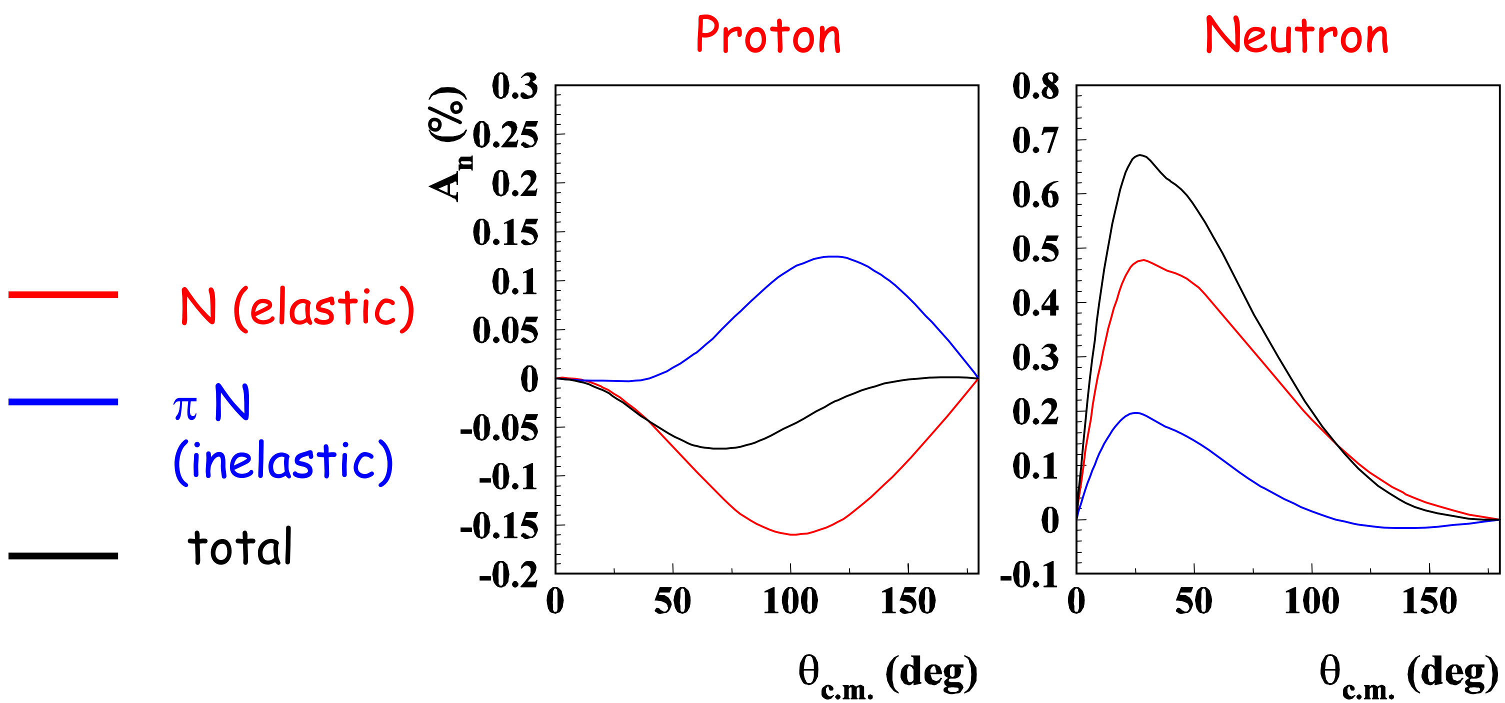


$$B_n = \frac{2\,m_e}{Q}\sqrt{2\,\varepsilon\,(1-\varepsilon)}\sqrt{1+\frac{1}{\tau}}\left(G_M^2 + \frac{\varepsilon}{\tau}G_E^2\right)^{-1}$$

$$\times \quad \left\{-\tau\,G_M\,\mathcal{I}m\left(\tilde{F}_3 + \frac{1}{1+\tau}\frac{\nu}{M^2}\tilde{F}_5\right) - G_E\,\mathcal{I}m\left(\tilde{F}_4 + \frac{1}{1+\tau}\frac{\nu}{M^2}\tilde{F}_5\right)\right\}$$

Target normal spin asymmetry

E_e = 0.570 GeV



$$A_n = \sqrt{\frac{2\,\varepsilon\,(1+\varepsilon)}{\tau}}\;\left(G_M^2 + \frac{\varepsilon}{\tau}G_E^2\right)^{-1}$$

$$\times \quad \left\{ -G_M\,\mathcal{I}m\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathcal{I}m\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

