Nucleon Form Factors Analyses: Status and Perspectives

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Analyses of (electromagnetic) nucleon form factors

- model independent analyses:
  - dispersion theoretical analysis
  - low Q^2: chiral perturbation theory
- lattice calculations of nucleon form factors:
  - chiral extrapolation from lattice data $M_\pi^2 \approx 0.3..1.2$ GeV^2 $\rightarrow$ physical pion mass $M_\pi^2 \approx 0.02$ GeV^2.
- model analyses of nucleon form factors:
  - SKYRME model with vector mesons
  - intrinsic or "core" form factor + VMD
  - "relativity" and "meson cloud" effects
- (constituent) quark model description of form factors
  - field theoretically based (constituent) quark models
    - quark-diquark Dyson-Schwinger–Bethe-Salpeter approach
    - Covariant CQM on the basis of the Salpeter equation
  - Poincaré-invariant prescriptions for calculating currents in quark models

for strange and/or weak form factor, see Wim van Oers;
for the discussion of the 2-photon exchange, see Marc Vanderhaeghen

Dispersion-theoretical analysis

H.-W. Hammer, U.-G. Meißner, Eur. Phys. J. A **20** (2004) 469

Definitions: $t := q^2 = (p' - p)^2\,, Q^2 = -t\,;\qquad G_E = F_1 + \frac{t}{4m^2}F_2\,,\quad G_M = F_1 + F_2$

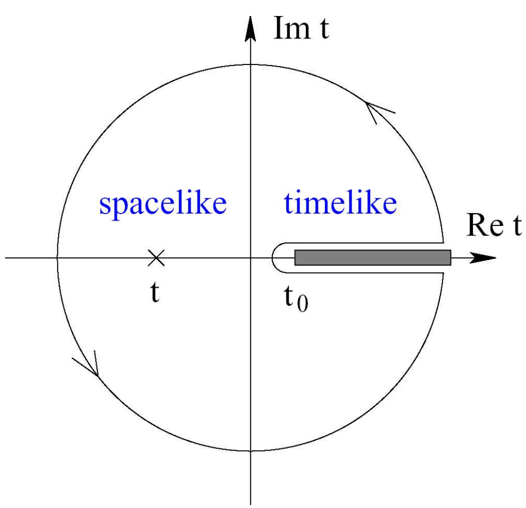
Isospin decomposition:

$$\langle N(p')|j_\mu^I|N(p)\rangle = \bar{u}(p')\left[F_1^I(t)\gamma_\mu + \frac{i}{2m}F_2^I(t)\sigma_{\mu\nu}q^\mu\right]u(p), \quad I = S, V,$$

Crossing: $\langle N(p')|j_\mu^I|N(p)\rangle \leftrightarrow \langle \bar{N}(\bar{p})N(p')|j_\mu^I|0\rangle$

Spectral representation:

$$\Im\langle \bar{N}(\bar{p})N(p')|j_\mu^I|0\rangle \quad \approx \quad \sum_n \langle \bar{N}(\bar{p})N(p')|n\rangle\langle n|j_\mu^I|0\rangle$$

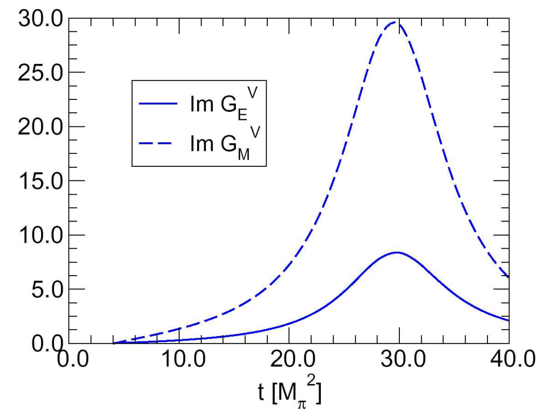


$$\Rightarrow F_i^I(Q^2) = \frac{1}{\pi}\int_{(\mu_o^I)^2}^{\infty} d\mu^2\,\frac{\sigma_i^I(\mu^2)}{\mu^2 + Q^2}, \quad i = 1, 2,\ I = S, V,$$

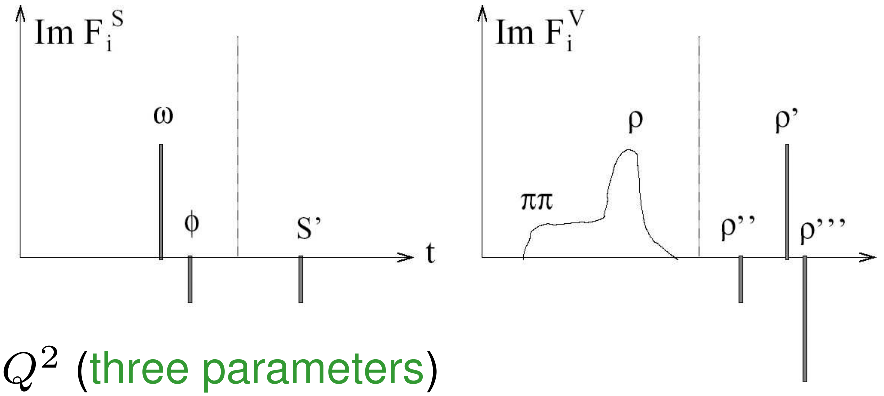
with the spectral function $\sigma_i^I(\mu^2) = \Im F_i^I(\mu^2)$,
and thresholds $\mu_0^S = 3M_\pi\,,\ \mu_0^V = 2M_\pi\,.$

Dispersion-theoretical analysis

Spectral functions σ_i^I:



- $I = V$: two-pion continuum (with ρ) is known up to $t \approx 40 M_\pi^2$
- higher states from VD-pole approximation:
  $F_i(t) \approx \sum_v \frac{a_i^v}{m_v^2 - t}$



- perturbative QCD behaviour at large Q^2 (three parameters)

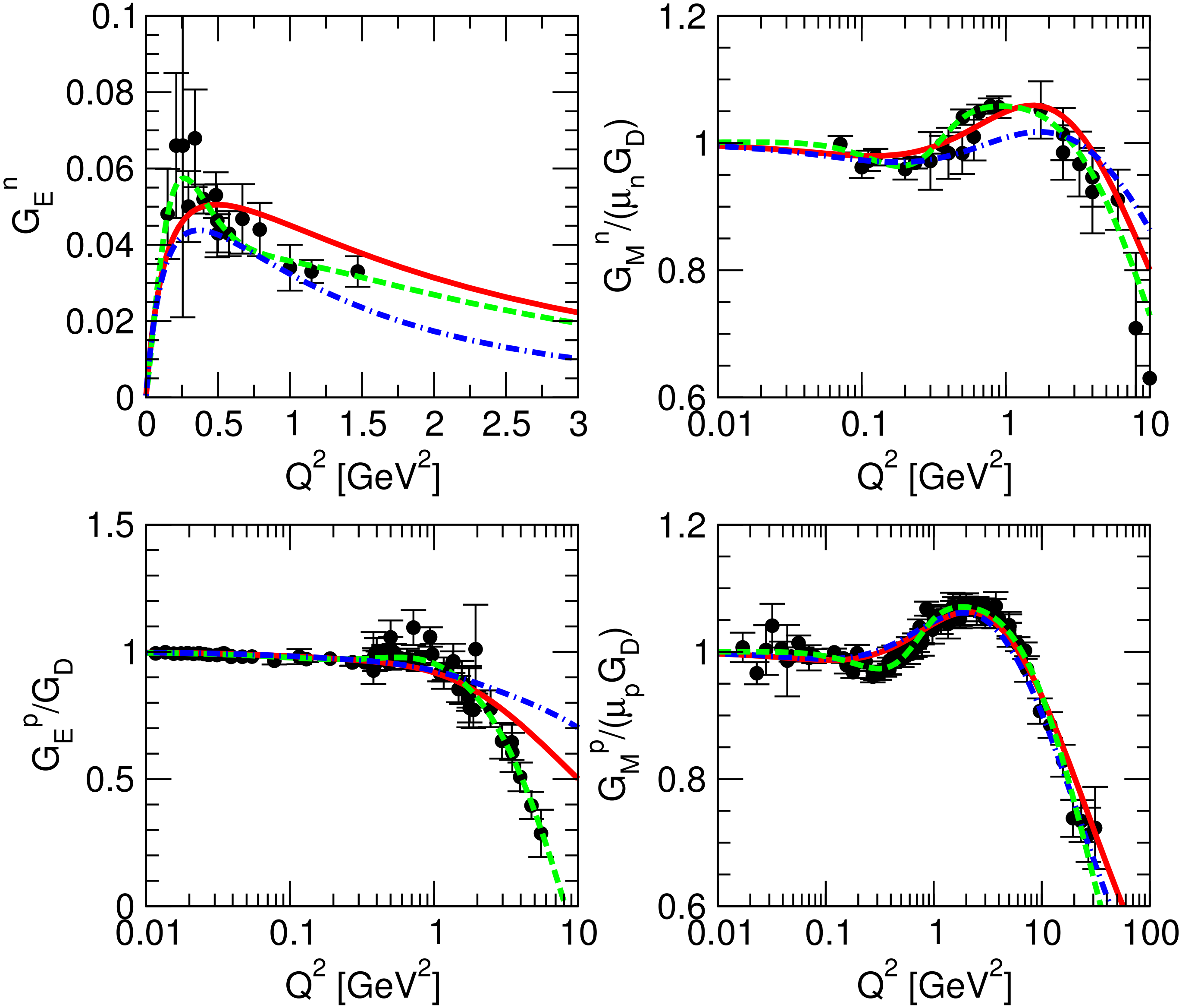
constrained by

- charges, magnetic moments and neutron charge radius
- superconvergence relations in the perturbative regime

fitted to all 4 form factors, data compiled by

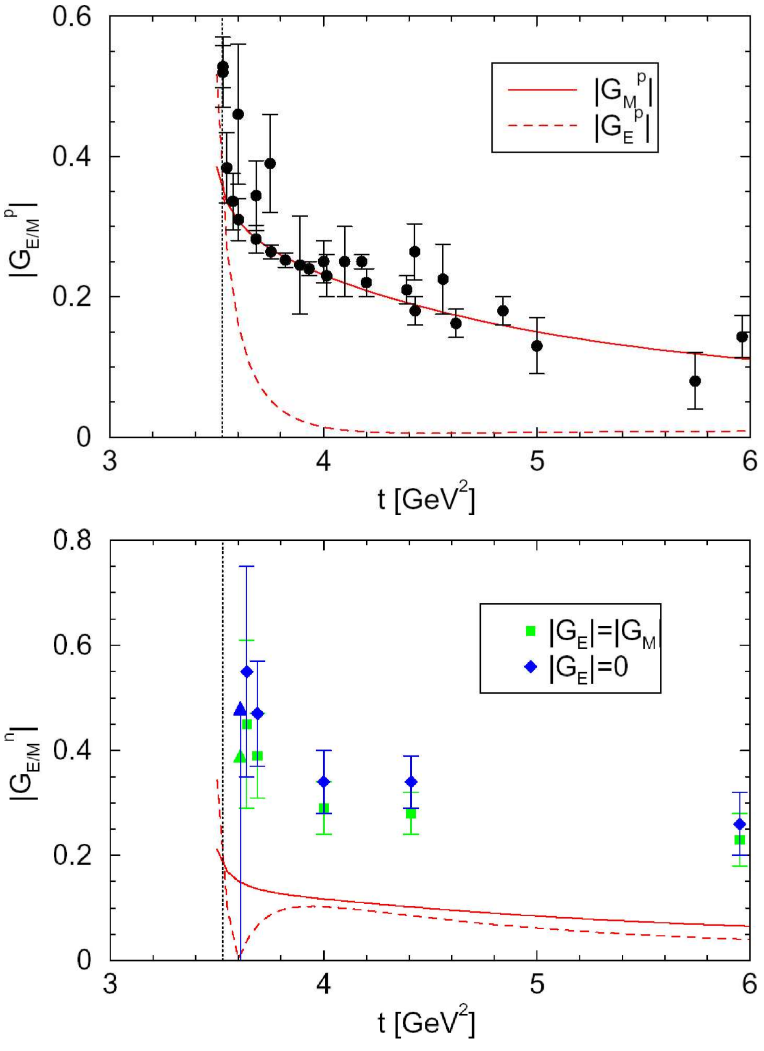
J. Friedrich, T. Walcher, Eur. Phys. J A **17** (2003) 607

Dispersion-theoretical analysis: results



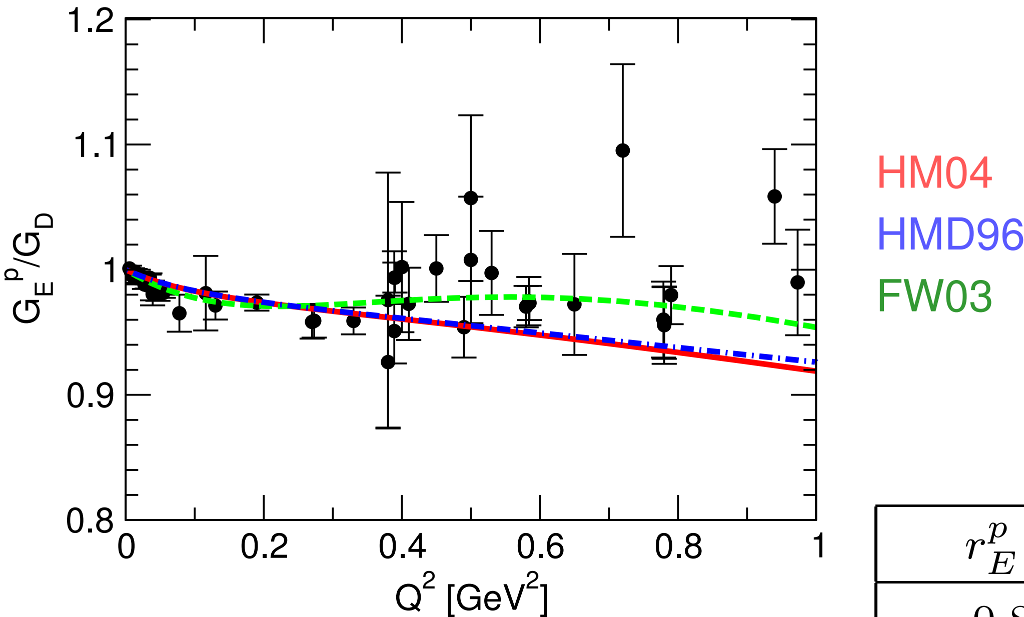
HM04
HMD96
FW03

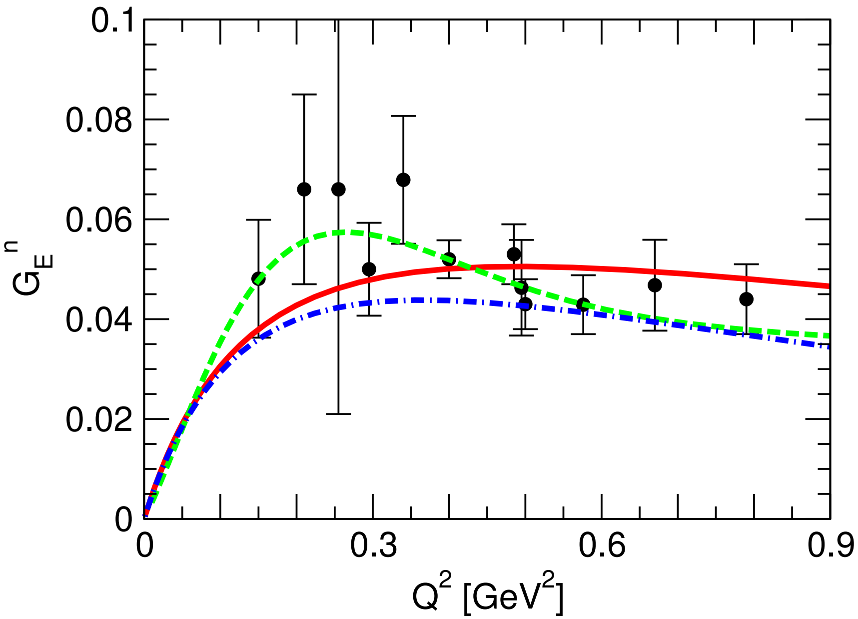
Dispersion-theoretical analysis: timelike FF



