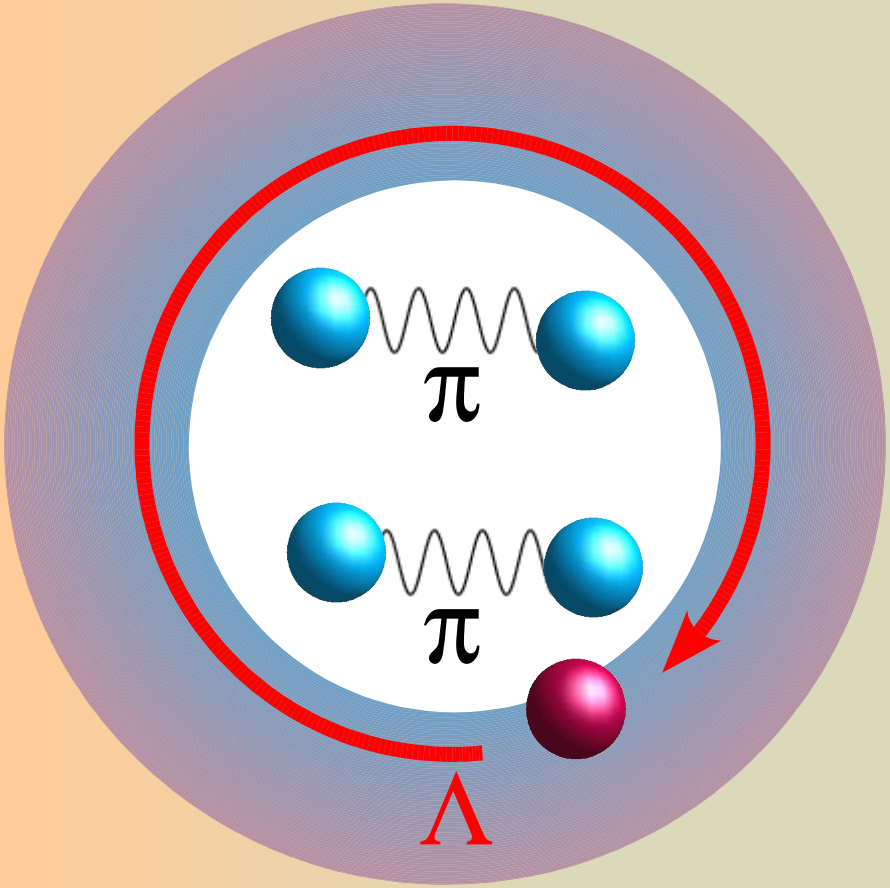
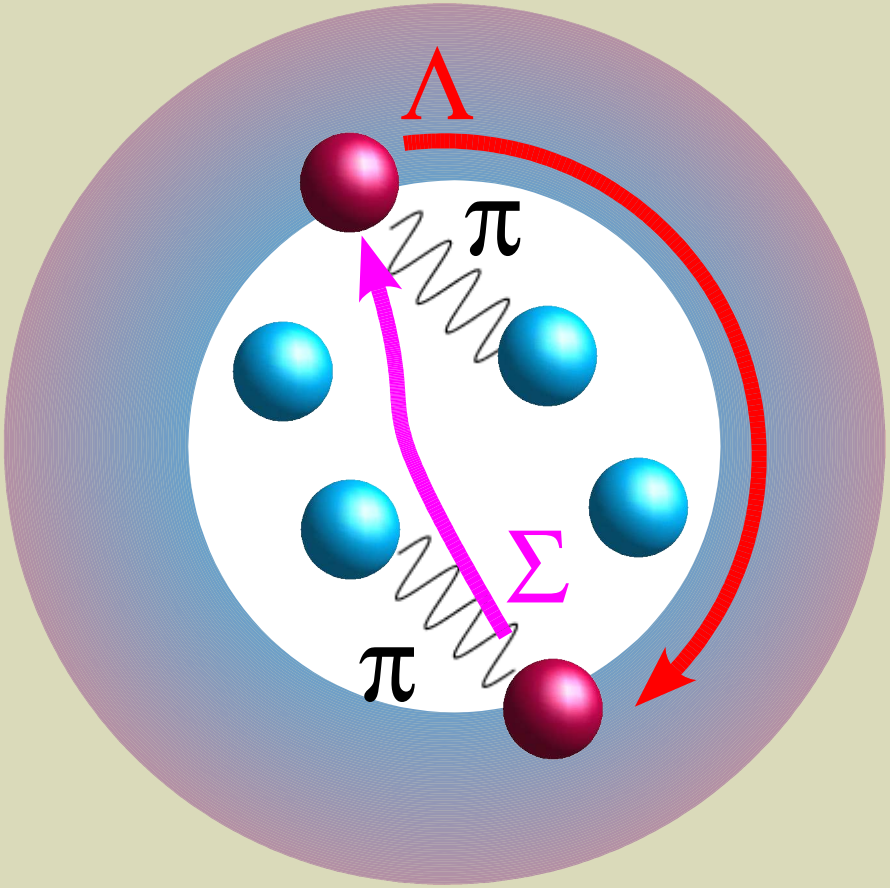
Light Hypernuclei

??Structure of ${}_{\Lambda}^{5}$He and tensor ΛN-ΣN coupling ??

H. Nemura

Institute of Particle and Nuclear Studies, KEK, Japan





Introduction:

Tensor interaction plays an important role for light normal nuclei.

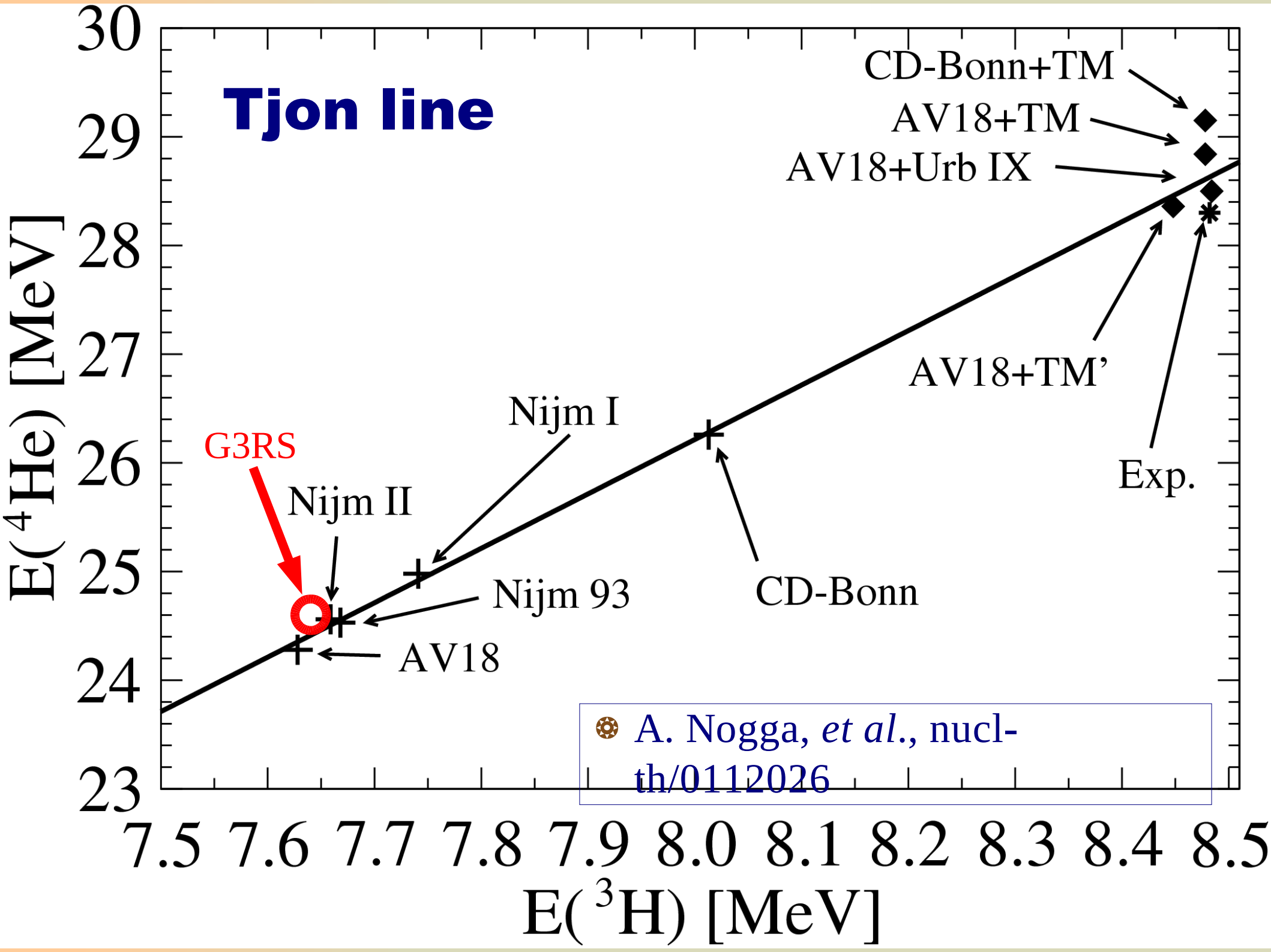
$$\Psi = \frac{1}{\sqrt{1+|\alpha|^2}}\left(\psi_S + \alpha\,\psi_D\right)$$

- $|\alpha| \sim 0.2$ (^{2}H)
- $|\alpha| \sim 0.3$ (^{3}H)
- $|\alpha| \sim 0.3$–0.4 (^{4}He)

Results for light nuclei

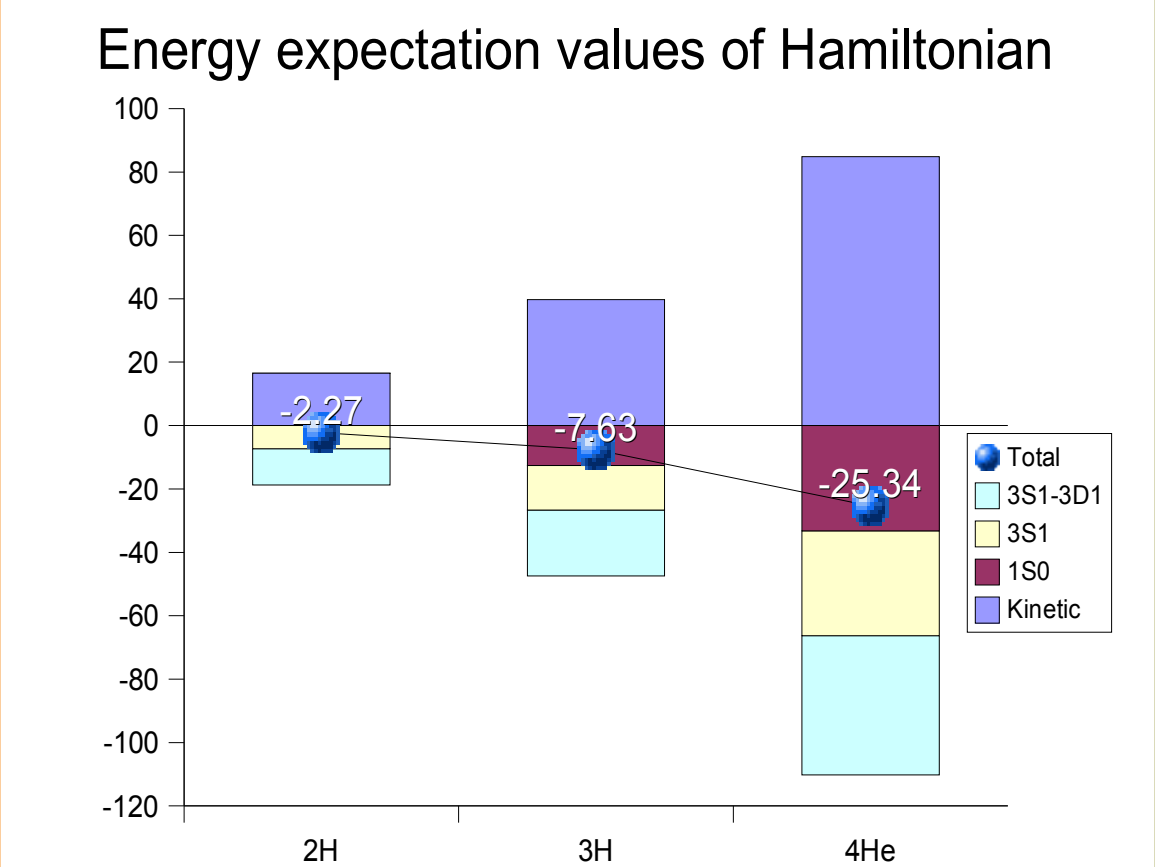
- *NN*: G3RS (Central+Tensor)