Target normal spin asymmetry : partonic calculation

two-photon
electron-quark
amplitude

$$
\left\{
\begin{aligned}
\mathcal{I}\left(\tilde{f}_1^{soft}\right) &= -\frac{e^2}{4\pi}\ln\left(\frac{\lambda^2}{\hat{s}}\right) \\
\mathcal{I}\left(\tilde{f}_1^{hard}\right) &= -\frac{e^2}{4\pi}\left\{\frac{Q^2}{2\,\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right)+\frac{1}{2}\right\} \\
\mathcal{I}\left(\tilde{f}_3\right) &= -\frac{e^2}{4\pi}\frac{1}{\hat{u}}\left\{\frac{\hat{s}-\hat{u}}{\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right)+1\right\}
\end{aligned}
\right.
$$

$$
\Rightarrow\; A_n = \sqrt{\frac{2\,\varepsilon\,(1+\varepsilon)}{\tau}}\,\frac{1}{\sigma_R}\left\{G_E\,\mathcal{I}\,(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}}\,G_M\,\mathcal{I}\,(B)\right\}
$$

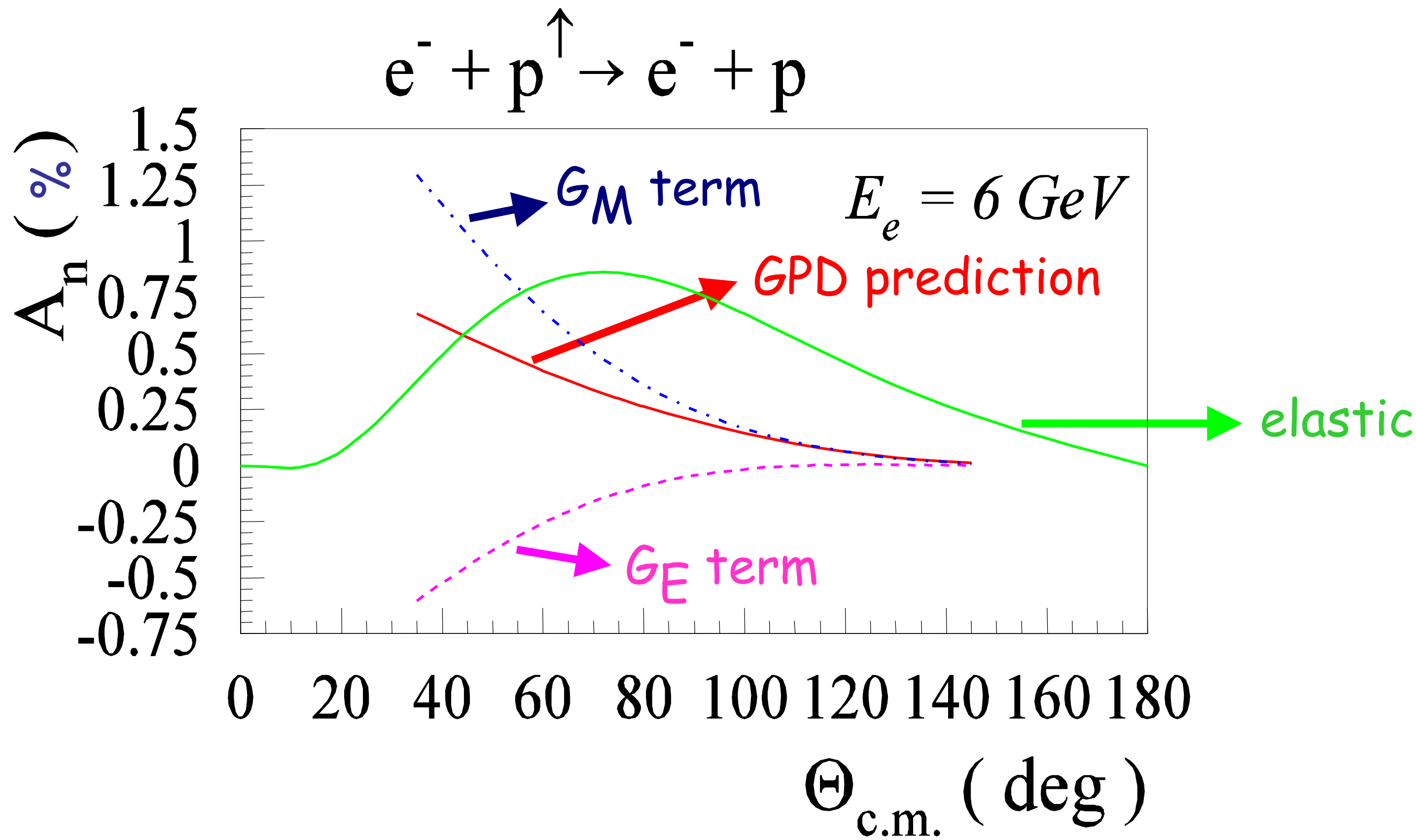
$$
A \equiv \int_{-1}^{1}\frac{dx}{x}\,\frac{\left[(\hat{s}-\hat{u})\,\tilde{f}_1^{hard}-\hat{s}\hat{u}\,\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\,\left(H^q+E^q\right)
$$