H.-W. Hammer, U.-G. Meißner, D. Drechsel, Phys. Lett. B **385** (1996) 343
update in preparation: M. Belushkin, H.-W. Hammer, U.-G. Meißner

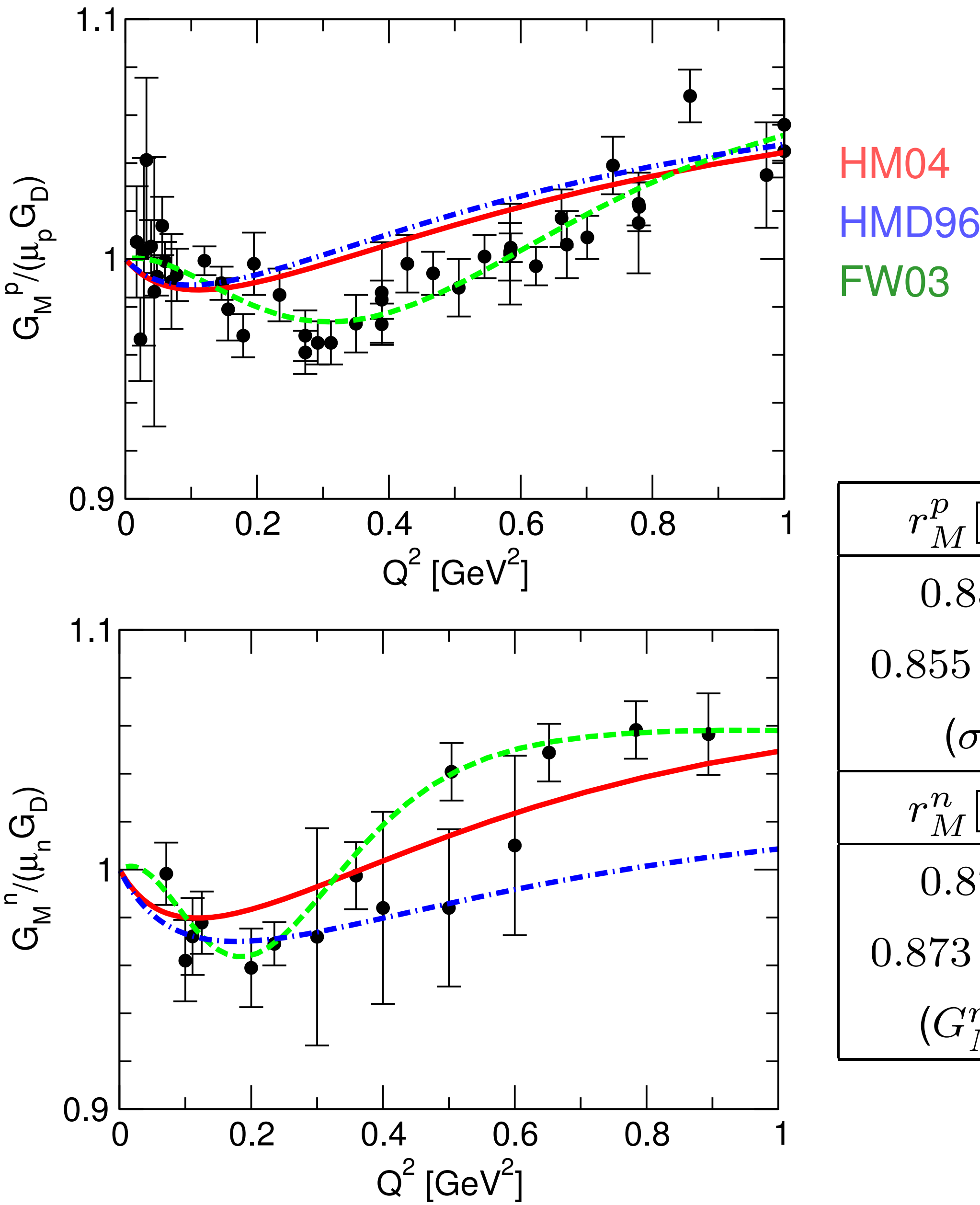
Electric nucleon form factors low Q^2





r_E^p [fm]	
0.848	HM04
0.880 ± 15	R. Rosenfelder,
(σ)	Phys. Lett. B **479** (2000) 381
0.895 ± 18	I. Sick,
(σ)	Phys. Lett. B **576** (2003) 62
0.883 ± 14	K. Melnikov, T. v. Ritbergen,
(Lamb shift)	Phys. Rev. Lett. **84** (2000) 1673

Magnetic nucleon form factors low Q^2



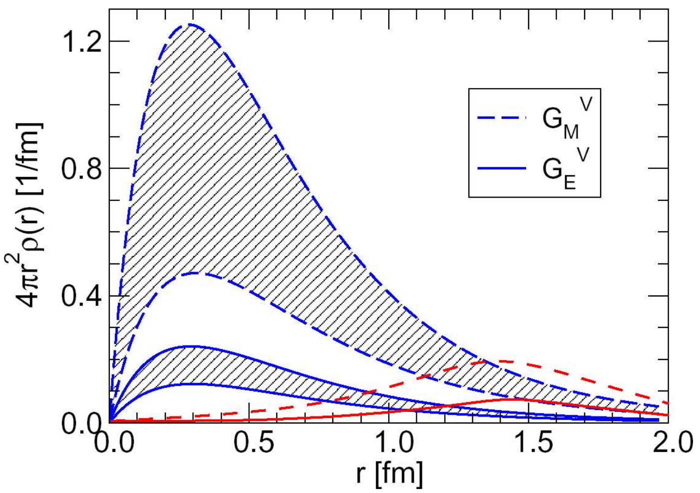
r_M^p [fm]	
0.857	HM04
0.855 ± 35	I. Sick,
(σ)	Phys. Lett. B **576** (2003) 62
r_M^n [fm]	
0.879	HM04
0.873 ± 11	G. Kubon *et al.*,
(G_M^n)	Phys. Lett. B **524** (2002) 26

2π-continuum and the "pion cloud"

- By associating the dipole-FF with a "bare nucleon" and an exponential contribution with the "pion cloud" Friedrich and Walcher arrived at an extremely long-ranged pion cloud with $r_{max} \approx 1.5$ fm.
- However, separating the ρ meson contribution from $\Im G^V$ and calculating the coordinate-space charge distribution by

$$\rho_i^V(r) = \frac{1}{4\pi^2} \int_{4M_\pi^2}^{40M_\pi^2} dt\, \Im G_i^V(t) \frac{\exp(-r\sqrt{t})}{r} \qquad i = E, M$$

Hammer and Meißner find $r_{max} \approx 0.3$ fm.



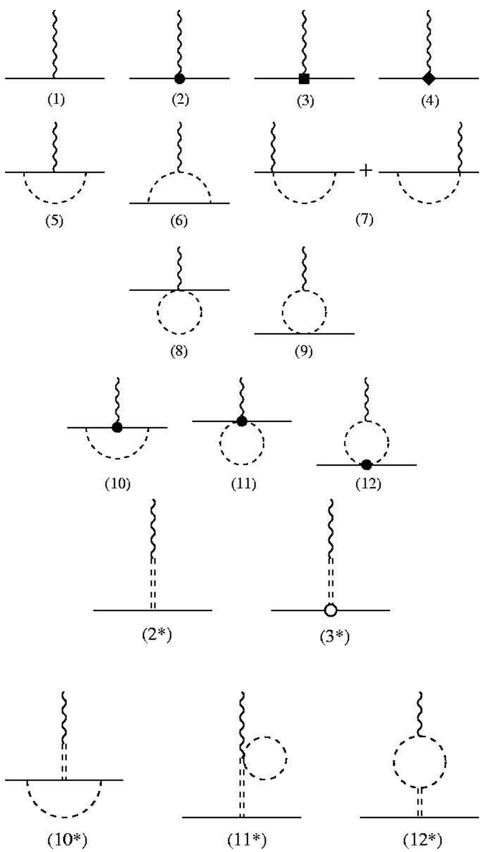
Summary:

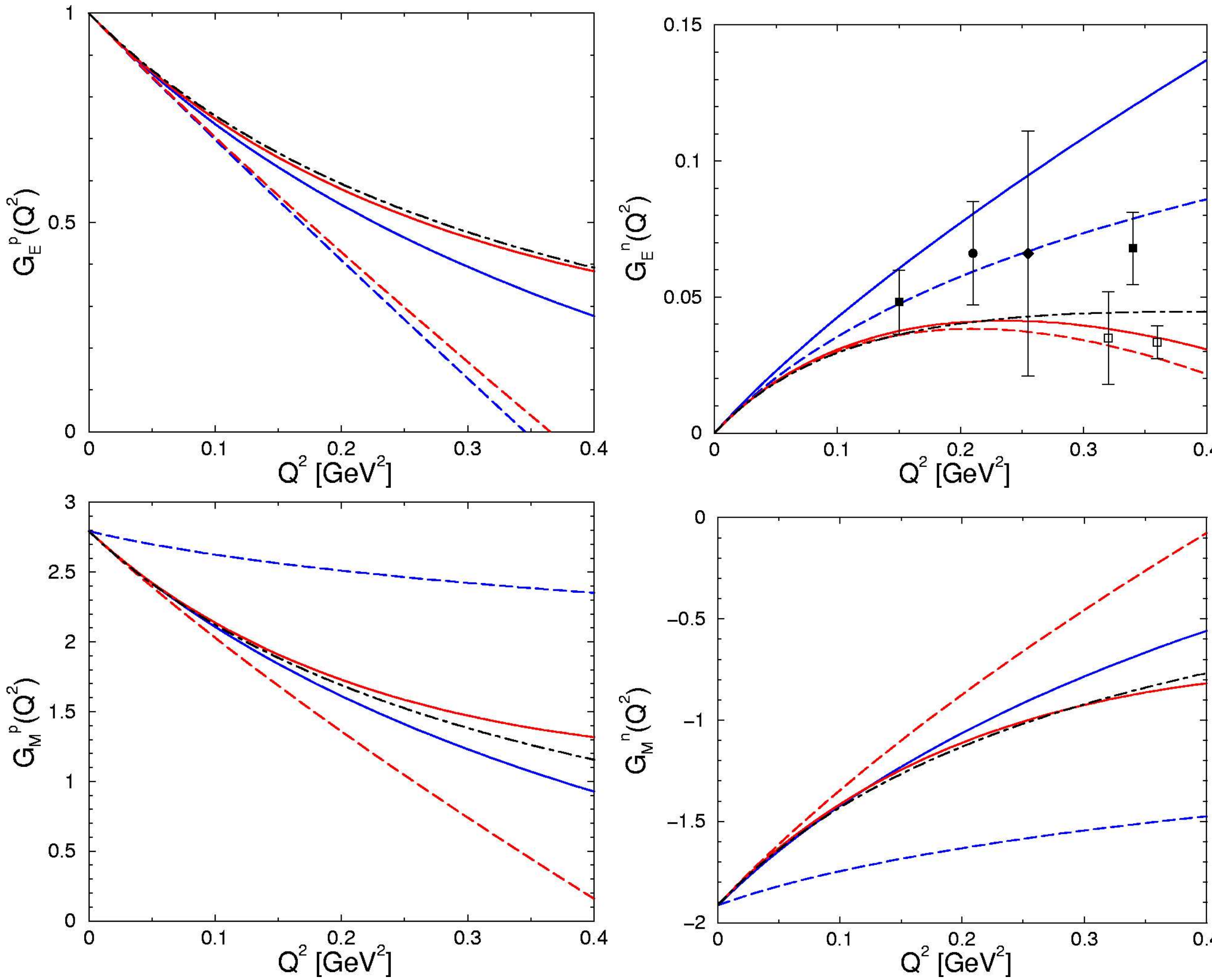
- except for the JLAB G_E^p/G_M^p data dispersion theory consistently accounts for all nucleon form factor data;
- from 2π-continuum: "pion cloud" $\sqrt{\langle r^2 \rangle} = 0.8 \ldots 1.0$ fm ;
- long range "pion cloud" $r \approx 1.5 \ldots 2.0$ fm incompatible with dispersion theory/ChPT .

Baryon chiral perturbation theory

B. Kubis, U.-G. Meißner, Nucl. Phys. **A679**, 698 (2001).

Calculations performed up to $\mathcal{O}(q^3)$ and $\mathcal{O}(q^4)$ in the framework of relativistic chiral perturbation theory (i.e. 1-pion loop) with infrared regularisation with (solid) and without (dashed) vector mesons. LEC from static properties of dispersion theoretical analysis (dashed-dot).