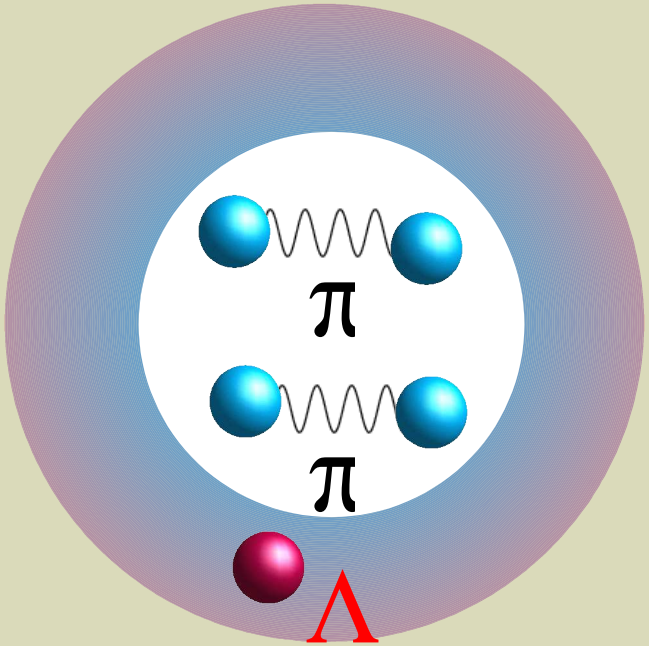
	^{2}H	^{3}H	^{4}He
B (MeV)	2.28	7.63	24.58
P_D	0.048	0.067	0.10
$\sqrt{r^2}$ (fm)	1.98	1.80	1.52


Tjon line
CD-Bonn+TM
AV18+TM
AV18+Urb IX
AV18+TM'
Exp.
Nijm I
G3RS
Nijm II
Nijm 93
CD-Bonn
AV18
E(^{4}He) [MeV]
E(^{3}H) [MeV]
30
29
28
27
26
25
24
23
7.5
7.6
7.7
7.8
7.9
8.0
8.1
8.2
8.3
8.4
8.5
A. Nogga, *et al.*, nucl-th/0112026

Introduction:

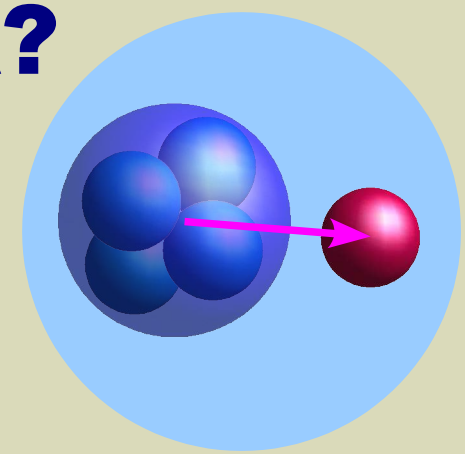
Tensor interaction plays an important role for light normal nuclei.





- G3RS *NN* potential is used.

What is the hypernuclear structure due to the presence of a Λ?



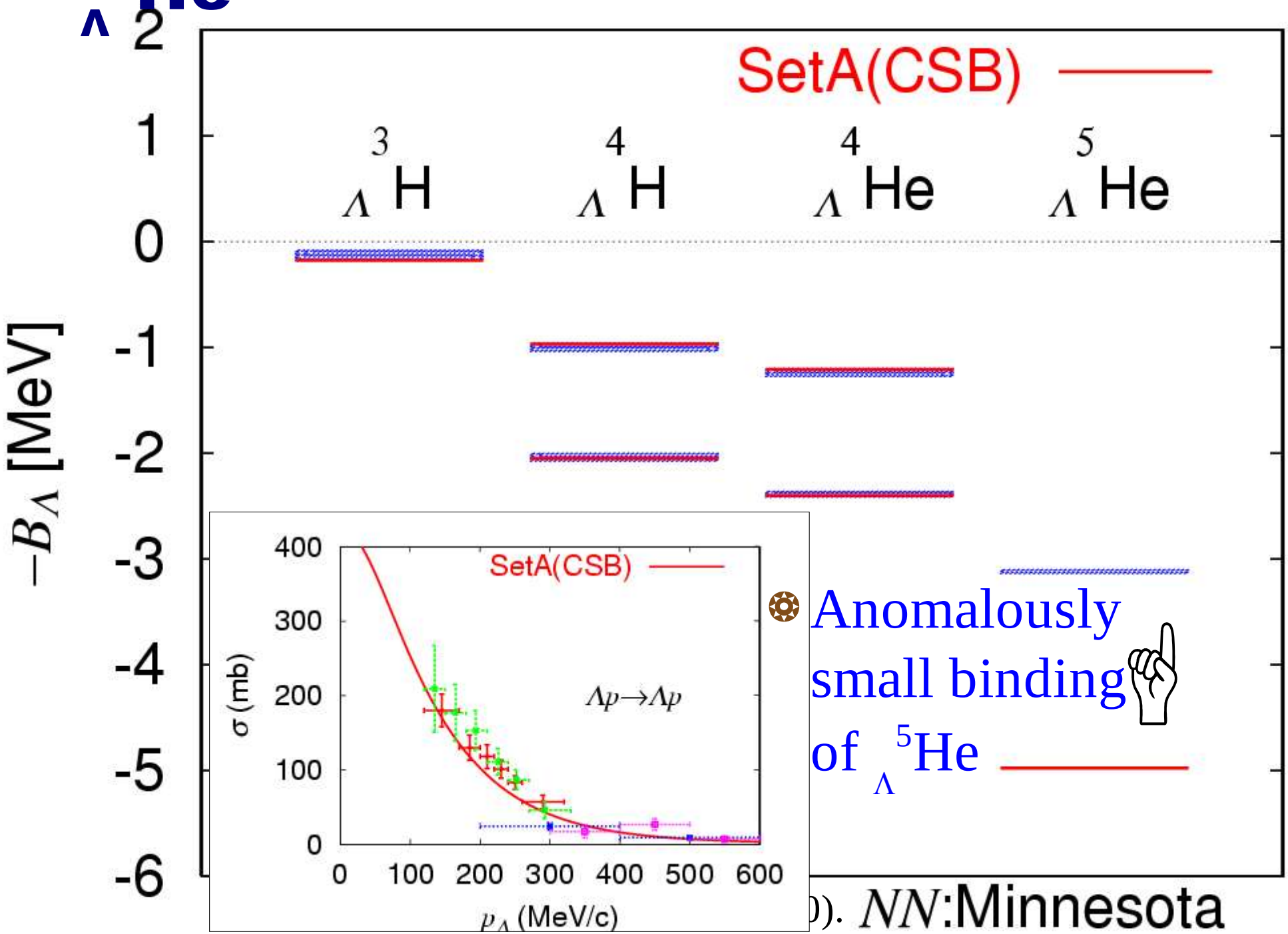
- $B(^4\mathrm{He})$ 28 MeV
- $B_\Lambda({}_\Lambda^5\mathrm{He})$ 3 MeV ➔ ${}_\Lambda^5\mathrm{He}$ α+Λ

Rigid core+Λ picture

- $J_c=0$ No tensor ΛN interaction
- $I_c=0$ No ΛN-ΣN coupling

- Is the conventional picture acceptable? ➔ No!
  - Anomalously small binding of ${}_\Lambda^5\mathrm{He}$

Anomalously small binding of $^{5}_{\Lambda}$He



Anomalously small binding of $^{5}_{\Lambda}$He

0). *NN*:Minnesota

Anomalously small binding of ${}_{\Lambda}^{5}$He

- A phenomenological ΛN potential reproducing $B_{\Lambda}({}_{\Lambda}^{3}\mathrm{H})$, $B_{\Lambda}({}_{\Lambda}^{4}\mathrm{H})$, $B_{\Lambda}({}_{\Lambda}^{4}\mathrm{He})$, $B_{\Lambda}({}_{\Lambda}^{4}\mathrm{H}^{*})$, and $B_{\Lambda}({}_{\Lambda}^{4}\mathrm{He}^{*})$ values as well as the Λp total cross section, predicts (about two times) larger $B_{\Lambda}({}_{\Lambda}^{5}\mathrm{He})$ value than the experimental value.

  Dalitz, *et al.*, NPB**47**, 109 (1972).

- The experimental $B_{\Lambda}({}_{\Lambda}^{5}\mathrm{He})$ implies that ΛN interaction in ${}_{\Lambda}^{5}\mathrm{He}$ is weaker than the ΛN interaction in free space. ➔ ${}_{\Lambda}^{5}\mathrm{He}$ anomaly

Ab initio Approach to *s*-Shell Hypernuclei ${}^{3}_{\Lambda}$H, ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He, and ${}^{5}_{\Lambda}$He with a ΛN-ΣN Interaction

H. Nemura,[1] Y. Akaishi,[1] and Y. Suzuki[2]

[1]*Institute of Particle and Nuclear Studies, KEK, Tsukuba 305-0801, Japan*

[2]*Department of Physics, Niigata University, Niigata 950-2181, Japan*

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Variational calculations for *s*-shell hypernuclei are performed by explicitly including Σ degrees of freedom. Four sets of *YN* interactions [SC97d(S), SC97e(S), SC97f(S), and SC89(S)] are used. The bound-state solution of ${}^{5}_{\Lambda}$He is obtained and a large energy expectation value of the tensor ΛN-ΣN transition part is found. The internal energy of the ^{4}He subsystem is strongly affected by the presence of a Λ particle with the strong tensor ΛN-ΣN transition potential.

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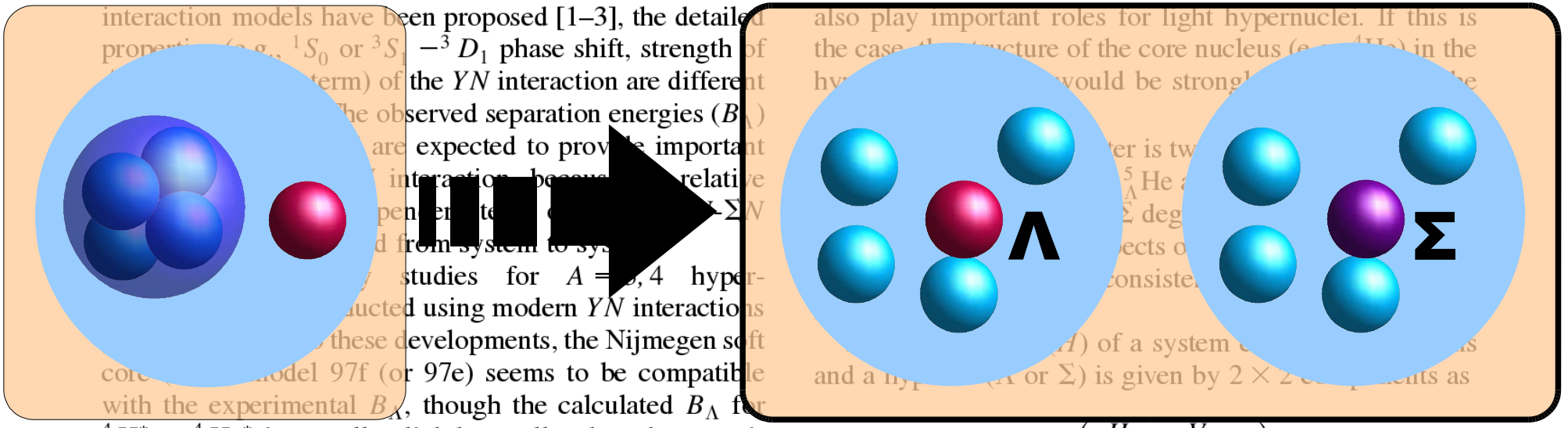
ΛN-ΣN coupling plays a significant role in hypernucleus ${}^{5}_{\Lambda}$He