“magnetic” GPD

$$
B \equiv \int_{-1}^{1}\frac{dx}{x}\,\frac{\left[(\hat{s}-\hat{u})\,\tilde{f}_1^{hard}-\hat{s}\hat{u}\,\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\,\left(H^q-\tau E^q\right)
$$

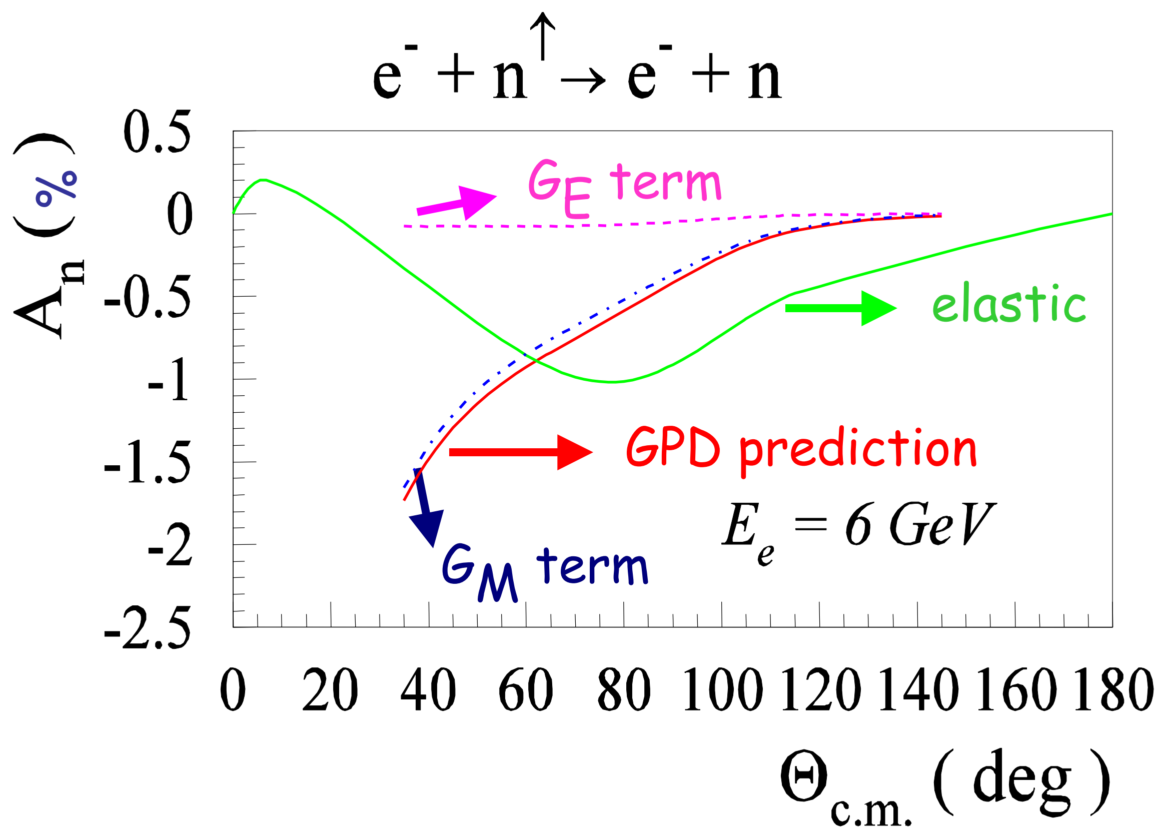
“electric” GPD

Target normal spin asymmetry : PROTON results



JLab proposals : could reach 0.1% precision

Target normal spin asymmetry : NEUTRON results



sizeable asymmetry on neutron :