Lattice calculations

J.D. Ashley, D.B. Leinweber, A.W. Thomas, R.D. Young; hep-lat/0308024
based on (quenched) QCDSF “data” also analysed by
M. Göckeler, *et al.* (QCDSF); hep-lat/0303019
Lattice data available for $M_\pi^2 \approx 0.3..1.2$ GeV^2
$\Rightarrow$ chiral extrapolation of isoscalar and isovector magnetic moments

$$\mu_i(M_\pi) = \frac{\mu_0}{1 - \frac{\chi_i}{\mu_0} M_\pi + c M_\pi{}^2}$$

and form factors parameterised by dipole masses

$$\left(\Lambda_M^v\right)^2 (M_\pi) = \frac{12(1 + A_1 M_\pi^2)}{A_0 + \frac{\chi_1}{M_\pi}\frac{2}{\pi}\arctan\left(\frac{\mu}{M_\pi}\right) + \frac{\chi_2}{2}\ln\left(\frac{M_\pi^2}{M_\pi^2+\mu^2}\right)}$$

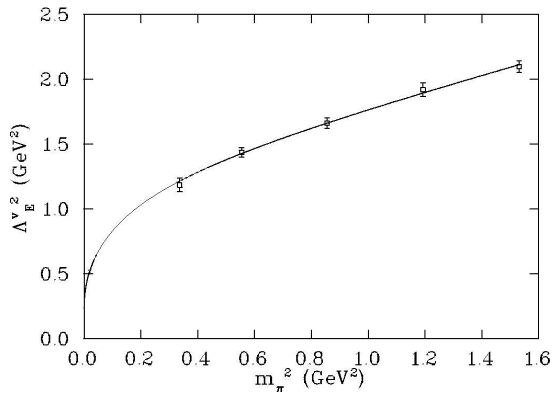
$$\left(\Lambda_E^v\right)^2 (M_\pi) = \frac{12(1 + B_1 M_\pi^2)}{B_0 + \frac{\chi_2}{2}\ln\left(\frac{M_\pi^2}{M_\pi^2+\mu^2}\right)}$$

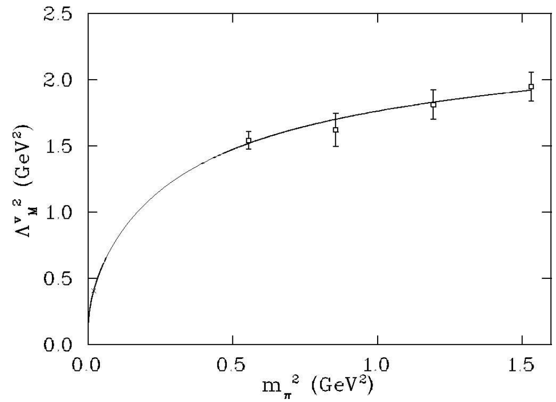
$$\left(\Lambda_i^s\right)^2 (M_\pi) = C_0^i + C_1^i M_\pi^2$$

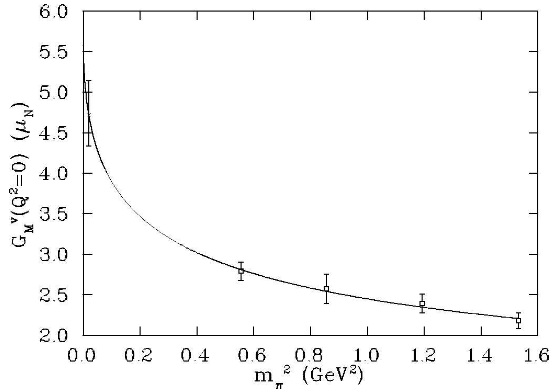
to the physical pion mass.

Lattice calculations, extrapolating parameters

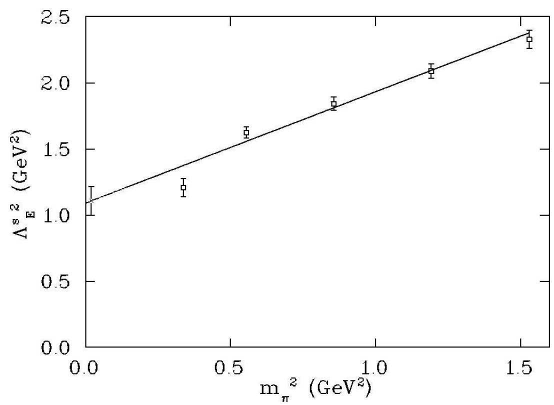
isovector (electric) dipole mass (magnetic) magnetic moment

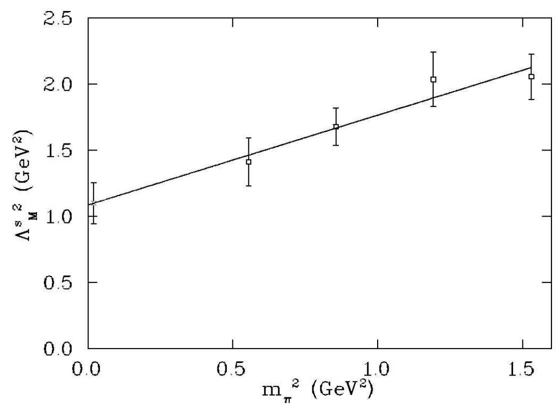


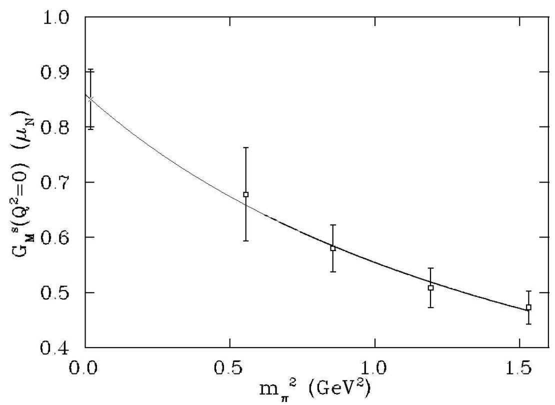




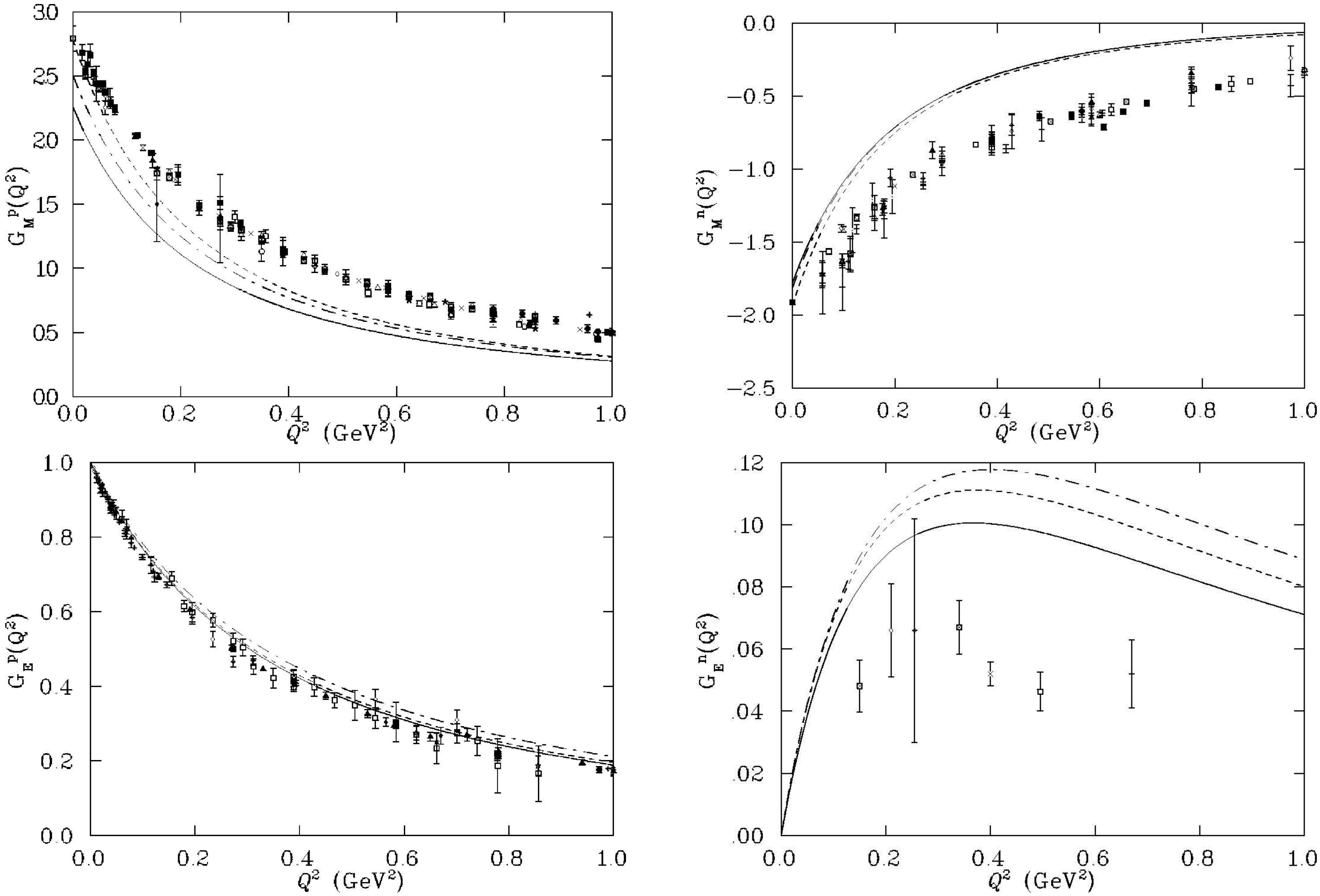
isoscalar (electric) dipole mass (magnetic) magnetic moment







Lattice calculations, form factors



Alternative (phenomenological) analyses

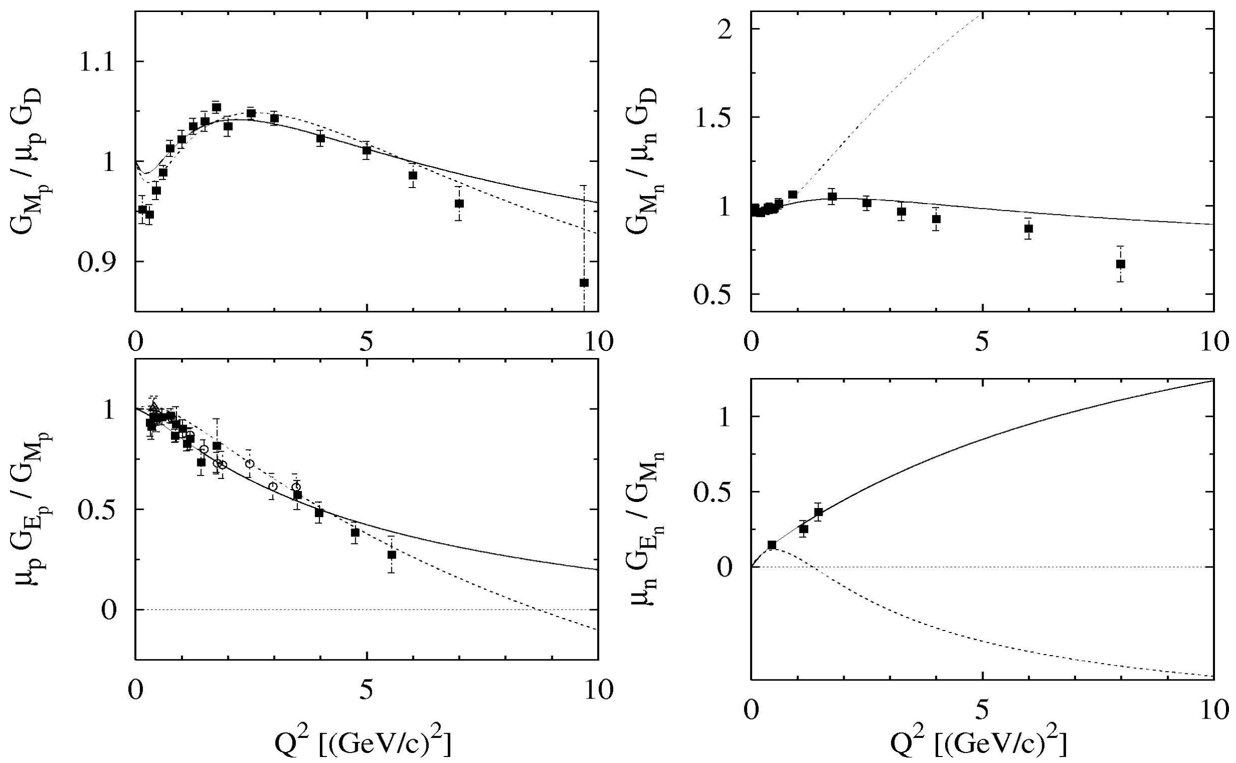
R. Bijker, F. Iachello, nucl-th/0405028: Intrinsic form factor and VMD:

$$F_i^I(Q^2) = \frac{f_i^I(Q^2)}{2(1+\gamma\, Q^2)^2}$$ with

$$f_1^I(Q^2) = \left[1 - \sum_{v_I} \beta_{v_I} \frac{Q^2}{m_{v_I}^2 + Q^2}\right], I = S, V, (v_S = \omega, \phi, v_V = \rho)$$

$$f_2^S(Q^2) = \left[\frac{(\mu_p + \mu_n - 1 - \alpha_\phi) m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2}\right], \quad f_2^V(Q^2) = \left[\frac{(\mu_p - \mu_n - 1 - \alpha_\rho)}{1+\gamma\, Q^2} + \alpha_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2}\right]$$

(ρ-width– and (possible) logarithmic pQCD– dependence)



(also time-like FF with complex γ)

Chiral soliton model

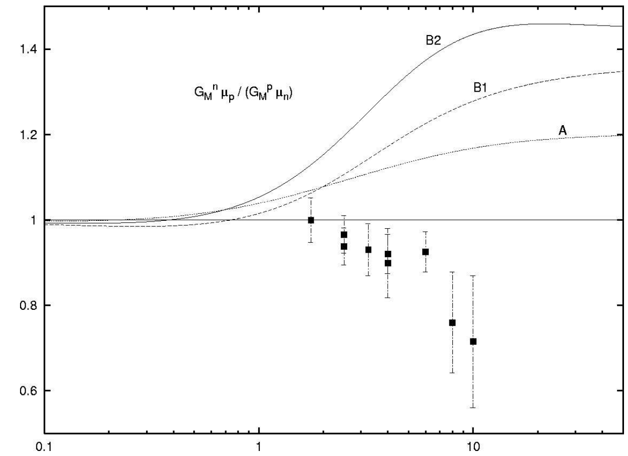
G. Holzwarth; hep-ph/0201138 $SU(2)$ pionic Skyrme model + vector meson effects:

(A) $\rho - \omega$–VMD with two free parameters

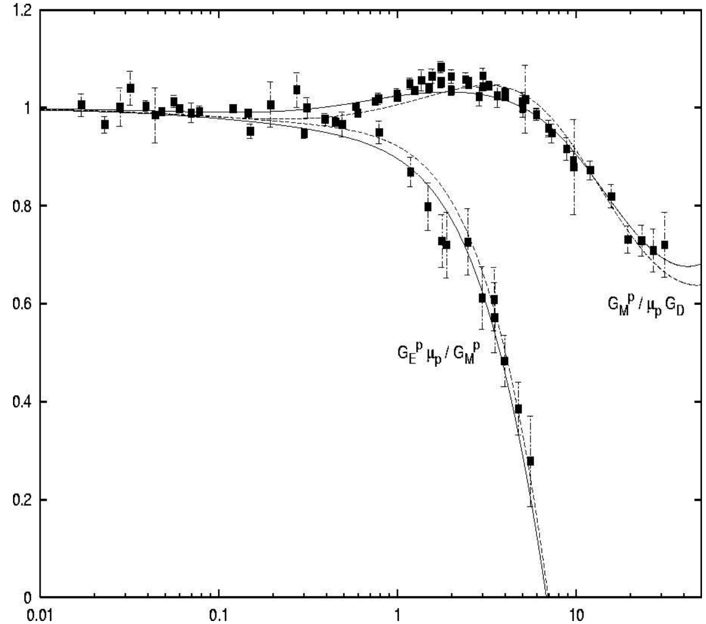
(B) Extension of the Skyrme-Lagrangian by $\rho - \omega$–d.o.f. (4 parameters)