Few-body calculations for *s*-shell hypernuclei with mass number $A = 3$–5 are important not only to explore exoti[illegible] freedom, but also to clarify the characteristic features of the hyperon-nucleon (*YN*) interaction. Although several interaction models have been proposed [1–3], the detailed [illegible] $^1S_0$ or $^3S_1$ $-^3D_1$ phase shift, strength of [illegible] term) of the *YN* interaction are different [illegible] The observed separation energies (B_Λ) [illegible] are expected to provide important [illegible] interaction because [illegible] relative [illegible] Λ-ΣN [illegible] from system to sys[illegible] studies for $A = 3, 4$ hyper[illegible] ducted using modern *YN* interactions [illegible] these developments, the Nijmegen soft core [illegible] model 97f (or 97e) seems to be compatible with the experimental B_Λ, though the calculated B_Λ for ${}^{4}_{\Lambda}$H* or ${}^{4}_{\Lambda}$He* is actually slightly smaller than the experi-

mechanisms of light nuclei. More than a third, or about one-half, of the interaction energy comes from the tensor [illegible] the ^{4}He [illegible]. Since the pion (or kaon) exchange also induces the ΛN-ΣN transition for the *YN* sector, both the *NN* and ΛN-ΣN tensor interactions may also play important roles for light hypernuclei. If this is the case, the structure of the core nucleus (e.g., ^{4}He) in the hy[illegible] would be strongl[illegible] the [illegible] ter is tw[illegible] ${}^{5}_{\Lambda}$He a[illegible] Σ deg[illegible] ects o[illegible] consiste[illegible]



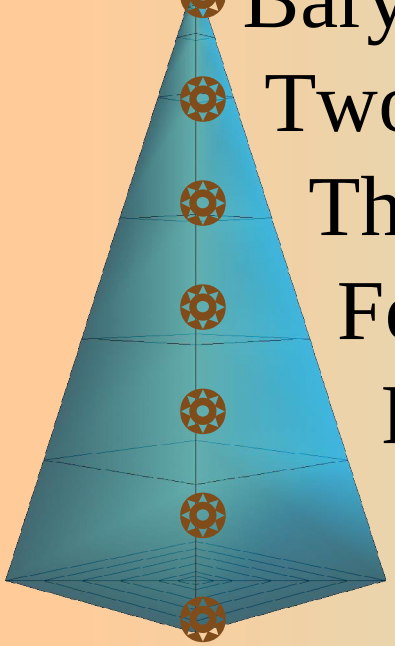
[illegible] H) of a system [illegible] and a hyp[illegible] (Λ or Σ) is given by 2×2 components as

$$H = \begin{pmatrix} H_\Lambda & V_{\Sigma-\Lambda} \\ V_{\Lambda-\Sigma} & H_\Sigma \end{pmatrix}, \quad (1)$$

The purpose of this work

- To describe an *ab initio* calculation of ${}_{\Lambda}^{5}\mathrm{He}$ as well as A=3, 4 hypernuclei explicitly including Σ degrees of freedom,
- To conduct a new view of the ${}_{\Lambda}^{5}\mathrm{He}$, due to taking account of explicit Σ admixture, beyond $\alpha+\Lambda$ model.
- We would also like to discuss,
  - Why the *YN* interaction in ${}_{\Lambda}^{5}\mathrm{He}$ is so weaker than that in free space or in A=3, 4 systems?

NN and YN potentials

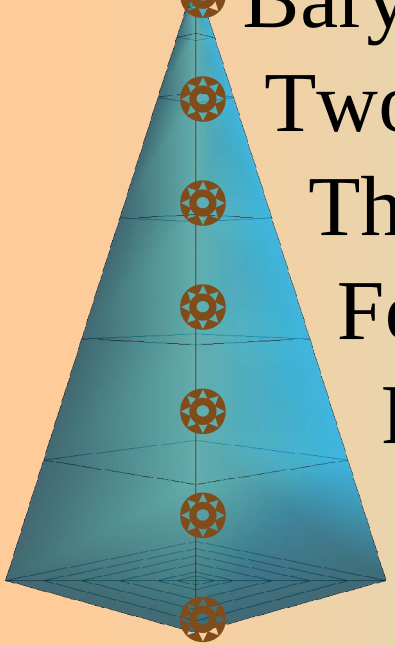


- Baryon-baryon interaction
  - Two-body system
    - Three-body system
      - Four-body system
        - Five-body system
          - ...
            - Many-body systems

Top-down approach

- In the nuclear physics,
  - *NN* potential is given by a modern interaction model, such as Nijmegen model.
    - Few-body calculation is made using the interaction.

NN and YN potentials



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        - Five-body system
          - ...
            - Many-body systems

Top-down and bottom-up approach

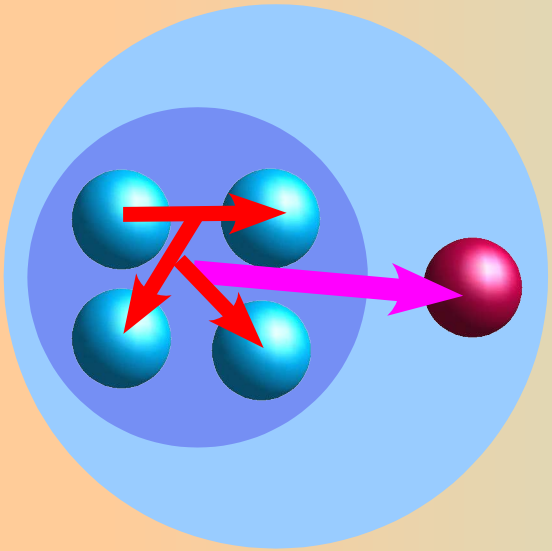
- In the hypernuclear physics, phase-shift analysis has not been confirmed yet.
- A phenomenological potential is used, which is phase-equivalent to the modern interaction model (e.g. Nijmegen model), and which reproduces the

NN and YN potentials

- *NN* interaction:
  - G3RS (central+tensor)
  - The *NN* interaction reproduces the low energy *NN* phase shifts.
- *YN* interaction:
  - SC97e(S) (central+tensor+spin-orbit; ΛN+ΣN); it is phase equivalent to the Nijmegen soft core model NSC97e.
  - The *YN* interaction reproduces the experimental B_Λ of A=3, 4 hypernuclei as well as the Λp total cross section.

Hamiltonian of a system comprising (A-1) nucleons and a hyperon

- Hamiltonian (H) is divided into the internal motion of the core nucleus (H_{core}) and relative motion between the core and the hyperon ($H_{Y\text{-core}}$).



$$H=\sum_{i=1}^{A}\left(m_i c^2+\frac{\boldsymbol{p}_i^2}{2}m_i\right)-T_{CM}+\sum_{i<j}^{A-1}v_{ij}^{(NN)}+\sum_{i=1}^{A-1}v_{iY}^{(NY)},$$

$$=H_{core}+H_{Y-core},$$

$$H_{core}=\sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2}m_N-\frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)m_N}+\sum_{i<j}^{A-1}v_{ij}^{(NN)},$$

$$=T_{core}+V_{NN},$$

$$H_{Y-core}=\frac{\pi_{Y-core}^2}{2\mu_Y}+(m_Y-m_\Lambda)c^2+\sum_{i=1}^{A-1}v_{iY}^{(NY)},$$

$$=T_{Y-core}+V_{YN}.$$

$$\mu_Y=\frac{(A-1)m_N m_Y}{(A-1)m_N+m_Y},\quad (Y=\Lambda,\Sigma).$$

Hamiltonian of a system comprising (A-1) nucleons and a hyperon

- If *rigid core* + Λ is good approximation for the hypernucleus, there is no rearrangement energy;

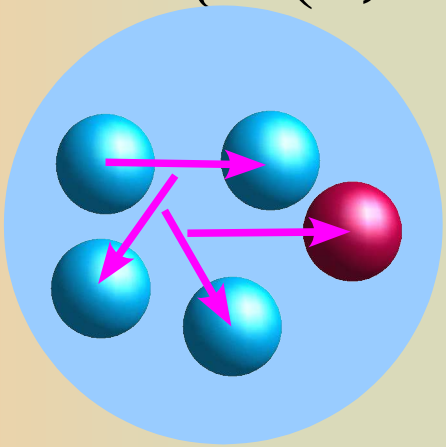
$$\left\langle H_{core} \right\rangle_{{}^{A}_{\Lambda}Z} \approx \left\langle H_{core} \right\rangle_{{}^{(A-1)}Z},$$

$$\left\langle H \qquad \right\rangle \approx -B_{\Lambda}({}^{A}_{\Lambda}Z). \qquad .$$

Ab initio calculation with stochastic variational method

- The variational trial function must be flexible enough to incorporate both
  - Explicit Σ degrees of freedom and
  - Higher orbital angular momenta.
- $\Psi = \Sigma_i c_i \Phi_{JMTM_T}(\boldsymbol{x}; \boldsymbol{A}_i, u_i)$
- $\Phi_{JMIM_I}(\boldsymbol{x}, \boldsymbol{A}_i, u_i)$

  $= \mathcal{A}\{G(\boldsymbol{x}; \boldsymbol{A}_i)[\theta_{(kl)i}(\boldsymbol{x}; u_i) \times \chi_{s_i}]_{JM} \eta_{IM_I}\}$



Complete five-body treatment

Ab initio calculation with stochastic variational method

- Correlated Gaussian

$$G(\boldsymbol{x};\boldsymbol{A}_i)=\exp\{-(1/2)\Sigma_{m<n}\alpha_{i,mn}(\boldsymbol{r}_m-\boldsymbol{r}_n)^2\}$$
$$=\exp\{-(1/2)\Sigma_{m,n}\boldsymbol{A}_{i,mn}\boldsymbol{x}_m\cdot\boldsymbol{x}_n\}$$

- Global vector representation

$$\theta_{(kl)i}(\boldsymbol{x};u_i)=v_i^{2k+l}Y_{li}(\boldsymbol{v}_i),\ \text{with}\ \boldsymbol{v}_i=\Sigma_m u_{i,m}\boldsymbol{x}_m$$

- Spin function $\chi_{s_i}=[[[\boldsymbol{s}_1\times\boldsymbol{s}_2]_{s_{12}}\times\ldots]_{s_{1234}}\times\boldsymbol{s}_5]_{s_i}\sim|\uparrow\downarrow\uparrow\downarrow\uparrow\rangle+\ldots$

- Isospin function $\eta_{IM_I}=[[[N_1\times N_4]_{I_{12}}\times\ldots]_{I_{1234}}\times Y]_{IM_I}$

$$\sim|pnpn\Lambda\rangle+\ldots,\ \text{or}\sim|pnpn\Sigma^0\rangle+\ldots$$

Ab initio calculation with stochastic variational method

- An example of spin function
  - The case of ${}_{\Lambda}^{3}\mathrm{H}$, ($J$=1/2, T=0)
  - $[(L=0)\times(S=1/2)]_{J=1/2}$
    - $\chi_{s=1/2}=(1/\sqrt{2})\ (|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle)$, or
    - $\chi_{s=1/2}=(1/\sqrt{6})\ (2|\uparrow\uparrow\downarrow\rangle-|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle)$
  - $[(L=2)\times(S=3/2)]_{J=1/2}$
    - $\chi_{s=3/2}=|\uparrow\uparrow\uparrow\rangle$

Ab initio calculation with stochastic variational method

- An example of isospin function
  - The case of ${}_{\Lambda}^{3}\mathrm{H}$, ($J$=1/2, I=0)
    - $\eta_{IM_I}=(1/\sqrt{2})(|pn\Lambda\rangle-|np\Lambda\rangle)$
    - $\eta_{IM_I}=(1/\sqrt{3})(|nn\Sigma^{+}\rangle+|pp\Sigma^{-}\rangle)$
      $-(1/\sqrt{6})(|pn\Sigma^{0}\rangle+|np\Sigma^{0}\rangle)$

Algorithm of SVM

- Step-by-step
  - [0] $k = 1$
    - (The $(k-1)$ basis functions $\{\Phi_1, \ldots, \Phi_{k-1}\}$ are already given.)
  - [1] n trial functions $\{\Phi_k^{(1)}, \ldots, \Phi_k^{(n)}\}$ are generated randomly.
    - (n $(A(A-1)/2) \times g$, g: number of possible spin-isospin functions)
  - [2] The n eigenvalue problems of k-dimension are solved and the corresponding energies $\{E_k^{(1)}, \ldots, E_k^{(n)}\}$ are determined.