no cancellation effect as on proton

Elastic electron-nucleon amplitudes

with electron helicity flip

$$T = T^{non-flip} + T^{flip}$$

$$\begin{aligned}
T^{non-flip}_{h'\lambda'_N,\, h\lambda_N} &= \frac{e^2}{Q^2}\, \bar{u}(k',h')\gamma_\mu u(k,h) \\
&\times \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma.K P^\mu}{M^2}\right) u(p,\lambda_N) \\
T^{flip}_{h'\lambda'_N,\, h\lambda_N} &= \frac{e^2}{Q^2}\frac{m_l}{M}\left[\bar{u}(k',h')u(k,h)\cdot \bar{u}(p',\lambda'_N)\left(\tilde{F}_4 + \tilde{F}_5 \frac{\gamma.K}{M}\right) u(p,\lambda_N)\right. \\
&\left. + \tilde{F}_6\, \bar{u}(k',h')\gamma_5 u(k,h)\cdot \bar{u}(p',\lambda'_N)\gamma_5 u(p,\lambda_N)\right]
\end{aligned}$$

In Born approximation :
$$\left\{\begin{aligned}
\tilde{G}_M^{Born}(\nu,Q^2) &= G_M(Q^2) \\
\tilde{F}_2^{Born}(\nu,Q^2) &= F_2(Q^2) \\
\tilde{F}_{3,4,5,6}^{Born}(\nu,Q^2) &= 0
\end{aligned}\right.$$

Elastic electron-quark amplitudes

with electron helicity flip

$$H^{hard}_{h'h,\,\lambda} \;=\; \frac{(e\,e_q)^2}{Q^2}\Big\{\bar{u}(k',h')\gamma_\mu u(k,h)\cdot\bar{u}(p'_q,\lambda)\left(\tilde{f}_1\,\gamma^\mu + \tilde{f}_3\,\gamma.K\,P_q^\mu\right)u(p_q,\lambda)$$
$$+\,m_l\;\tilde{f}_5\;\bar{u}(k',h')u(k,h)\cdot\bar{u}(p'_q,\lambda)\gamma.K\,u(p_q,\lambda)\Big\}$$

lepton mass

new amplitude

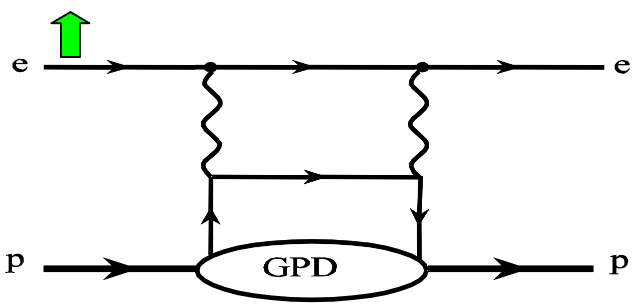
$$\mathcal{I}\left(\tilde{f}_1^{soft}\right) = -\frac{e^2}{4\pi}\ln\left(\frac{\lambda^2}{\hat{s}}\right)$$

$$\mathcal{I}\left(\tilde{f}_1^{hard}\right) = -\frac{e^2}{4\pi}\left\{\frac{Q^2}{2\,\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right)+\frac{1}{2}\right\}$$

$$\mathcal{I}\left(\tilde{f}_3\right) = -\frac{e^2}{4\pi}\,\frac{1}{\hat{u}}\left\{\frac{\hat{s}-\hat{u}}{\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right)+1\right\}$$

$$\mathcal{I}\left(\tilde{f}_5\right)_{2\gamma} = -\frac{e^2}{4\pi}\,\frac{Q^2}{\hat{u}}\left\{-\frac{1}{\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right)-\frac{1}{\hat{s}}\right\}$$

Beam normal spin asymmetry : partonic calculation



$$
\begin{aligned}
B_n &= \frac{2\,m_l}{Q}\sqrt{2\,\varepsilon\,(1-\varepsilon)}\left(G_M^2 + \frac{\varepsilon}{\tau}G_E^2\right)^{-1} \\
&\times \left\{ G_M \left[\frac{\sqrt{1+\varepsilon}\sqrt{\tau}}{\sqrt{1+\varepsilon}\sqrt{1+\tau}+\sqrt{1-\varepsilon}\sqrt{\tau}}\,\mathcal{I}(A) - \sqrt{\frac{1+\varepsilon}{2\,\varepsilon}}\sqrt{\frac{\tau}{1+\tau}}\,\mathcal{I}(A') \right] \right. \\
&+ \left. \frac{1}{\tau} G_E \left[\frac{\sqrt{1+\varepsilon}\sqrt{\tau}+\sqrt{1-\varepsilon}\sqrt{1+\tau}}{\sqrt{1+\varepsilon}\sqrt{1+\tau}+\sqrt{1-\varepsilon}\sqrt{\tau}}\sqrt{\frac{1+\varepsilon}{2\,\varepsilon}}\,\mathcal{I}(B) - \sqrt{\frac{\tau}{1+\tau}}\,\mathcal{I}(B') \right] \right\} \\
&+ \mathcal{O}(e^4)
\end{aligned}
$$