- Static model $\Rightarrow$ boost to Breit-frame (mass-parameter: $M_b \approx 2$ GeV)

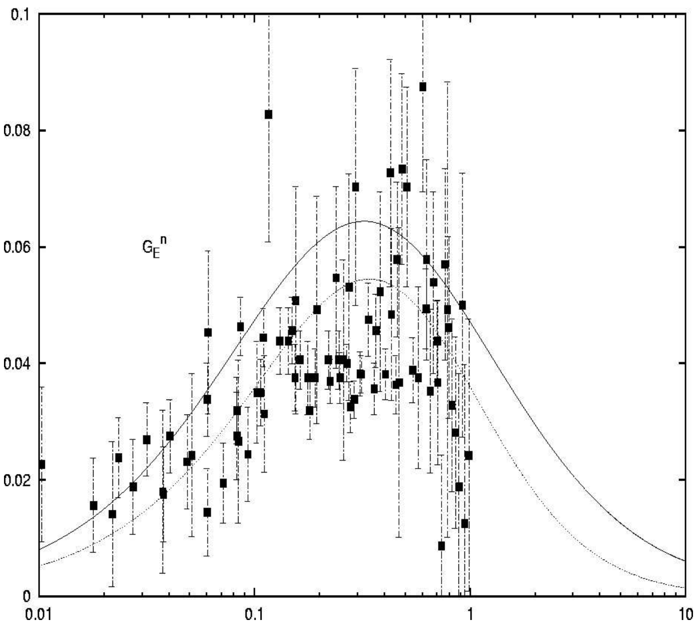
$G_M^n \mu_p / G_M^p \mu_n$



$G_M^p / \mu_p G_D$



G_E^n



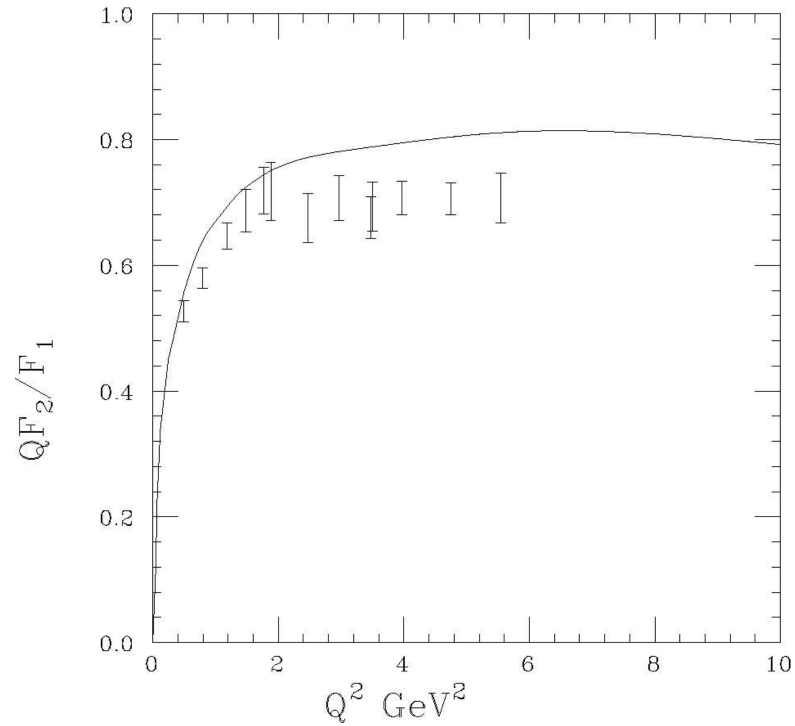
$G_E^p \mu_p / G_M^p$

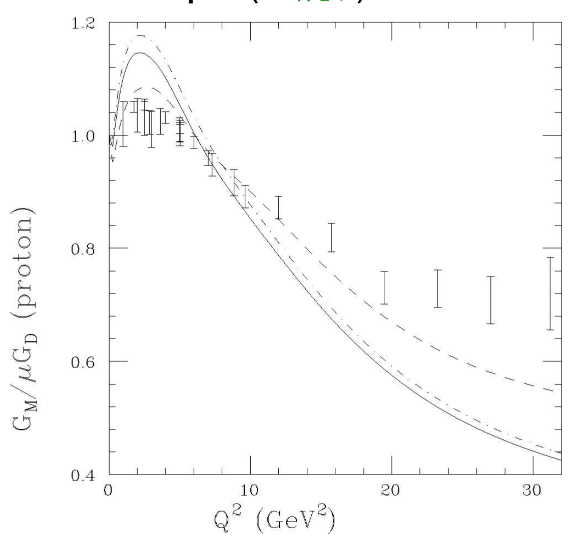
Poincaré covariant light-cone calculation

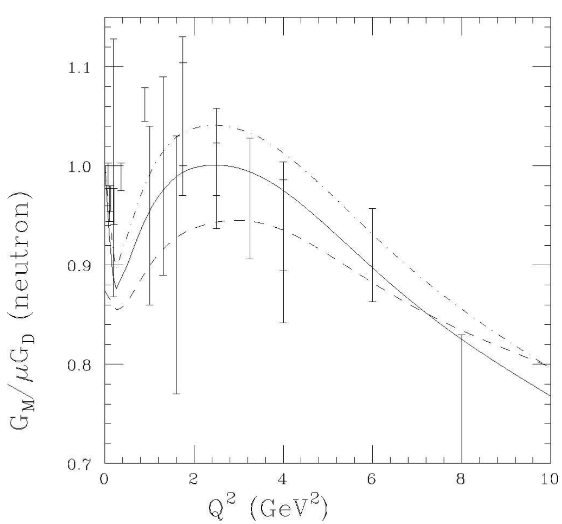
G. Miller, nucl-th/0301041;

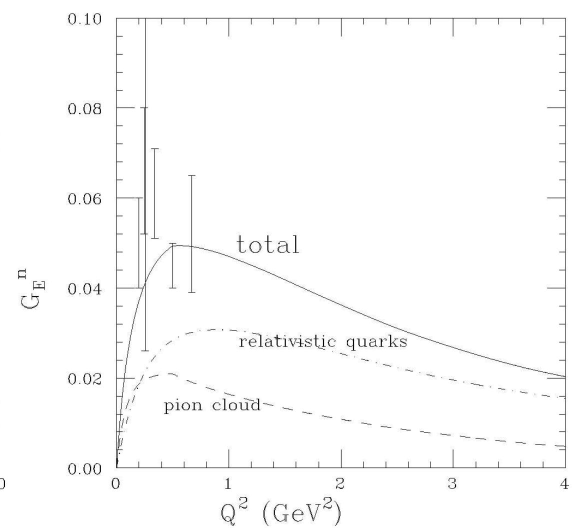
1. q^3 light-cone wave function of the Schlumpf-type: $\Phi(M_0(\{p_i\}, m)) = N\,(M_0^2 + \beta^2)^{-\gamma}$ makes $Q^4 G_M(Q^2)$ virtually constant at large Q^2 and predicts $G_E(Q^2)/G_M(Q^2)$ to drop;

- analytic arguments (lower components of Dirac spinors) for QF_2/F_1 to be constant for large Q^2
- including e.m. couplings with pion loops ($\Lambda_{\pi N}$)





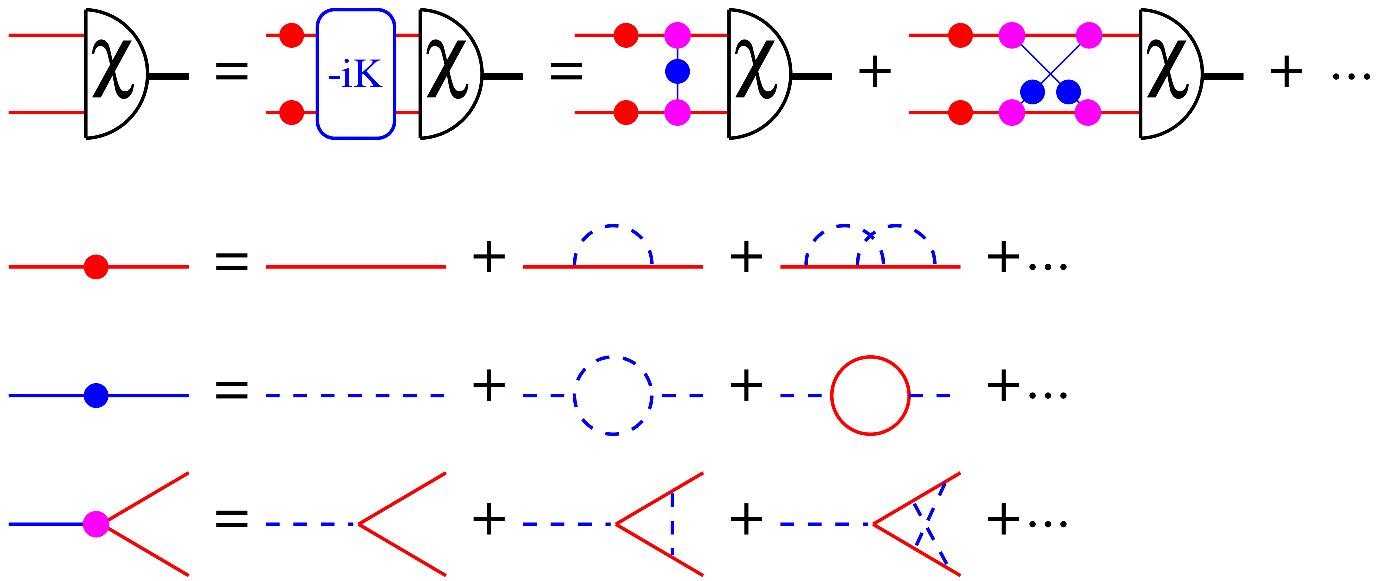




Bound states of 4-momentum $\bar{P}$ ($\bar{P}^2 = M^2$) described by BETHE-SALPETER-amplitude

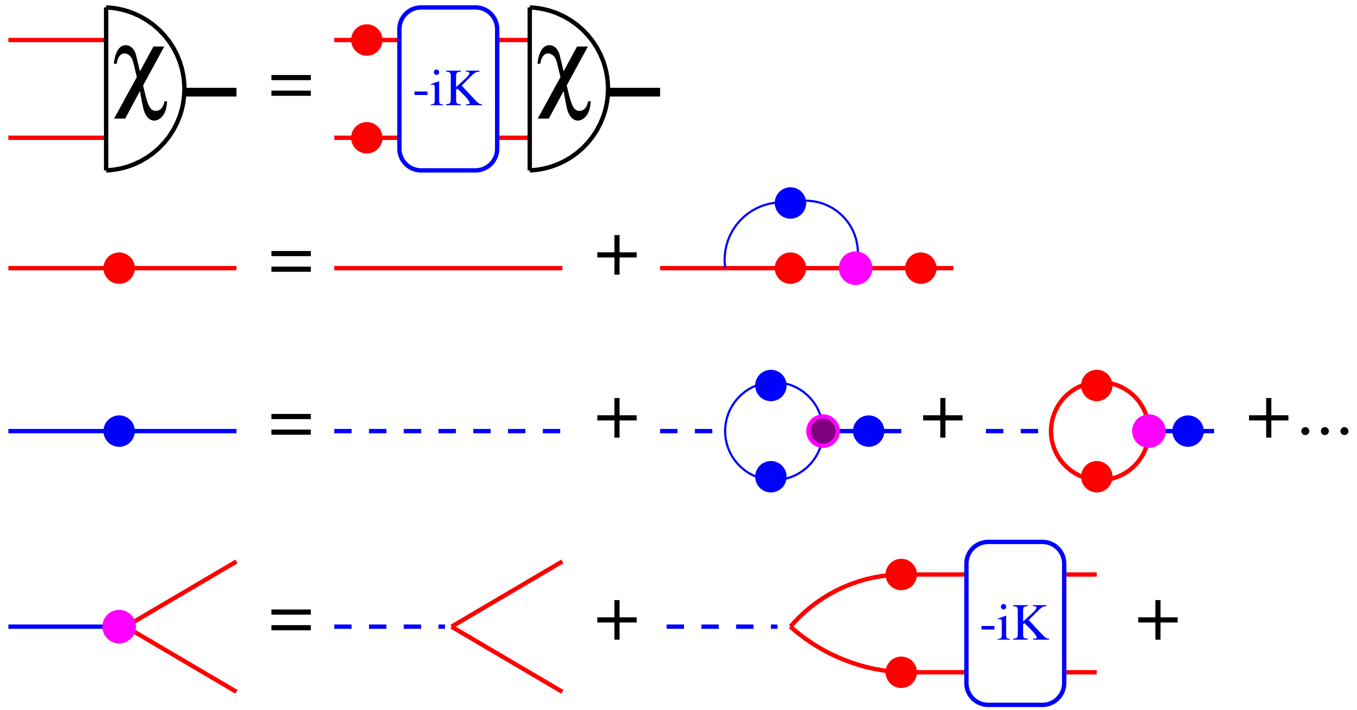
$$\chi_{\alpha\beta}(x_1, x_2) := \langle 0 | \, T \left[\psi^1_\alpha(x_1) \bar{\psi}^2_\beta(x_2) \right] | \bar{P} \rangle$$

fulfil the homogeneous BETHE-SALPETER equation:



and involve full (dressed) propagators for fermions, exchange bosons and full (dressed) vertex functions: This leads to the skeleton-expansion: *i.e.* an infinite set of coupled DYSON-SCHWINGER- and BETHE-SALPETER-equations:

Skeleton-expansion, approximations



In order to solve this in practise one truncates this expansion, makes an *Ansatz* for some n-point function and solves the equations (BETHE-SALPETER-equation for two particles or the DYSON-SCHWINGER-equation for the self-energy) of lower order.

⇒ renormalisation-group-improved rainbow-ladder approach (DSE) based on an effective gluon propagator with a specific infrared behaviour

P. Maris, C.D. Roberts: "Dyson-Schwinger Equations: A tool for hadron physics", Int. J. Mod. Phys. E**12** (2003) 297; nucl-th/0301049, (2003)

Poincaré covariant quark-diquark Fadeev approach

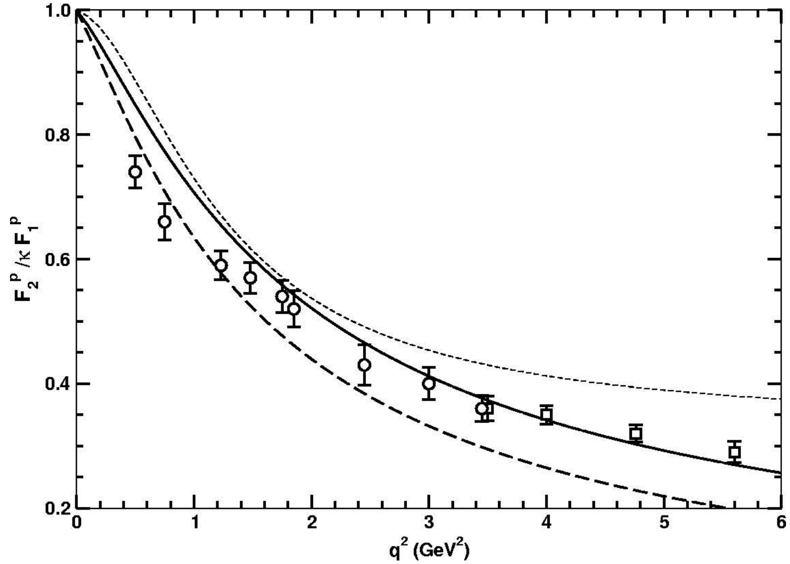
J.C.R. Bloch, A. Krassnigg, C.D. Roberts, Few Body Syst. **33** (2003) 219; nucl-th/0306059

1. dressed quark propagators from DSE: $S(p) = Z(p^2)\left[i\gamma^\mu p_\mu + M(p^2)\right]^{-1}$, effectively parameterising previous (meson) numerical results;
2. quark–(scalar) diquark (qD) Fadeev amplitude for the baryon in terms of a pseudoparticle diquark propagator and a qD Bethe-Salpeter amplitude, also effective parameterising previous numerical results;
3. dressed-quark–photon coupling parametrised according to a solution to the inhom. Bethe-Salpeter equation for the 3-point function, in fact determined by the dressed quark propagator (spectator approx.).