$$\{\Phi_1, \ldots, \Phi_{k-1}, \Phi_k^{(1)}\} \Rrightarrow E_k^{(1)}$$

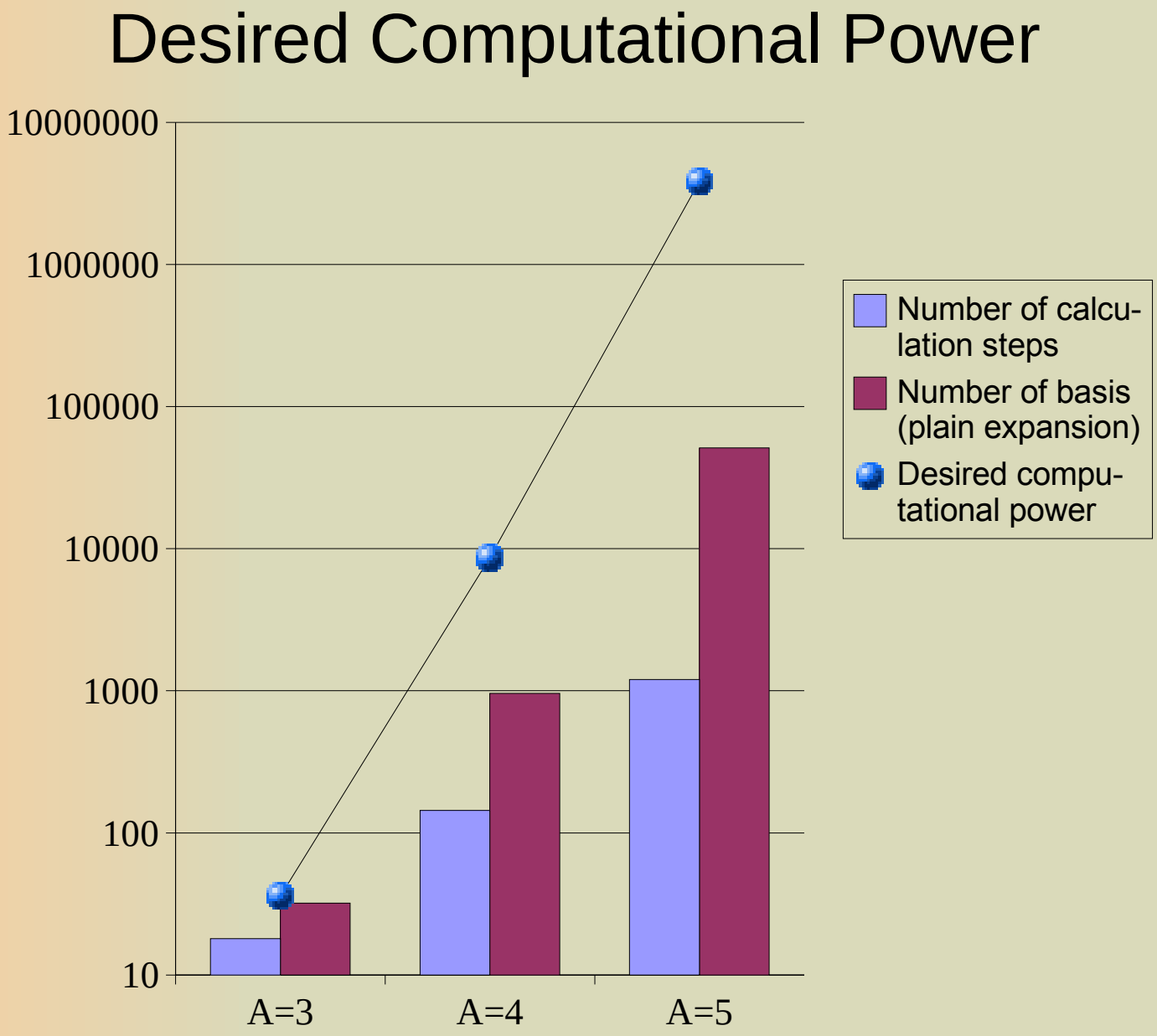
$$\{\Phi_1, \ldots, \Phi_{k-1}, \vdots\} \Rrightarrow \vdots$$

$$\{\Phi_1, \ldots, \Phi_{k-1}, \Phi_k^{(n)}\} \Rrightarrow E_k^{(n)}$$

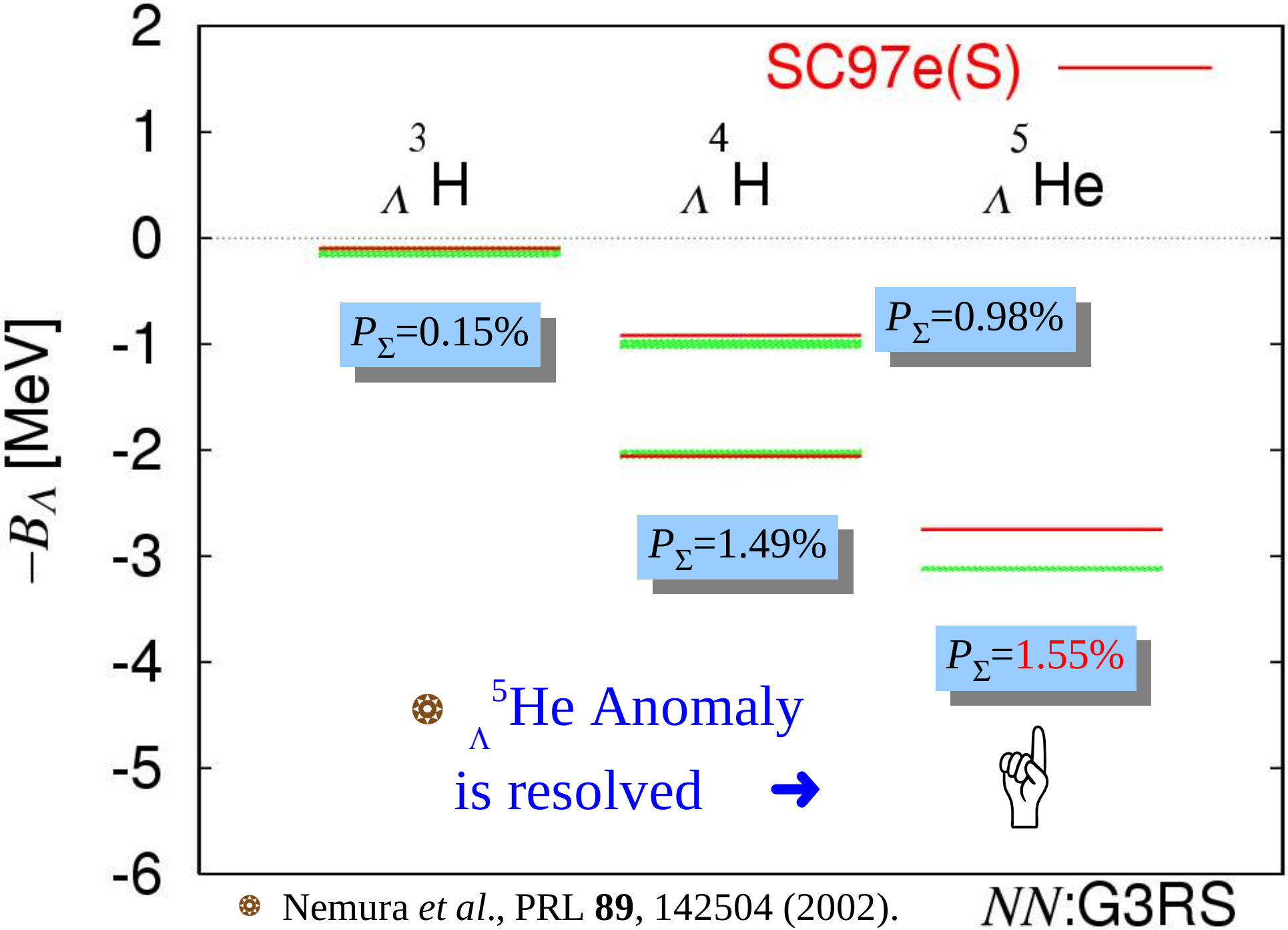
  - [3] The trial function Φ_k that produces the lowest energy among the set $\{E_k^{(1)}, \ldots, E_k^{(n)}\}$ is selected to be the k-th basis.
  - [4] $k = k+1$, go to [1].

Ab initio calculation with SVM

- SVM is capable of handling the massive calculation.

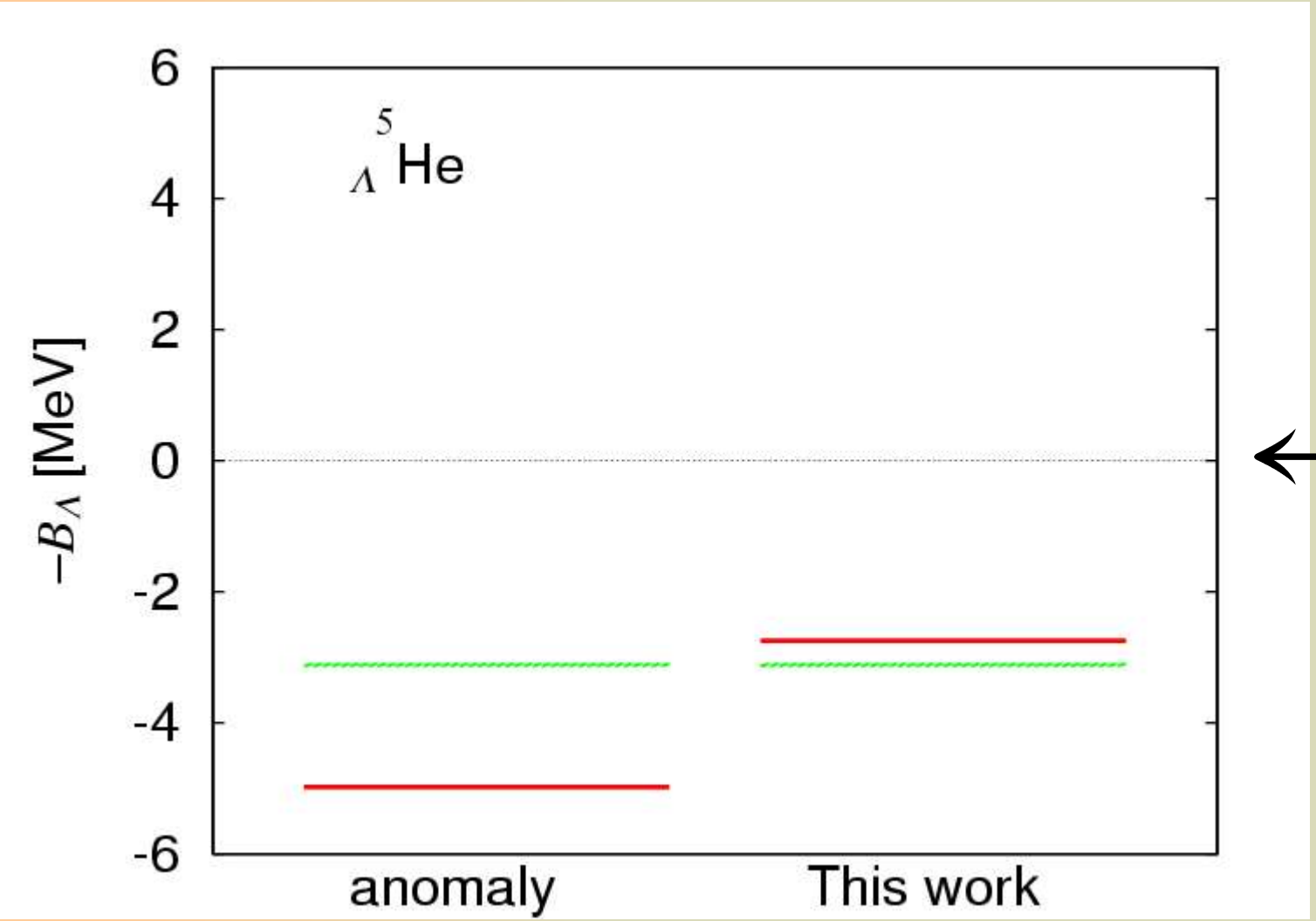


Results



- Nemura *et al.*, PRL **89**, 142504 (2002).