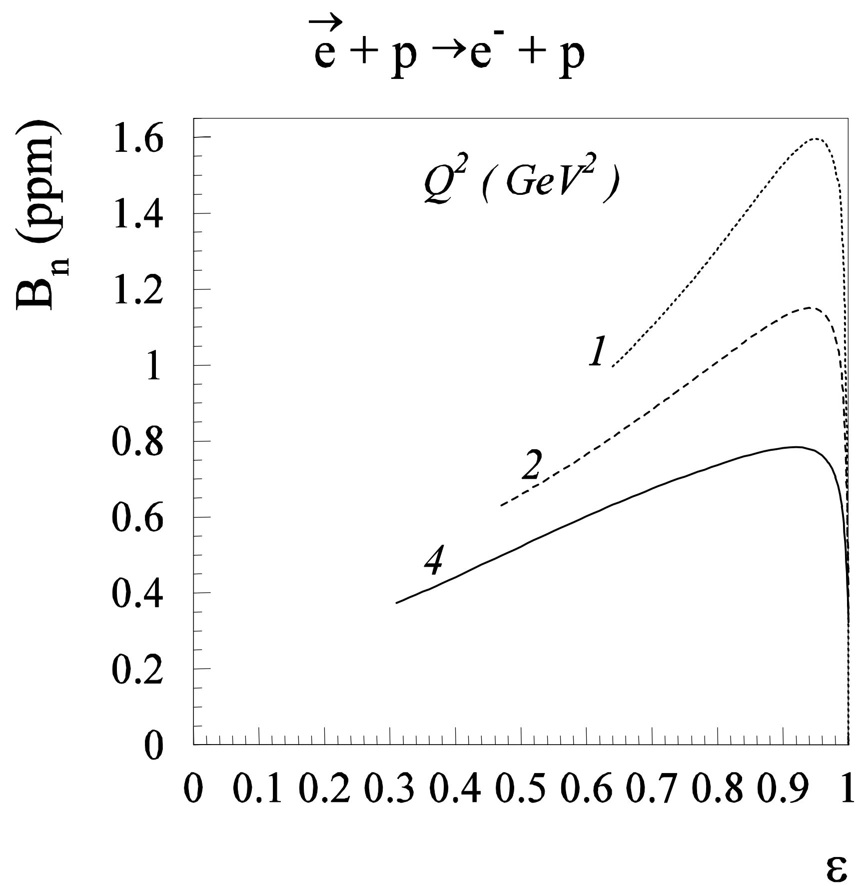
$$A \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\,(H^q + E^q)$$ "magnetic" GPD

$$B \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)}\sum_q e_q^2\,(H^q - \tau E^q)$$ "electric" GPD

$$A' \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\sqrt{-\hat{s}\hat{u}}}{2}\left[\frac{2}{\hat{s}}\tilde{f}_1^{hard} + \tilde{f}_3 + \tilde{f}_5\right]\sum_q e_q^2\,(H^q + E^q)$$ "magnetic" GPD

$$B' \equiv \int_{-1}^{1}\frac{dx}{x}\frac{\sqrt{-\hat{s}\hat{u}}}{2}\left[\frac{2}{\hat{s}}\tilde{f}_1^{hard} + \tilde{f}_3 + \tilde{f}_5\right]\sum_q e_q^2\,(H^q - \tau E^q)$$ "electric" GPD

Beam normal spin asymmetry : proton results



Results of GPD calculation

Note : elastic contribution to Bn is negligibly small

Present PV experimental set-ups (0.1 ppm precision) : opportunity to measure this asymmetry

Conclusions

- Developed the formalism to describe elastic electron-nucleon scattering beyond the one-photon exchange approximation
- Performed a partonic calculation of two-photon exchange contribution in terms of generalized parton distributions of nucleon (handbag calculation)
- Able to resolve existing discrepancy between Rosenbluth and polarization transfer observables quantitatively
- SSA in elastic electron-nucleon scattering :

  promising observables to access doubly (spacelike) virtual Compton scattering