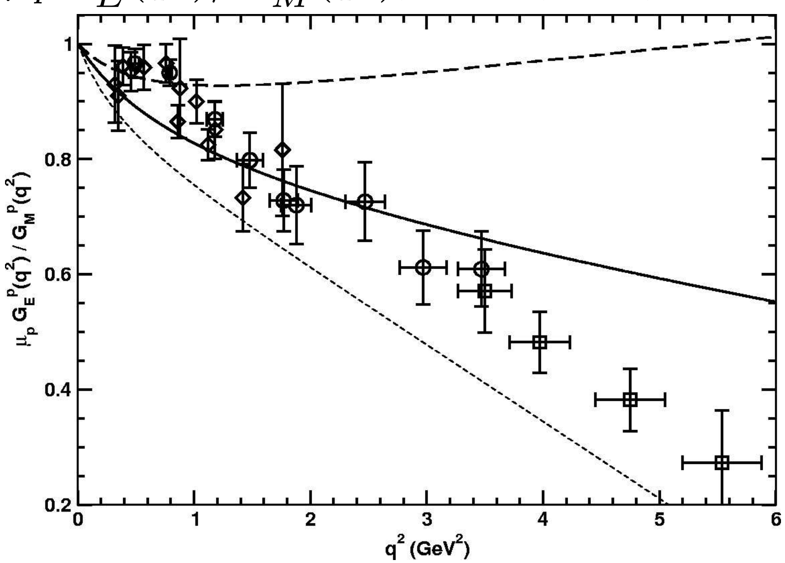
- 4 parameters: m_0, ω_0 for the diquark structure and $\omega_{q\{qq\}}$ ($\sim$ quark-diquark separation), fitted to a dipole-like electric proton form factor for various R ($\sim$ importance of lower components)

Poincaré covariant quark-diquark Fadeev approach

$F_2^p(q^2)/\kappa F_1^p(q^2), R = 0, 0.25, 0.50$



$\mu_p G_E^p(q^2)/G_M^p(q^2), R = 0, 0.25, 0.50$



- sizeable relativistic (lower) components in the Nucleon Fadeev Amplitude;
- few GeV^2 region determined by non-perturbative effects, such as sizes of composite bound states, dressing of quarks (vanishing for large Q^2), meson cloud effects.
- $\mu_p G_E^p(q^2)/G_M^p(q^2)$ is very sensitive to details: with increasing Q^2 $G_E^p(q^2)$ is a difference of small quantities!

M. Oettel, R. Alkofer, hep-ph/0204178

- also include axial-vector diquarks
- full structure of diquarks and e.m. coupling
- radii and G_E^n too small, $\mu_p G_E^p(q^2)/G_M^p(q^2)$ too steep
- some sensitivity to dressed quark propagator

BSE-based relativistic covariant quark model

...pretend to describe baryons in the framework of quantum field theory...

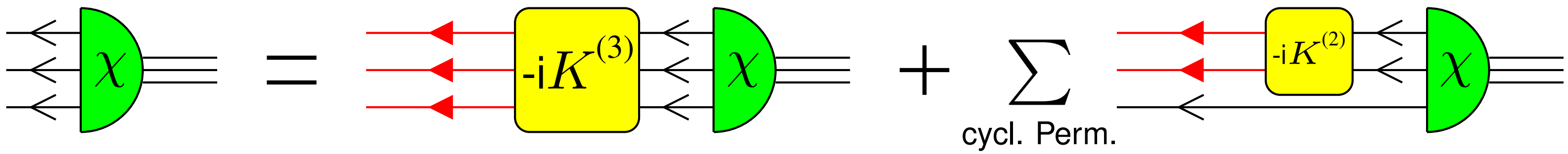
Three-particle Bethe-Salpeter Equation with instantaneous interaction kernels describing confinement and an instanton-induced (flavour dependent) hyperfine interaction

Issues

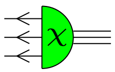
- **(Ir)regularities** in the spectra
  - Regge trajectories: $M^2 \propto J$
  - Parity doublets
  - Low lying scalar/isoscalar excitations (Roper & Co)
- **“testing” amplitudes** in (simple) (sub)processes
  - photon couplings/ helicity amplitudes; (transition) form factors
  - weak semi-leptonic decays; weak form factors
  - strong two-body decays: $B' \to B\,M$
- **“undetected” resonances**

U. Löring *et al.* Eur.Phys. J. A **10** (2001) 309, *ibid.* 395, *ibid.* 447.

Bethe-Salpeter-Equation

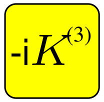


describes bound states of mass $M^2 = \bar{P}^2$ and total momentum $\bar{P} = p_1 + p_2 + p_3$, where:

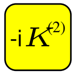
- 

$:= \langle 0|T\psi(x_1)\,\psi(x_2)\,\psi(x_3)|\bar{P}\rangle$, Bethe-Salpeter-Amplitude

- 
$= \langle 0|T\psi(x)\,\bar{\psi}(x')|0\rangle = S_F(x - x')$, full quark propagator

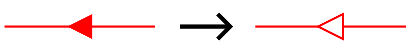
- 

irreducible **three**-particle kernel

- 

irreducible **two**-particle kernel

Approximations

- **free** propagators:  : $S_F^j(p_j) \equiv \frac{\mathrm{i}}{\not{p}_j - m_j + \mathrm{i}\epsilon}$

  $\Rightarrow$ **effective** quark masses m_j

- instantaneous approximation: interaction kernels do not depend on relative energies p_ξ^0 and p_η^0 in the baryon rest frame:

$$\boxed{K_P^{(3)}}(p_\xi, p_\eta; p'_\xi, p'_\eta)\Big|_{\bar{P}=(M,\vec{0})} = \boxed{V^{(3)}}(\vec{p}_\xi, \vec{p}_\eta; \vec{p}'_\xi, \vec{p}'_\eta)$$

$$\boxed{K_{P_{ij}}^{(2)}}(p_{\xi_k}, p_{\eta_k})\Big|_{\bar{P}=(M,\vec{0})} = \boxed{V^{(2)}}(\vec{p}_{\xi_k}, \vec{p}_{\eta_k})$$

$\Rightarrow$ retardation effects are neglected

(inspired by the NRCQM)

$\Rightarrow$ Reduction of the 8-dimensional BSE to the 6-dimensional Salpeter-Equation by integrating over p_ξ^0- and p_η^0.

Salpeter-Equation

$$\mathcal{H}\Phi_M^\Lambda = M\Phi_M^\Lambda$$

Eigenvalue equation for baryon mass M with:

- Salpeter-Amplitude: $\Phi_M(\vec{p}_\xi, \vec{p}_\eta) := \int \frac{\mathrm{d}p_\xi^0}{2\pi} \frac{\mathrm{d}p_\eta^0}{2\pi} \chi_M(p_\xi, p_\eta)$

  Projection: $\Phi_M^\Lambda := \left[\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ + \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right] \Phi_M$
- Φ_M^Λ in baryon rest frame, $\overline{M} = (M, \vec{0})$
- Salpeter-Hamilton-Operator: $\mathcal{H} = \mathcal{H}\left(\boxed{V^{(3)}}, \boxed{V^{(2)}}\right)$

Norm: $\langle \Phi_M^\Lambda | \Phi_M^\Lambda \rangle = \int \frac{\mathrm{d}p_\xi^3}{2\pi} \frac{\mathrm{d}p_\eta^3}{2\pi} \Phi_M^{\Lambda\,\dagger}(p_\xi, p_\eta)\, \Phi_M^\Lambda(p_\xi, p_\eta) = 2M$

$\Rightarrow$ induces positive definite scalar product $\langle \Phi_1 | \Phi_2 \rangle$

Salpeter Hamiltonian

. . . approximate treatment of $V^{(2)}$ . . .:

$$
\begin{aligned}
(\mathcal{H}\Phi_M)(\mathbf{p}_\xi,\mathbf{p}_\eta) &= \sum_{i=1}^{3} H_i\ \Phi_M(\mathbf{p}_\xi,\mathbf{p}_\eta) \\
&+ \quad \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ + \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right) \\
&\qquad \gamma^0 \otimes \gamma^0 \otimes \gamma^0 \int \frac{d^3p'_\xi}{(2\pi)^3}\ \frac{d^3p'_\eta}{(2\pi)^3}\ V^{(3)}(\mathbf{p}_\xi,\mathbf{p}_\eta,\mathbf{p}'_\xi,\mathbf{p}'_\eta)\ \Phi_M(\mathbf{p}'_\xi,\mathbf{p}'_\eta) \\
&+ \quad \left(\Lambda_1^+ \otimes \Lambda_2^+ \otimes \Lambda_3^+ - \Lambda_1^- \otimes \Lambda_2^- \otimes \Lambda_3^-\right) \\
&\qquad \gamma^0 \otimes \gamma^0 \otimes \mathbb{1} \int \frac{d^3p'_\xi}{(2\pi)^3}\ \left[V^{(2)}(\mathbf{p}_\xi,\mathbf{p}'_\xi) \otimes \mathbb{1}\right]\ \Phi_M(\mathbf{p}'_\xi,\mathbf{p}_\eta) \\
&\qquad + \ \text{cycl. perm. (123)}
\end{aligned}
$$

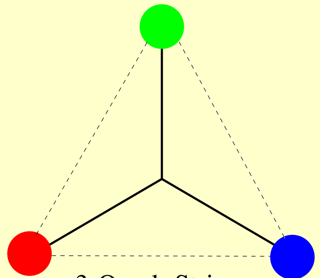
- $\Lambda_i^\pm(\mathbf{p_i}) := \frac{\omega_i \pm H_i}{2\omega_i}$ Energy projectors
- $H_i(\mathbf{p_i}) := \gamma^0\ (\boldsymbol{\gamma}\cdot\mathbf{p_i} + m_i)$ Dirac Hamiltonian

... solved by diagonalisation in a large finite basis...

Confinement

Quark confinement is realized by a phenomenological string potential for 3 quarks (*Ansatz* similar to NRCQM):

$$V^{(3)}_{\text{Conf}}(\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}) = \mathbf{A_3} + \mathbf{B_3}\sum_{\mathbf{i<j}} |\mathbf{x_i} - \mathbf{x_j}|$$



In contrast to the nonrelativistic quark model the relativistic quasi-potential $V^{(3)}_{\mathrm{Conf}}$ depends on the Dirac structure for three quarks.
We choose:

$$\mathbf{A}_3 = a\,\frac{3}{4}\left[\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\gamma^0\otimes\mathbb{1}\otimes\gamma^0+\mathbb{1}\otimes\gamma^0\otimes\gamma^0\right]$$

$$\mathbf{B}_3 = b\,\frac{1}{2}\left[-\mathbb{1}\otimes\mathbb{1}\otimes\mathbb{1}+\gamma^0\otimes\gamma^0\otimes\mathbb{1}+\gamma^0\otimes\mathbb{1}\otimes\gamma^0+\mathbb{1}\otimes\gamma^0\otimes\gamma^0\right]$$

$\longrightarrow$ Spin-orbit effects are small (compatible with exp.)

$\longrightarrow$ Regge trajectories are quantitatively correct.

The instanton-induced qq-interaction

In $qq, C = \bar{3}$-channel $\rightarrow$ instantaneous potential:
't Hooft's interaction (induced by instantons):

$$V^{(2)}_{\text{'t Hooft}}(\mathbf{x_1} - \mathbf{x_2}) = \underbrace{\delta^{(3)}(\mathbf{x_1} - \mathbf{x_2})}_{\text{point-interaction}} \cdot$$

$$-4 \underbrace{\left(g_{nn}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(nn) + g_{ns}\, \mathcal{P}^{\mathcal{F}}_{\mathcal{A}}(ns)\right)}_{\text{flavour-dependent coupling}} \left[\mathbb{I} \otimes \mathbb{I} + \gamma^5 \otimes \gamma^5\right] \mathcal{P}^{\mathcal{D}}_{S_{12}=0}$$

G. 't Hooft, Phys. Rev. D 14, 3432 (1976)

M. A. Shifman, A. I. Vainstein, V. I. Zakharov, Nucl. Phys. B 163, 46 (1980)

- flavour-dependent: $\mathcal{P}^{\mathcal{F}}_{\mathcal{A}}$: flavour-antisymmetric quark pairs.
- spin-dependent: $\mathcal{P}^{\mathcal{D}}_{S_{12}=0}$: antisymmetric in spin $S_{12} = 0$.
- point-interaction: $\delta^{(3)}(\mathbf{x_1} - \mathbf{x_2}) \longrightarrow \frac{\mathbf{1}}{\lambda^{\mathbf{3}} \pi^{\frac{\mathbf{3}}{\mathbf{2}}}}$ exp $\left(-\frac{|\mathbf{x_1} - \mathbf{x_2}|^{\mathbf{2}}}{\lambda^{\mathbf{2}}}\right)$

$\Rightarrow$ does <u>not</u> act on: flavour-decuplet, spin-symmetric states;

$\Rightarrow$ no $\vec{L} \cdot \vec{S}$, no tensor forces.