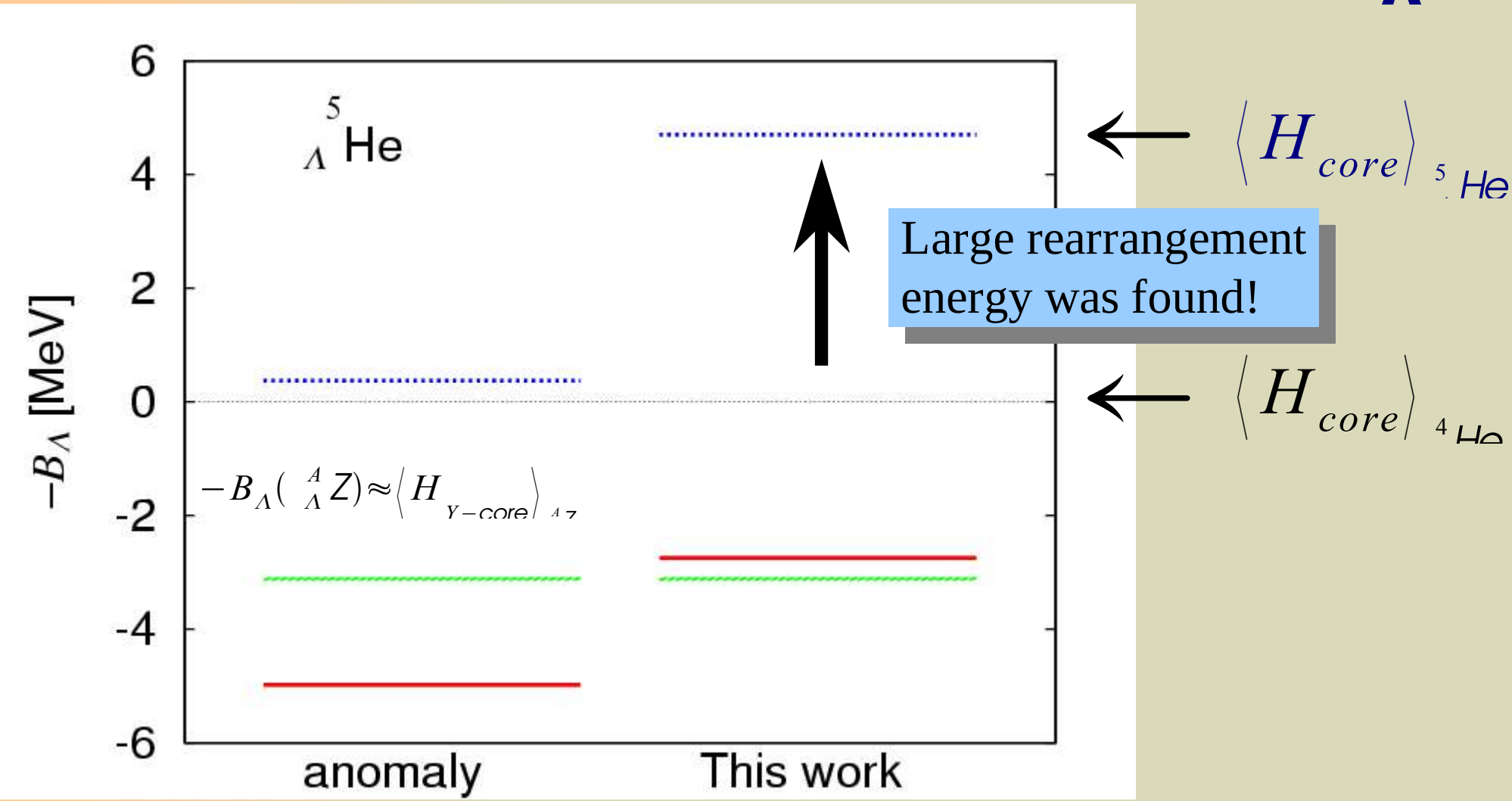
Rearrangement energy of ^{4}He in $_\Lambda^5$He



$\langle H_{core} \rangle_{^4He}$

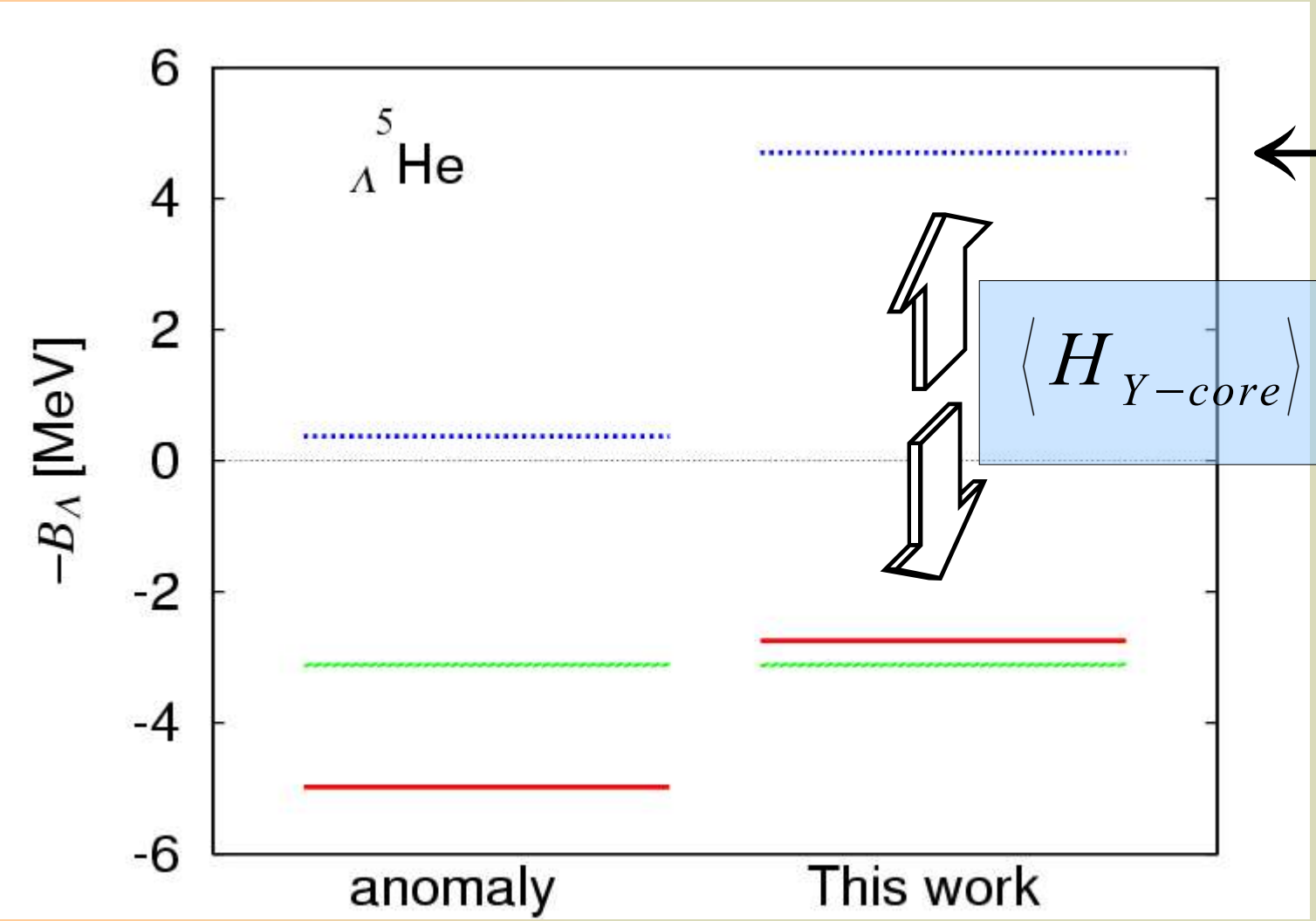
- $_\Lambda^5$He anomaly is resolved by taking account of explicit Σ degrees of freedom.

Rearrangement energy of ^{4}He in $_\Lambda^5$He



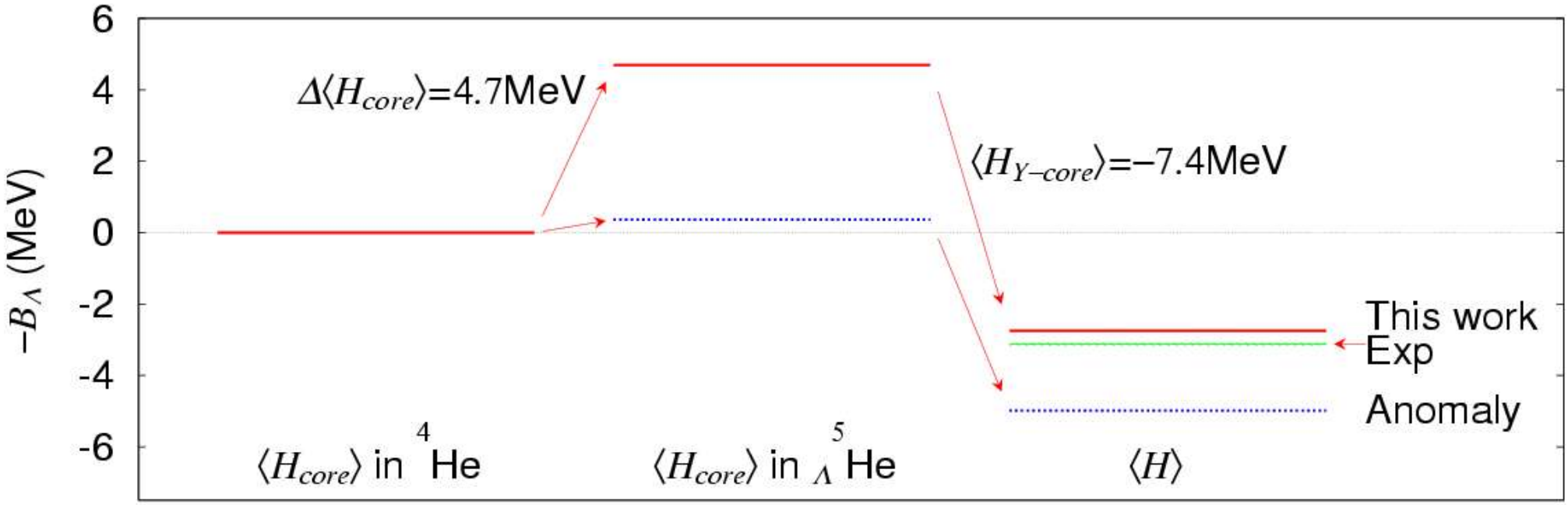
- Taking account of explicit Σ admixture, particularly using tensor Λ*N*-Σ*N* interaction, rearrangement energy is significant.

Rearrangement energy of ^{4}He in $_\Lambda^5$He



- $\langle H_{Y\text{-core}} \rangle$ = –7.4 MeV (with tensor ΛN-ΣN interaction)
- The YN interaction in $_\Lambda^5$He is much stronger than what the experimental B_Λ (=3.12 MeV) implies.

Rearrangement effect of ${}^{5}_{\Lambda}$He



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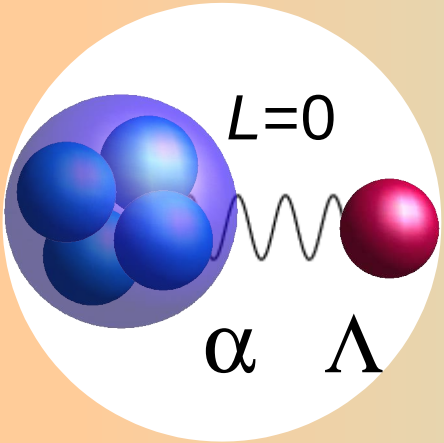
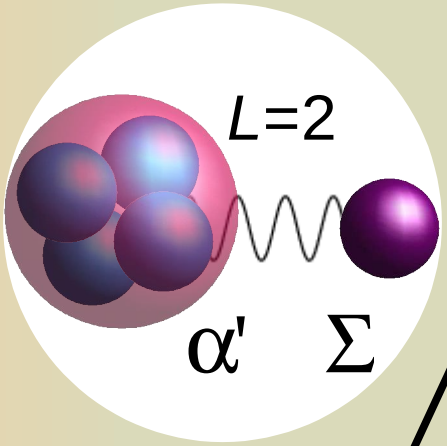
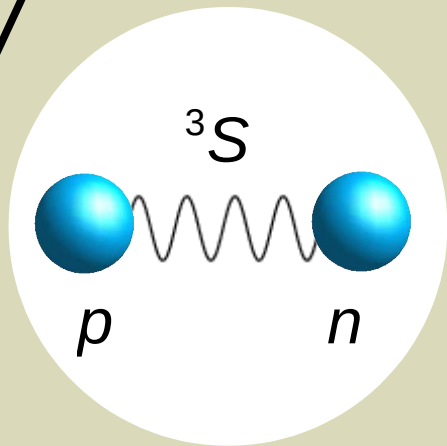
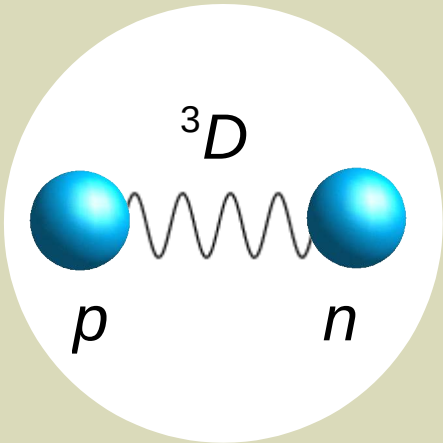
$$H_{core}=\sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2}m_N-\frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)m_N}+\sum_{i<j}^{A-1}v_{ij}^{(NN)}=T_{core}+V_{NN}\ .$$