Model parameters

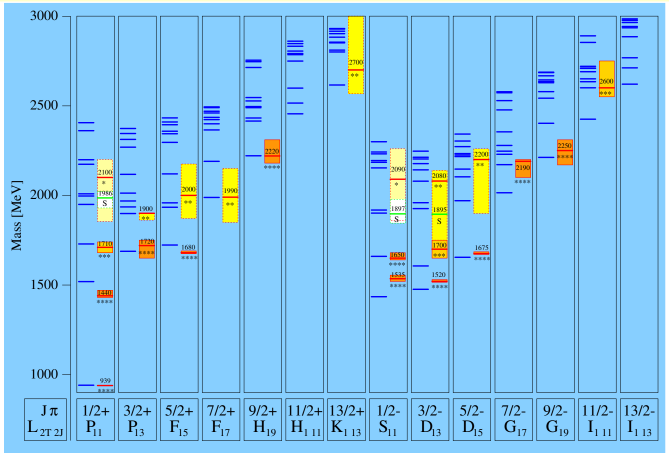
		parameter	value
quark-	’nonstrange’	m_n	330 Mev
masses	’strange’	m_s	670 Mev
confinement	offset	a	-744 MeV
	slope	b	470 MeV fm^{-1}
’t Hooft’s	nn-coupling	g_{nn}	136.0 MeV fm^3
force	ns-coupling	g_{ns}	94.0 MeV fm^3
	effective range	λ	0.4 fm

Parameters are fixed by

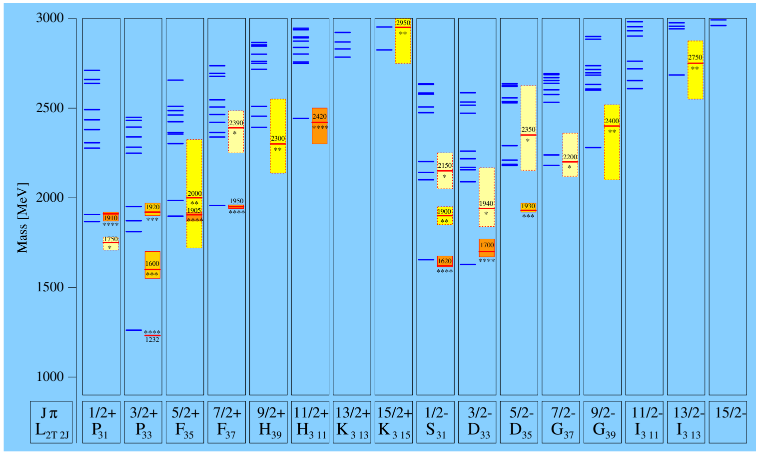
- the Δ-Regge trajectory
  $\longrightarrow$ Confinement parameters a, b and m_n
- baryon ground-states (octet and decuplet)
  $\longrightarrow$ g_{nn}, g_{ns}, λ and m_s

Light-flavoured Baryons

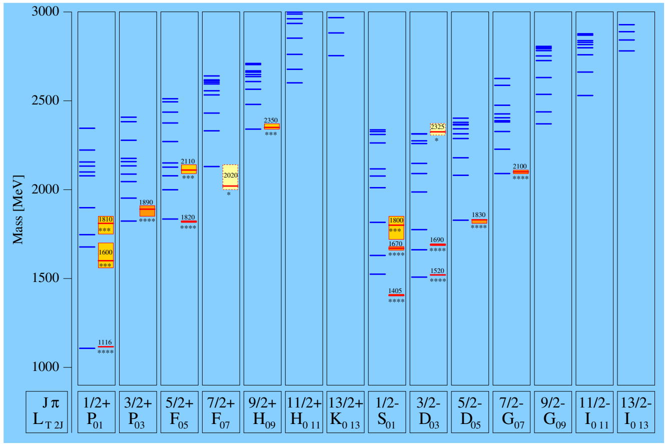
N:



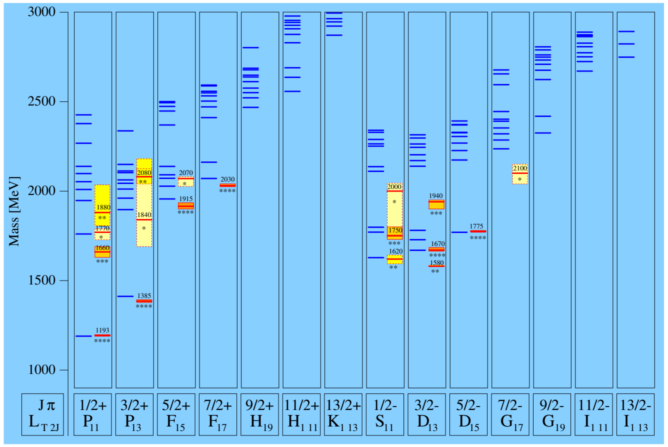
Δ :



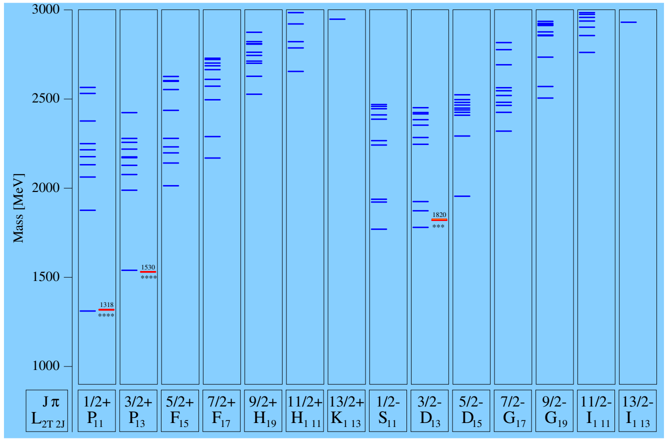
Λ :



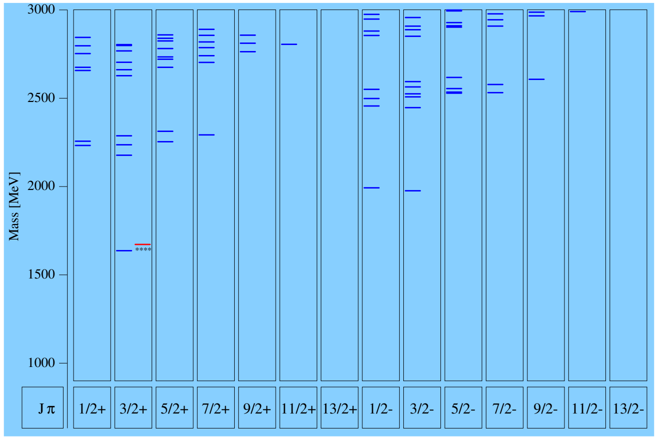
Σ :



Ξ :

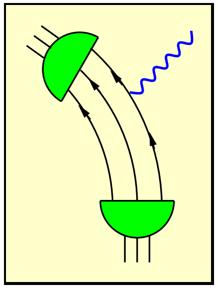


Ω :



Electromagnetic and strong coupling amplitudes

Mandelstam formalism: Matrix element in initial state rest frame (IA):



$$\langle B\bar{P}_B|J^\mu(0)|B^*M\rangle = -3\int\frac{\mathrm{d}^4p_\xi}{(2\pi)^4}\frac{\mathrm{d}^4p_\eta}{(2\pi)^4}$$

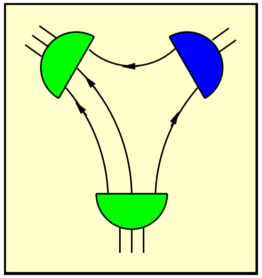
$$\overline{\Gamma}_{\bar{P}}(p'_\xi,p'_\eta)\,S^1_F\,(p_1)\otimes S^2_F\,(p_2)\otimes S^3_F\,(p'_3)\;\hat{Q}\gamma^\mu S^3_F\,(p_3)\,\Gamma_M(\vec{p}_\xi,\vec{p}_\eta)$$

- Reconstruction of the vertex function in the rest frame

$$\Gamma_M(\vec{p}_\xi,\vec{p}_\eta) = -i\int\frac{\mathrm{d}^3p'_\xi}{(2\pi)^3}\,\frac{\mathrm{d}^3p'_\eta}{(2\pi)^4}\,V(\vec{p}_\xi,\vec{p}_\eta,\vec{p}'_\xi,\vec{p}'_\eta)\Phi_M(\vec{p}'_\xi,\vec{p}'_\eta)$$

- Boost $(M,\vec{0})\rightarrow(\sqrt{M^2+\vec{P}^2},\vec{P})$

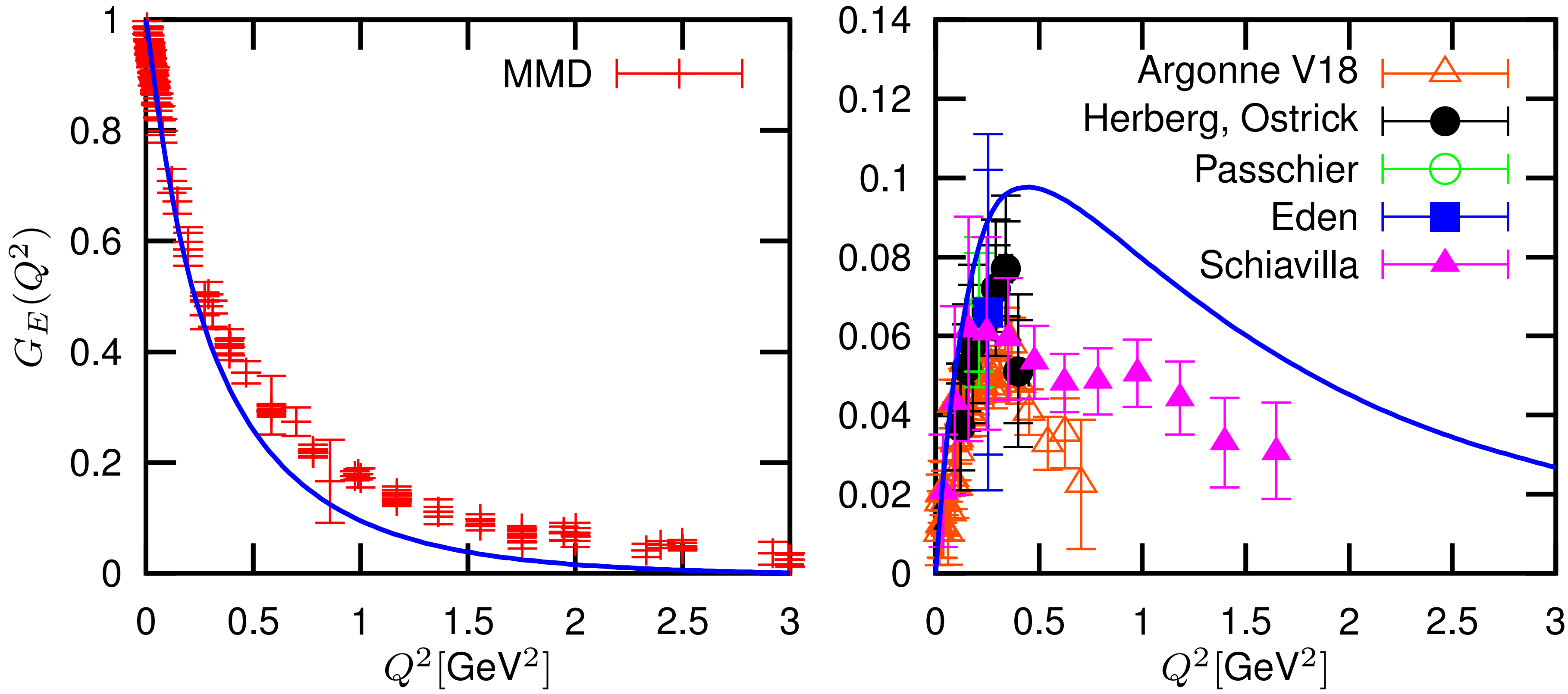
$$\langle B\bar{P}_B,\pi\bar{P}_\pi|S|B^*M\rangle = -3\int\frac{\mathrm{d}^4p_\xi}{(2\pi)^4}\frac{\mathrm{d}^4p_\eta}{(2\pi)^4}$$



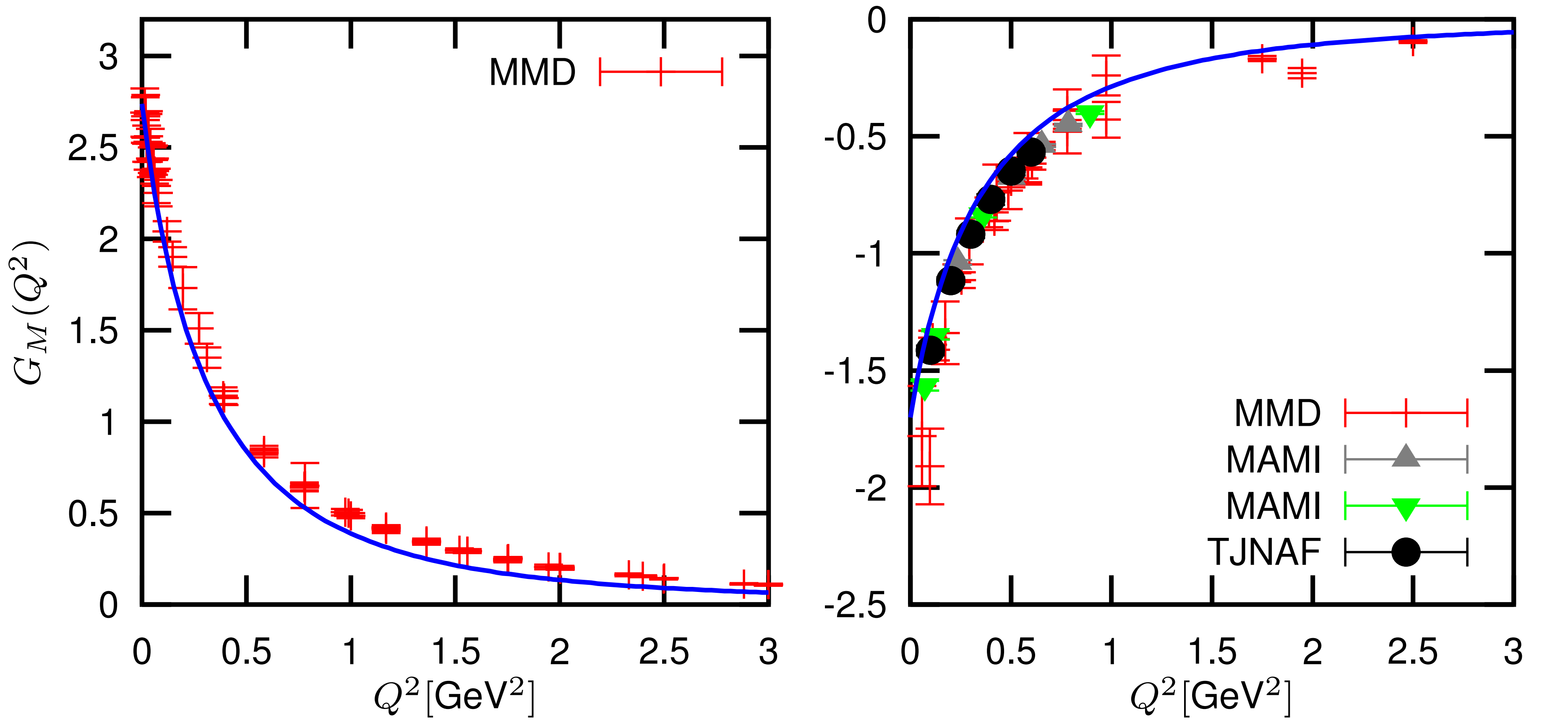
$$\overline{\Gamma}_{\bar{P}}(p'_\xi,p'_\eta)\,S^1_F\,(p_1)\otimes S^2_F\,(p_2)\otimes S^3_F\,(p'_3)\;\bar{\Gamma}(p)S^3_F\,(p_3)\,\Gamma_M(\vec{p}_\xi,\vec{p}_\eta)$$

mesonic Bethe-Salpeter amplitudes from
(M.Koll et al., Eur. Phys. J. A 9, 73 (2000))

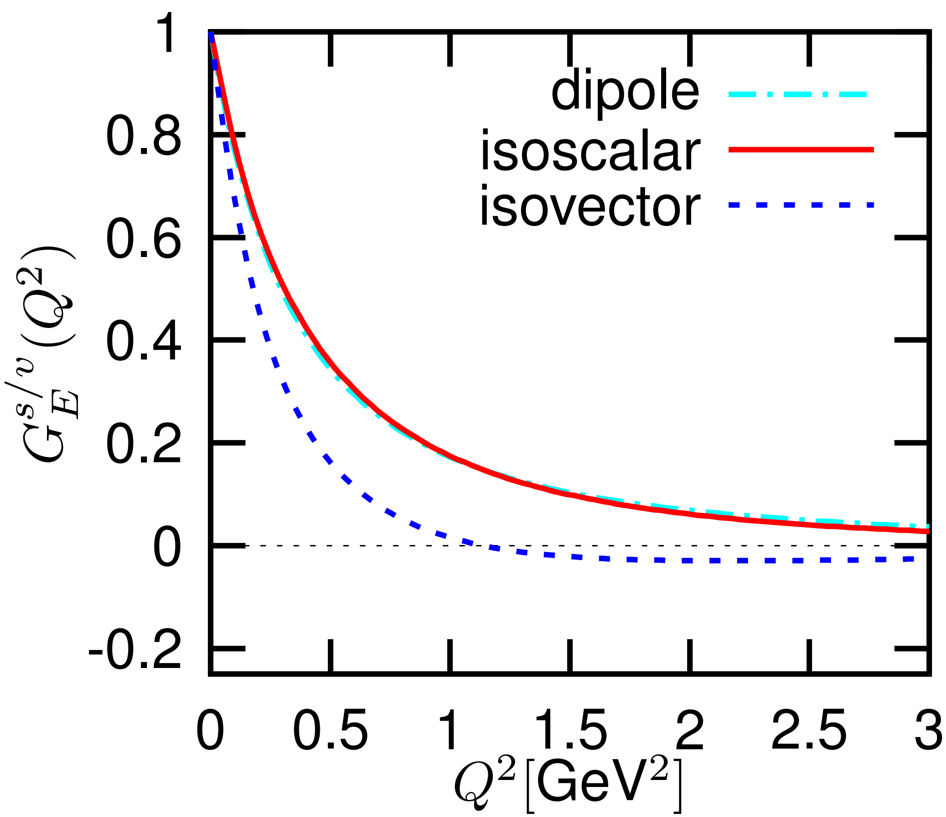
RCQM electric nucleon form factors



RCQM magnetic nucleon form factors

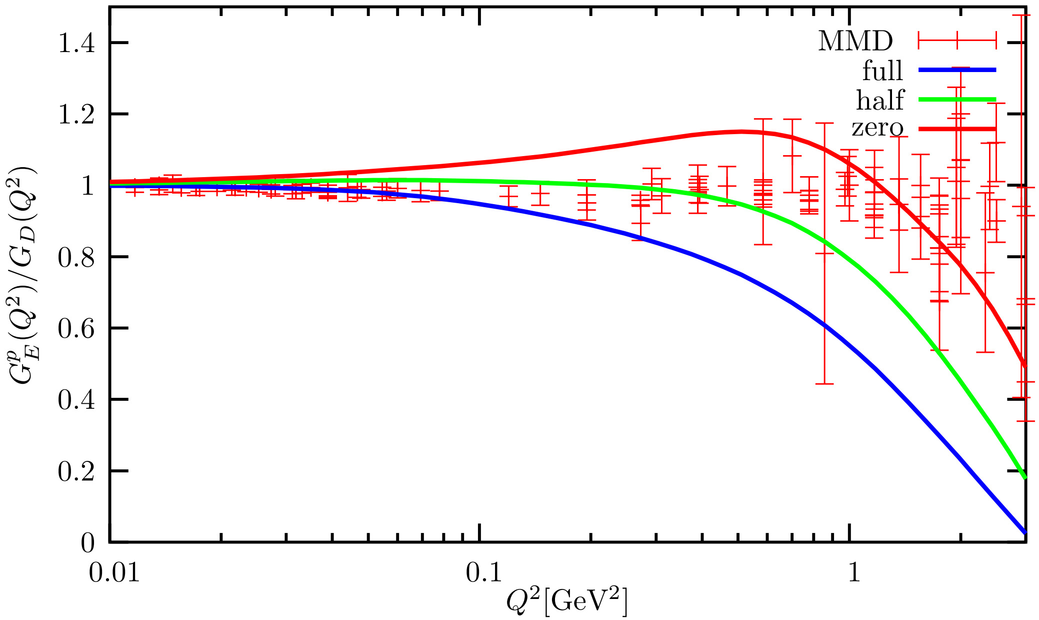


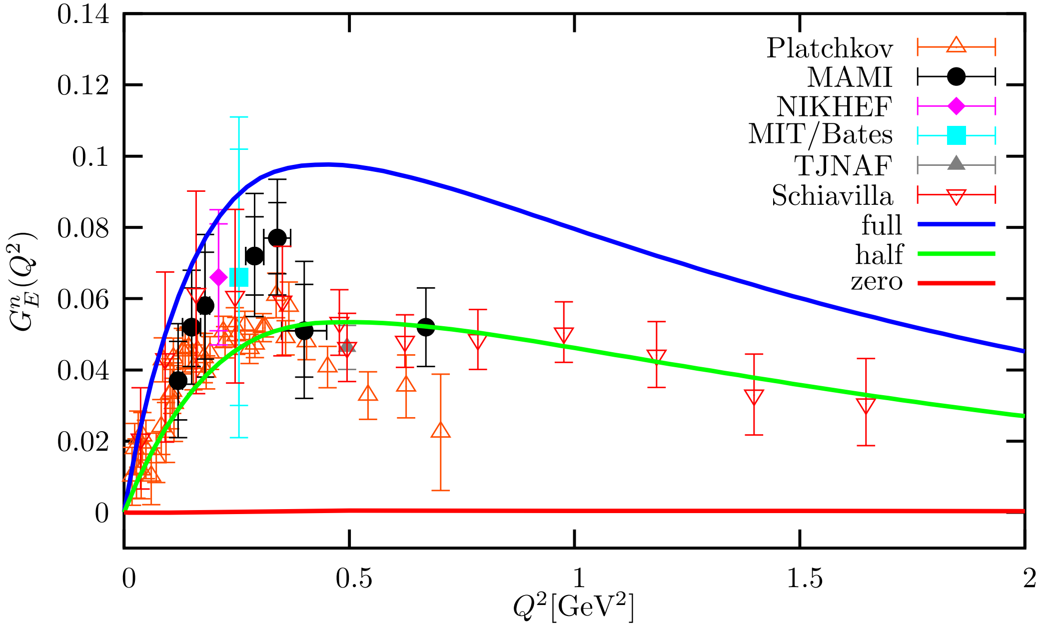
RCQM: isovector $\leftrightarrow$ isoscalar



isoscalar electric form factor: dipole shape

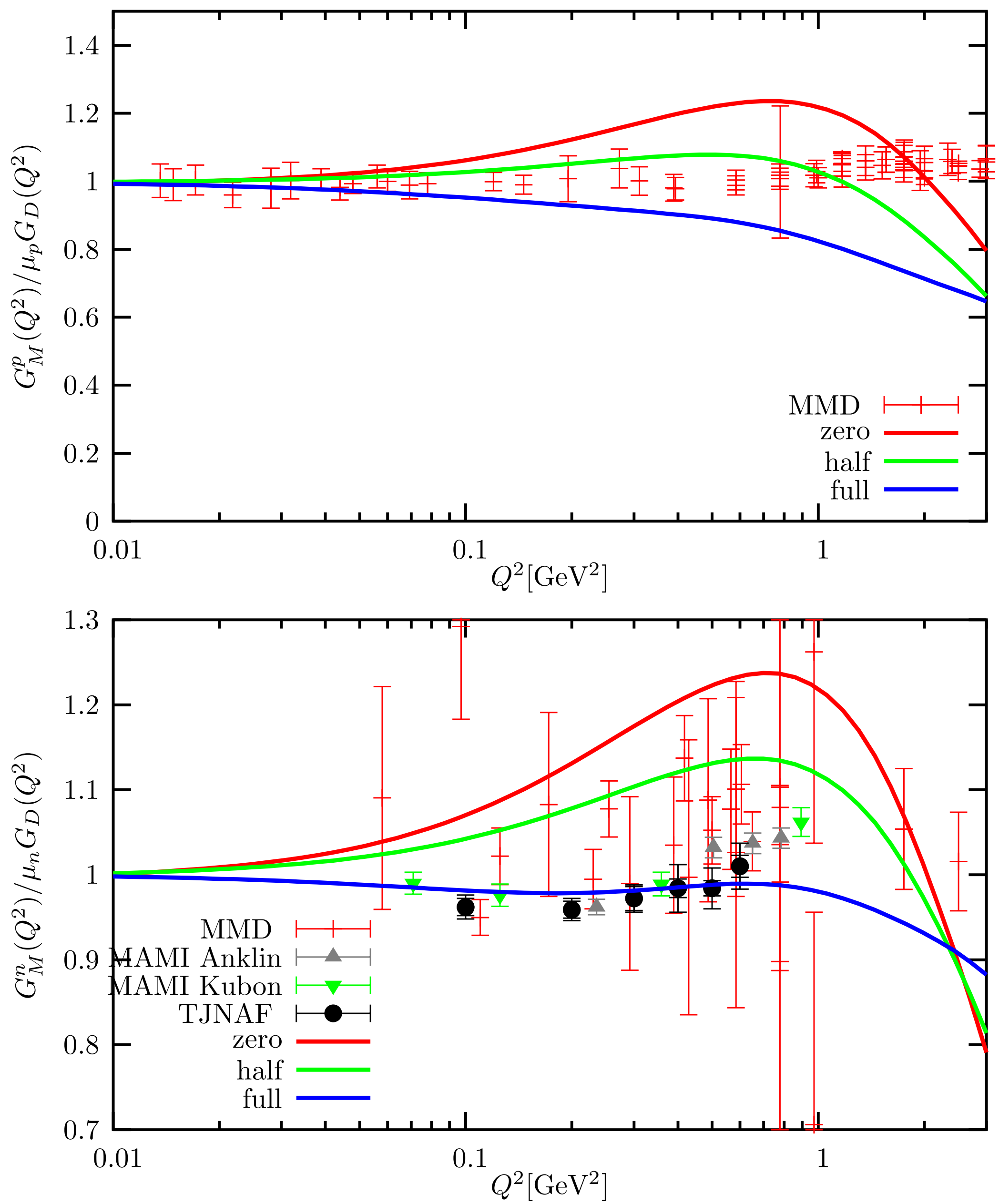
RCQM nucleon electric form factors





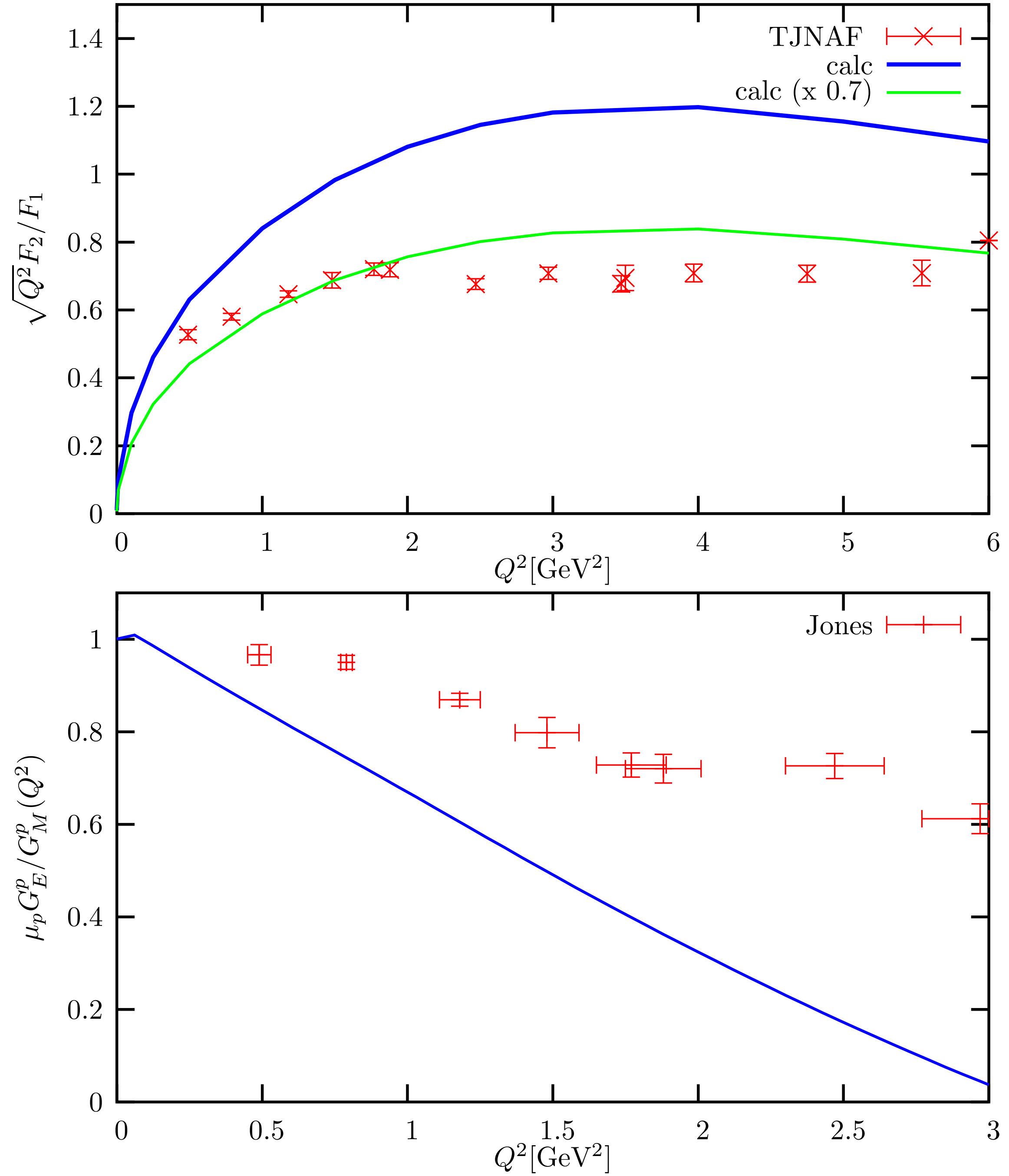
varying the strength of the instanton induced spin-flavour dependent interaction: 0.0, 0.5, 1.0 of the value determined by the spectrum