Comparison between core+Y and d=p+n









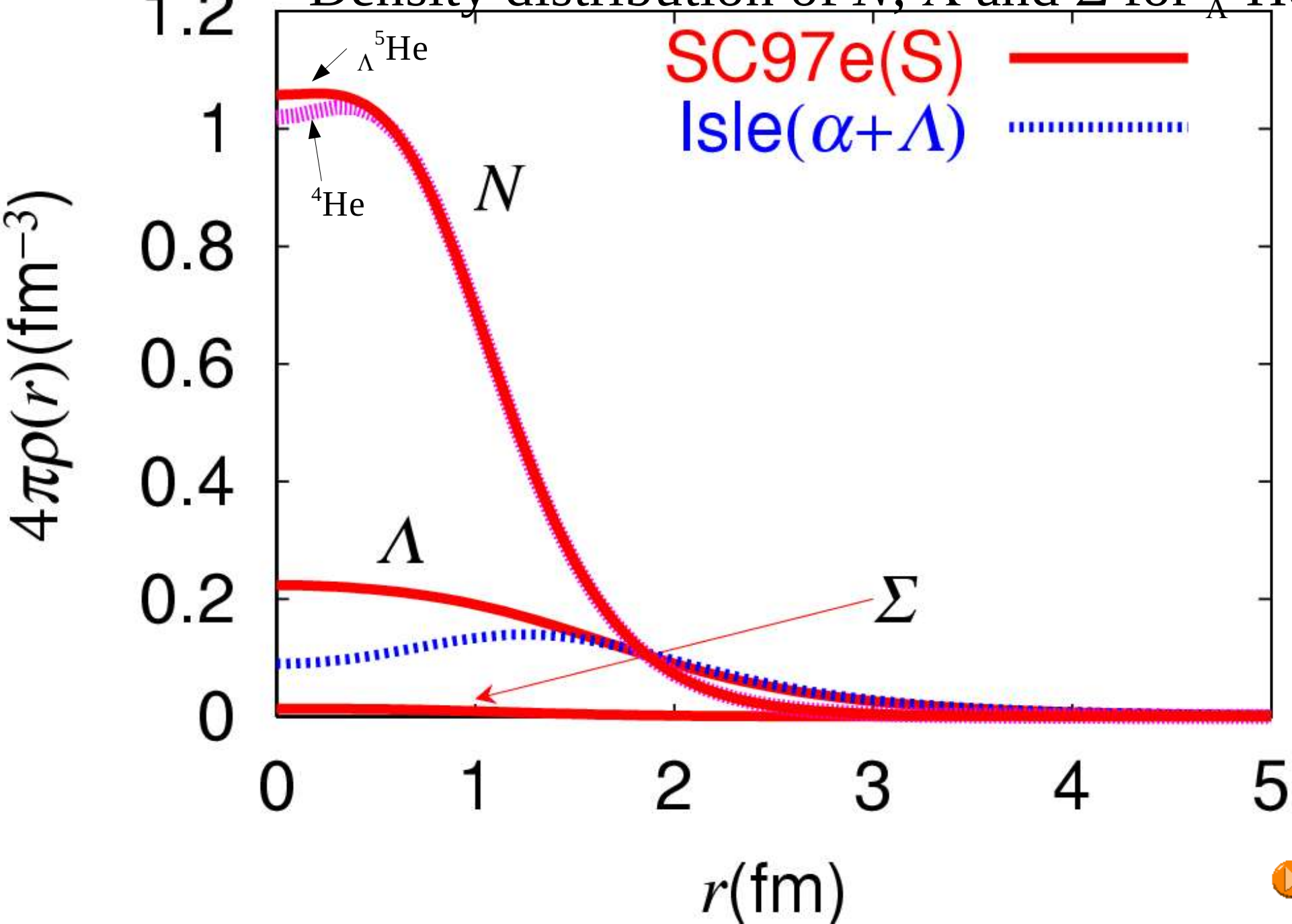
Phase shift

	$T_{Y\text{-c}\,\Lambda}$ (MeV)	$T_{Y\text{-c}\,\Sigma}+\Delta\ H_C$ (MeV)	V_{YN}(remainder) (MeV)	$2\ V_{\Lambda N\text{-}\Sigma N}$(tensor) (MeV)
${}_{\Lambda}^{5}$He	9.11	3.88+4.68	−0.86	−19.51
${}_{\Lambda}^{4}$H*	5.30	2.43+2.02	0.01	−10.67
${}_{\Lambda}^{4}$H	7.12	2.94+2.16	−5.05	−9.22

	T_S	T_D	V_{NN}(central)	V_{NN}(tensor)	V_{NN}(LS)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

Results for light hypernuclei

- Density distribution of N, Λ and Σ for ${}_{\Lambda}^{5}$He



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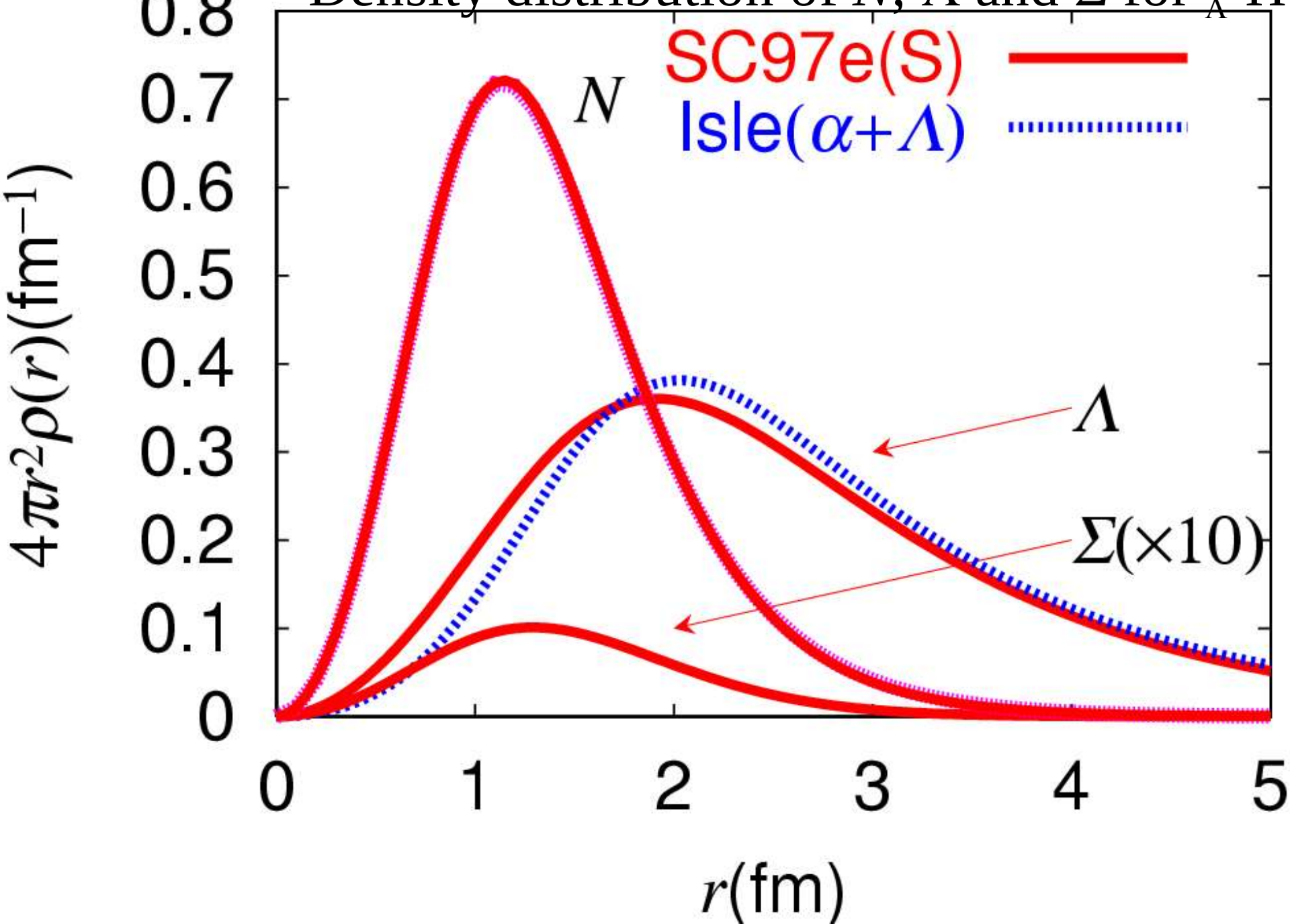
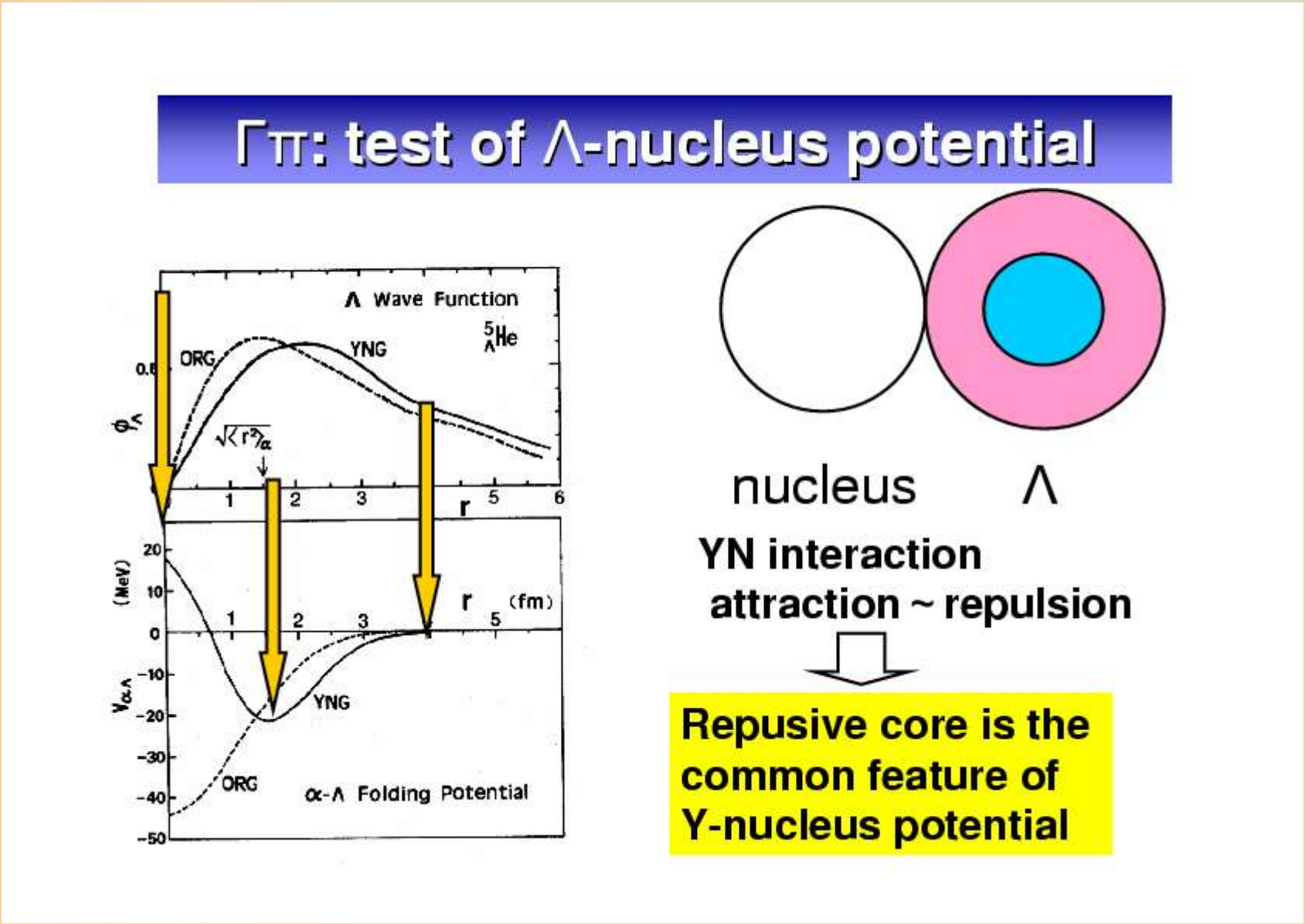


Recent weak decay experiments:

- KEK-PS E462 collaboration
  - Presented by H. Outa (RIKEN) at HYP2003 (JLAB)



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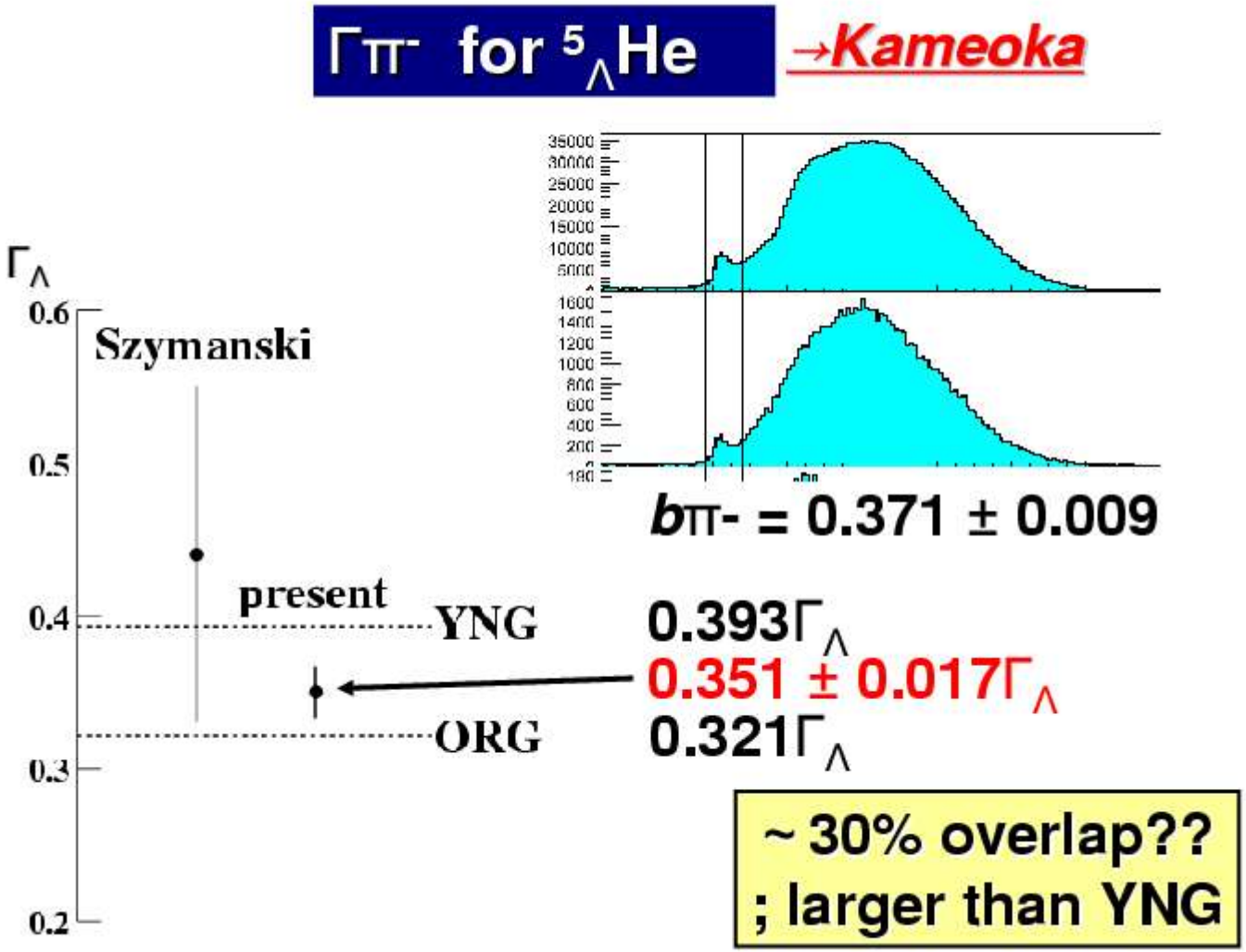
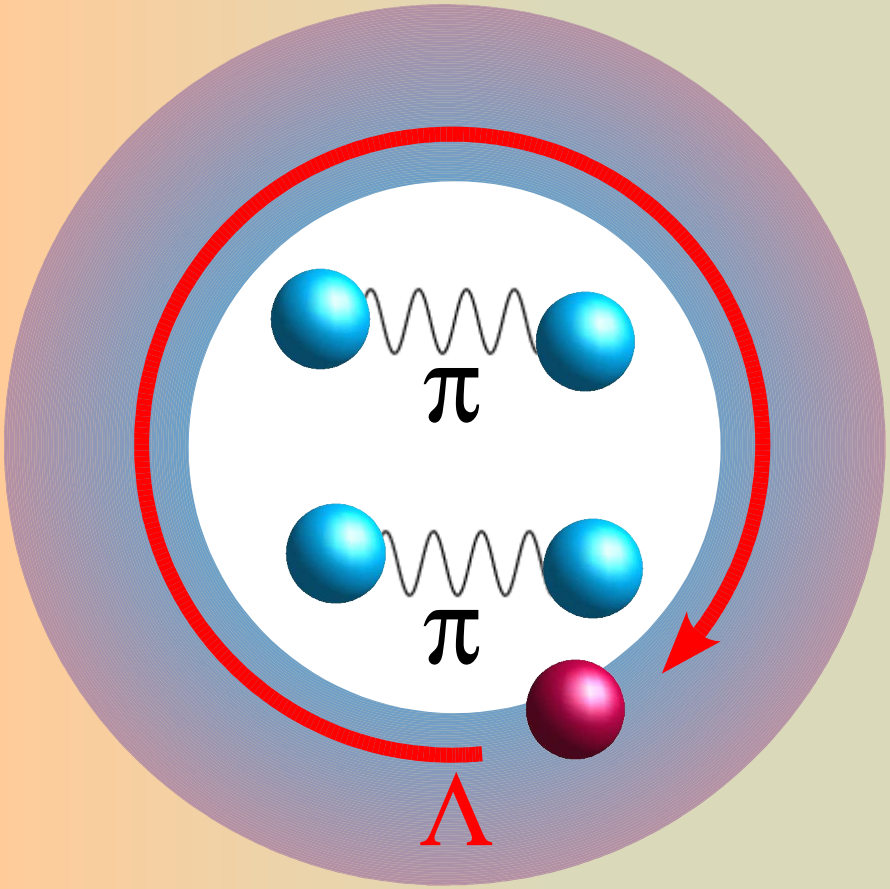
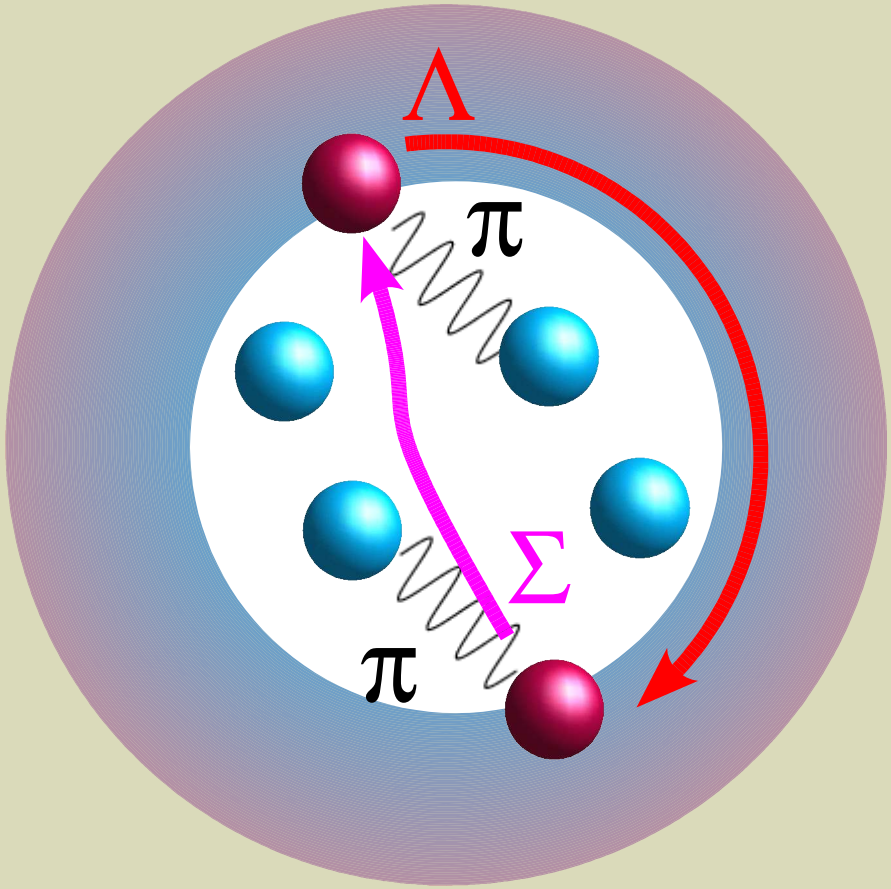


Rearrangement effect of ${}_{\Lambda}^{5}$He

- Left: A conventional picture of ${}_{\Lambda}^{5}$He
- Right: A new picture due to strong ΛN-ΣN tensor coupling





Summary

- Five-body calculation of ${}_{\Lambda}^{5}$He was performed with a *YN* interaction explicitly including Σ degrees of freedom.
- This is the first *ab initio* calculation of ${}_{\Lambda}^{5}$He using tensor Λ*N*–Σ*N* interaction which gives a bound-state solution.
- A new view of ${}_{\Lambda}^{5}$He:
  - We found the large rearrangement energy for the core nucleus in ${}_{\Lambda}^{5}$He; $\Delta\langle H_{\mathrm{core}}\rangle = 4.7$ MeV

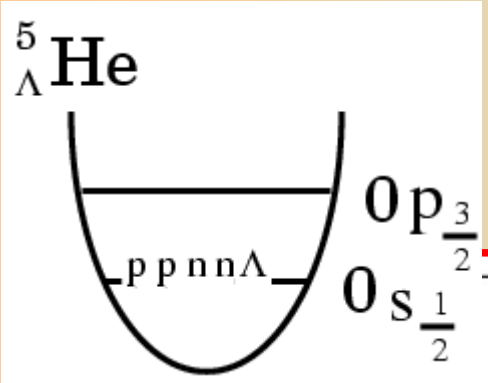
    $-B_{\Lambda} \neq \langle H_{Y\text{-core}}\rangle = -7.4$ MeV
  - Tensor Λ*N*-Σ*N* interaction is strongly attractive and affects the internal energy of the core nucleus.
  - ^{4}He is no longer rigid in interacting with a Λ particle.

In the future study:

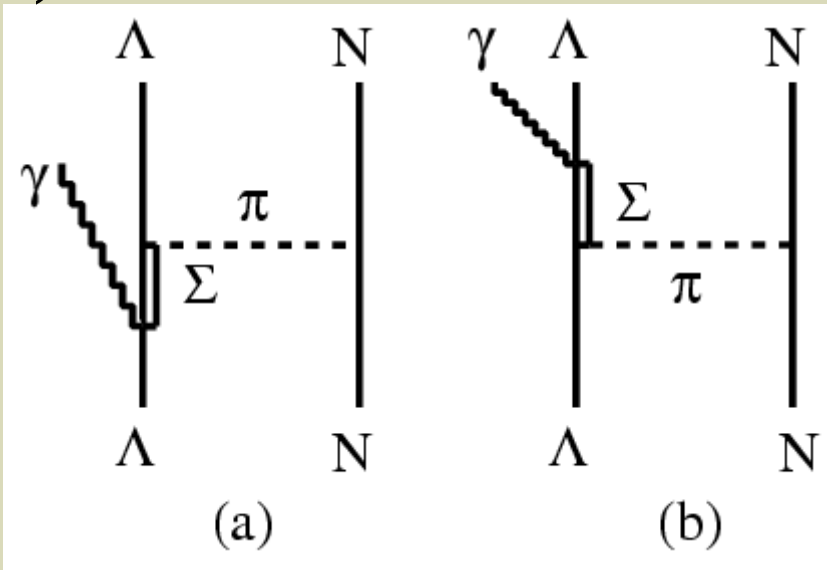
- What is the structure of ${}_{\Lambda\Lambda}^{6}\mathrm{He}$?
- Magnetic moment of ${}_{\Lambda}^{5}\mathrm{He}$
- Neutron-rich hypernuclei, e.g., ${}_{\Lambda}^{6}\mathrm{H}$

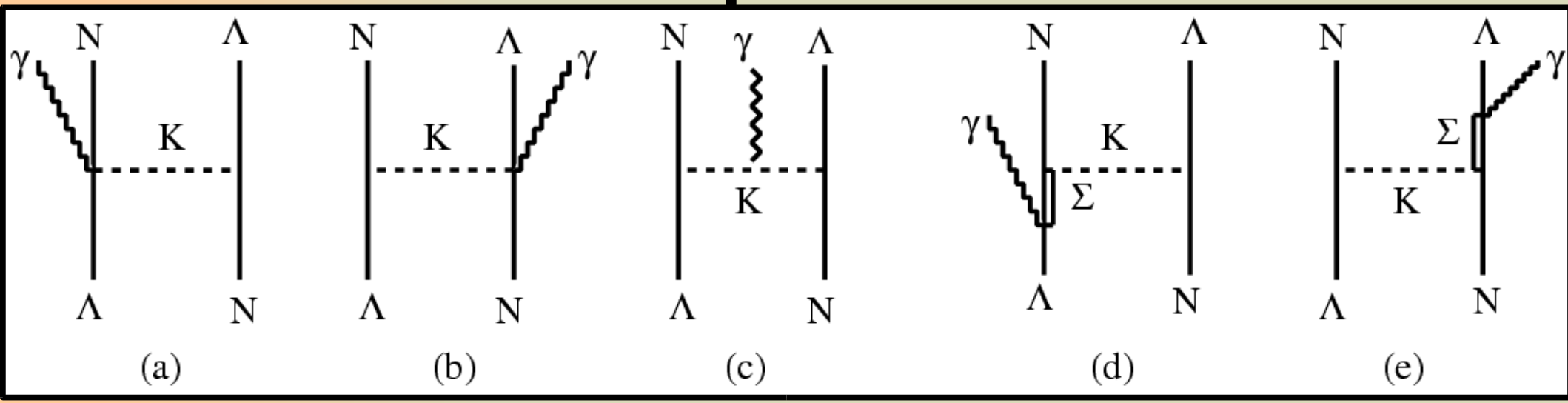
In the future study: Magnetic moments

- Exchange currents for hypernuclear magnetic moments,
  K.Saito, M. Oka and T. Suzuki,
  NPA **625**, 95 (1997).



	$^5_\Lambda$He
μ(single − particle)	-0.613μ_N
$\delta\mu$(^{4}He-Λ($s_{\frac{1}{2}}$))	0.054μ_N (-8.8%)
$\delta\mu$(Λ($s_{\frac{1}{2}}$)-N($p_{\frac{3}{2}}$))	
$\delta\mu$(^{4}He-N($p_{\frac{3}{2}}$))	

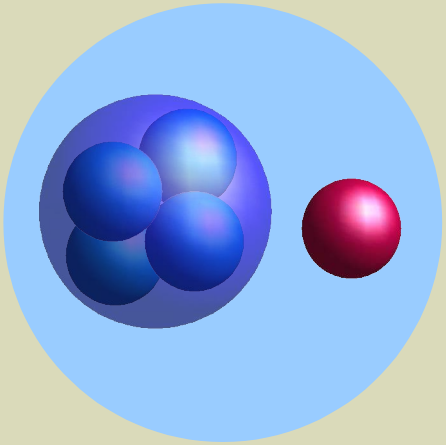




In the future study:

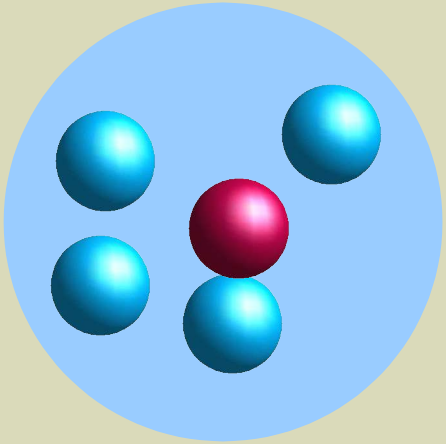
What is the structure of ${}_{\Lambda\Lambda}^{6}$He?

- $B(\text{total}) = B({}^{4}\text{He}) + B_{\Lambda}({}_{\Lambda}^{5}\text{He})$



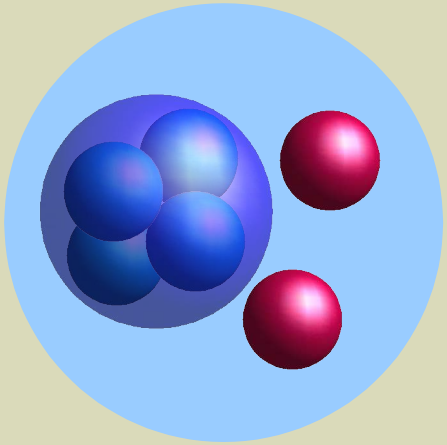
- A conventional picture:
  $B(\text{total})$
  $= B({}^{4}\text{He}) + B_{\Lambda}({}_{\Lambda}^{5}\text{He})$
  $= 28+3$ MeV.

- A new picture:
  $B(\text{total})$
  $= (B({}^{4}\text{He}) - \Delta E_{c}) + (B_{\Lambda}({}_{\Lambda}^{5}\text{He}) + \Delta E_{c})$
  $= 23+8$ MeV.



In the future study:
What is the structure of ${}_{\Lambda\Lambda}{}^{6}$He?

- $B(\text{total}) = B({}^{4}\text{He}) + 2B_{\Lambda}({}_{\Lambda}{}^{5}\text{He}) + \Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\text{He})$
- A conventional picture:



$B(\text{total})$

$= B({}^{4}\text{He}) + 2B_{\Lambda}({}_{\Lambda}{}^{5}\text{He}) + \Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\text{He})$

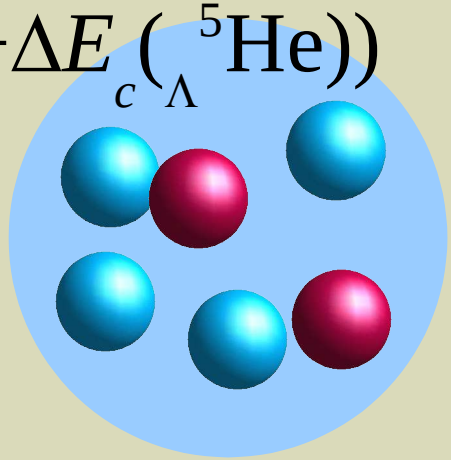
$= 28 + 2\times3 + 1 = 35$ MeV.

- A conjecture: $\Delta E_{c}({}_{\Lambda\Lambda}{}^{6}\text{He}) = 2\Delta E_{c}({}_{\Lambda}{}^{5}\text{He})$,

$B(\text{total})$

$= (B({}^{4}\text{He}) - 2\Delta E_{c}({}_{\Lambda}{}^{5}\text{He})) + 2(B_{\Lambda}({}_{\Lambda}{}^{5}\text{He}) + \Delta E_{c}({}_{\Lambda}{}^{5}\text{He}))$

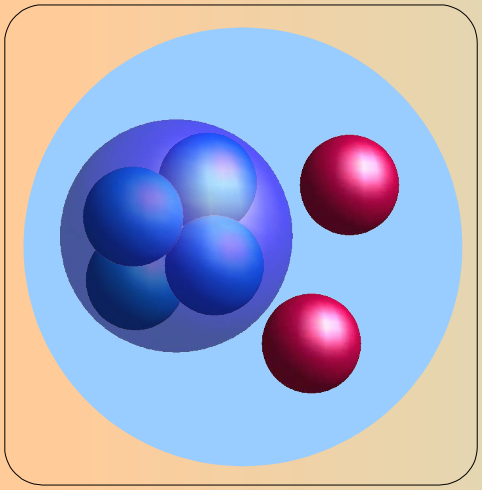
$+ \Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^{6}\text{He})$

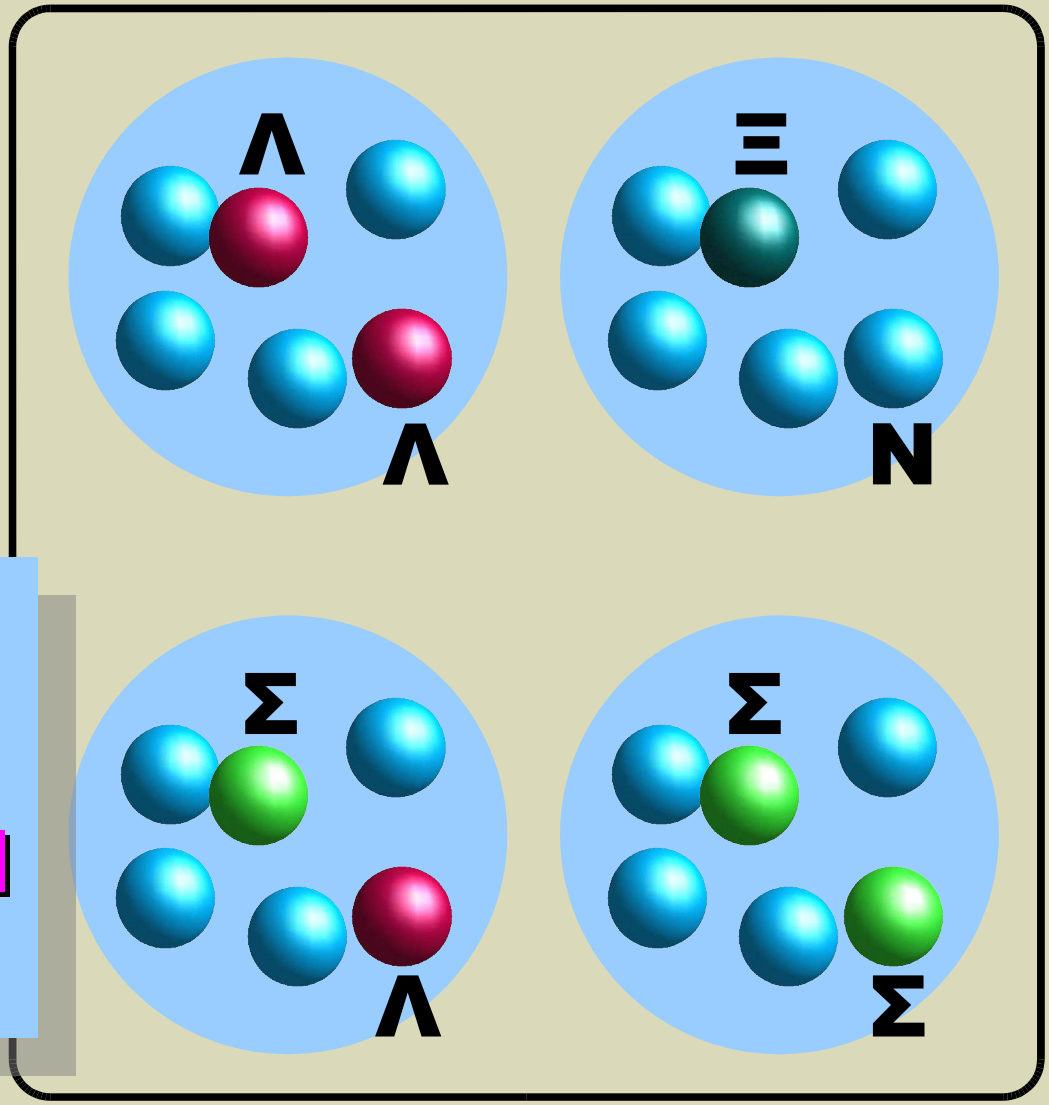


$= 18 + 2\times8 + 1 = 35$ MeV,

(or $= 19 + 2\times7 + 2 = 35$ MeV)

Ab initio calculation of S=-2 hypernucleus $_{\Lambda\Lambda}{}^{6}$He in a fully coupled channel scheme





Complete six-body

+

Full-coupled channel treatment