RCQM nucleon magnetic form factors

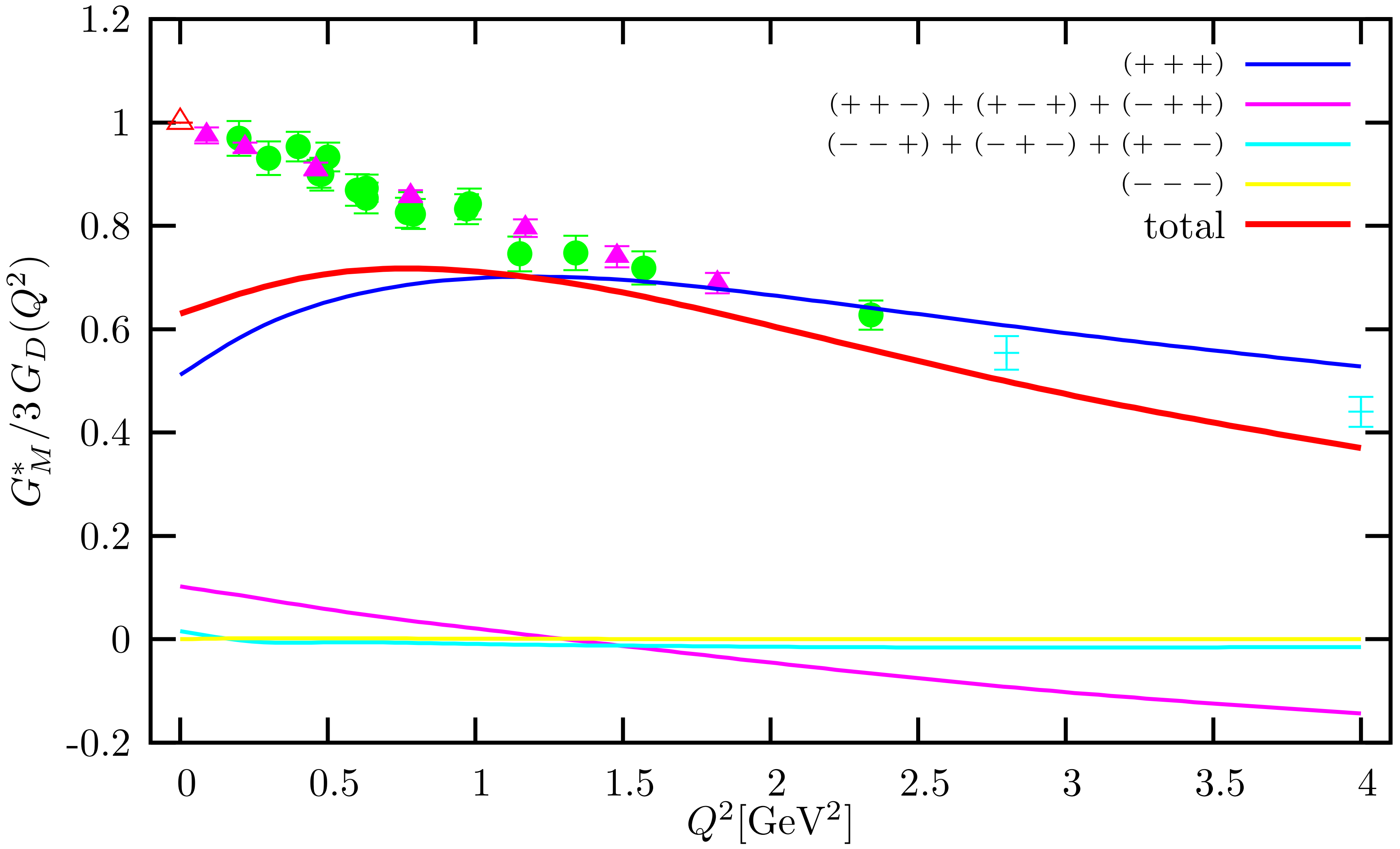


varying the strength of the instanton induced spin-flavour dependent interaction: 0.0, 0.5, 1.0 of the value determined by the spectrum

RCQM G_E^p/G_M^p and F_2/F_1 at large Q^2



RCQM $N - \Delta$ magnetic transition form factor



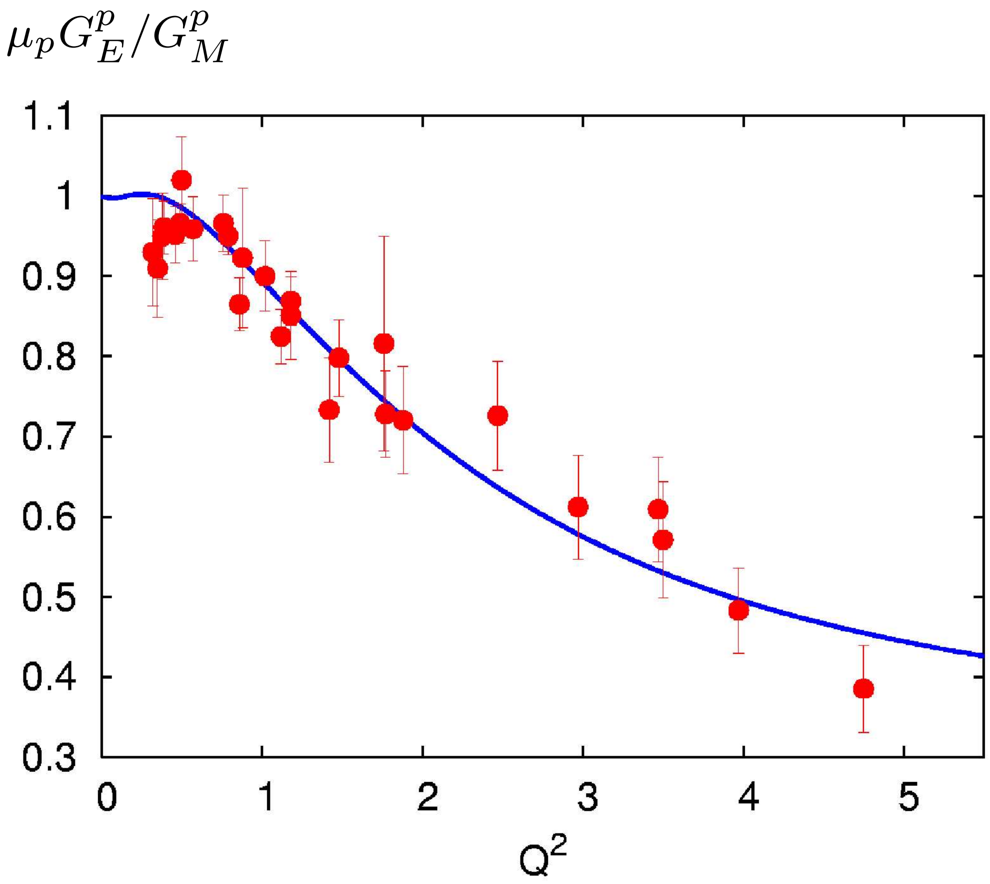
lower components of Dirac-spinors:
relevant for large Q^2 behaviour

help a bit at low $Q^2 \Rightarrow$ “pion cloud”?

Hypercentric constituent quark model

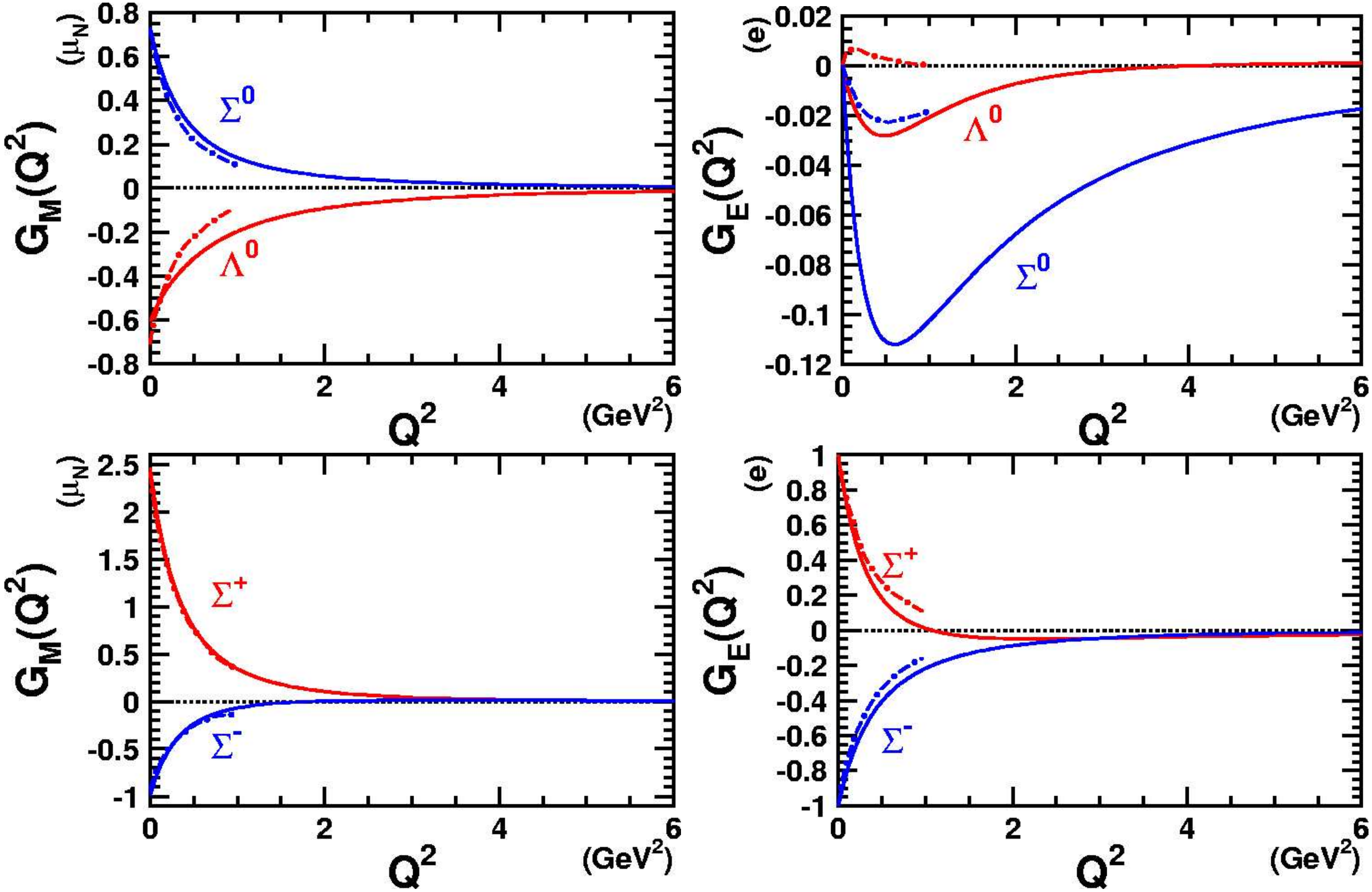
M. De Sanctis, M.M. Giannini, E. Santopinto, A. Vassallo; hep-ph/0402042

- constituent quark model with hypercentric Coulomb and linear confinement
- relativistic kinetic energy and hyperfine interactions
- boosts and relativistic (free) currents
- ((small) constituent quark form factors)



RCQM hyperon form factors

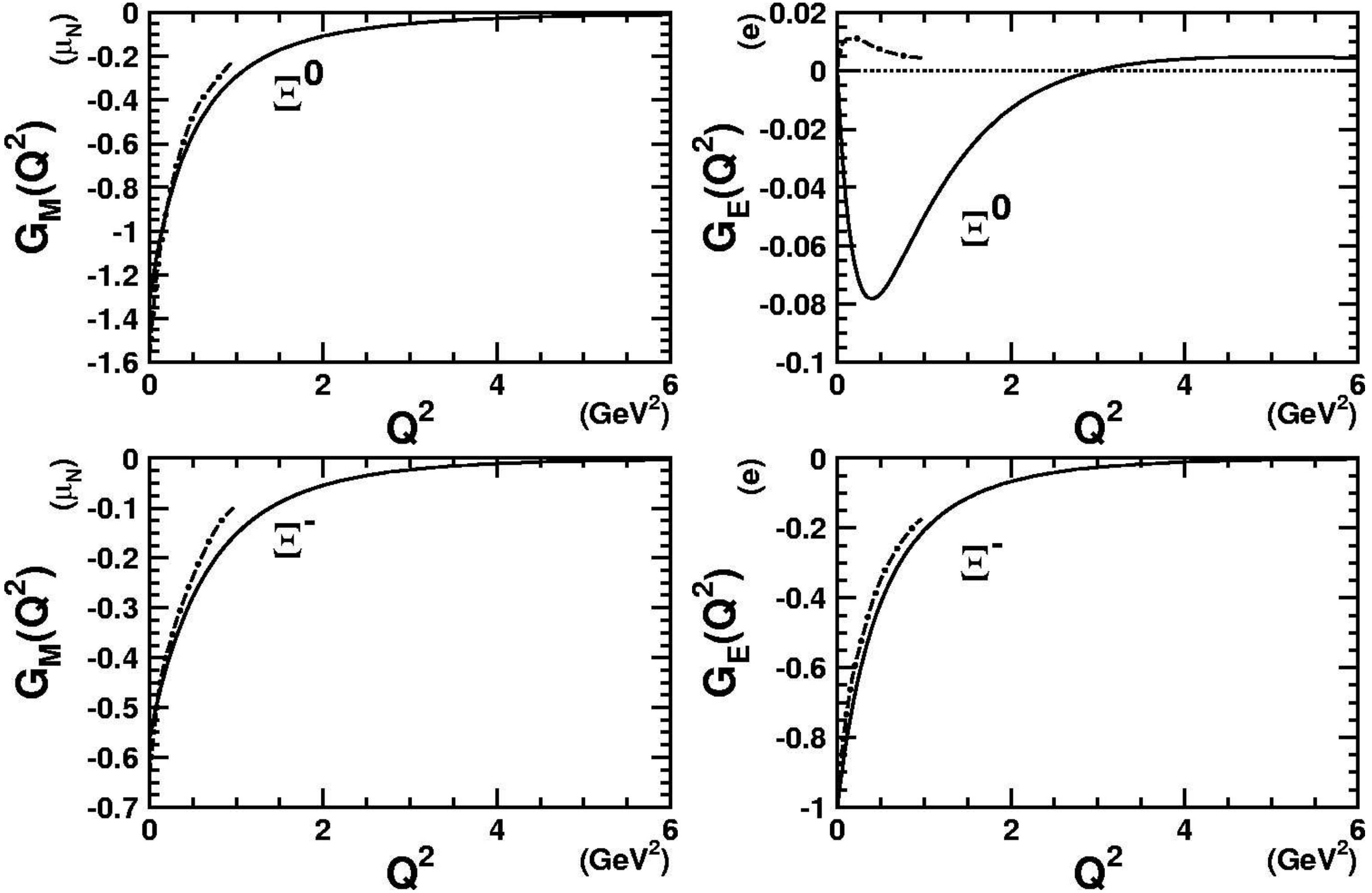
context: electromagnetic coupling to hyperons to be used as guidlines in hadronic models for strange meson photoproduction



BSE (solid): T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283

ChQSM (dashed-dotted): H. Ch. Kim *et al.*, Phys. Rev. **D53** (1996) 4013

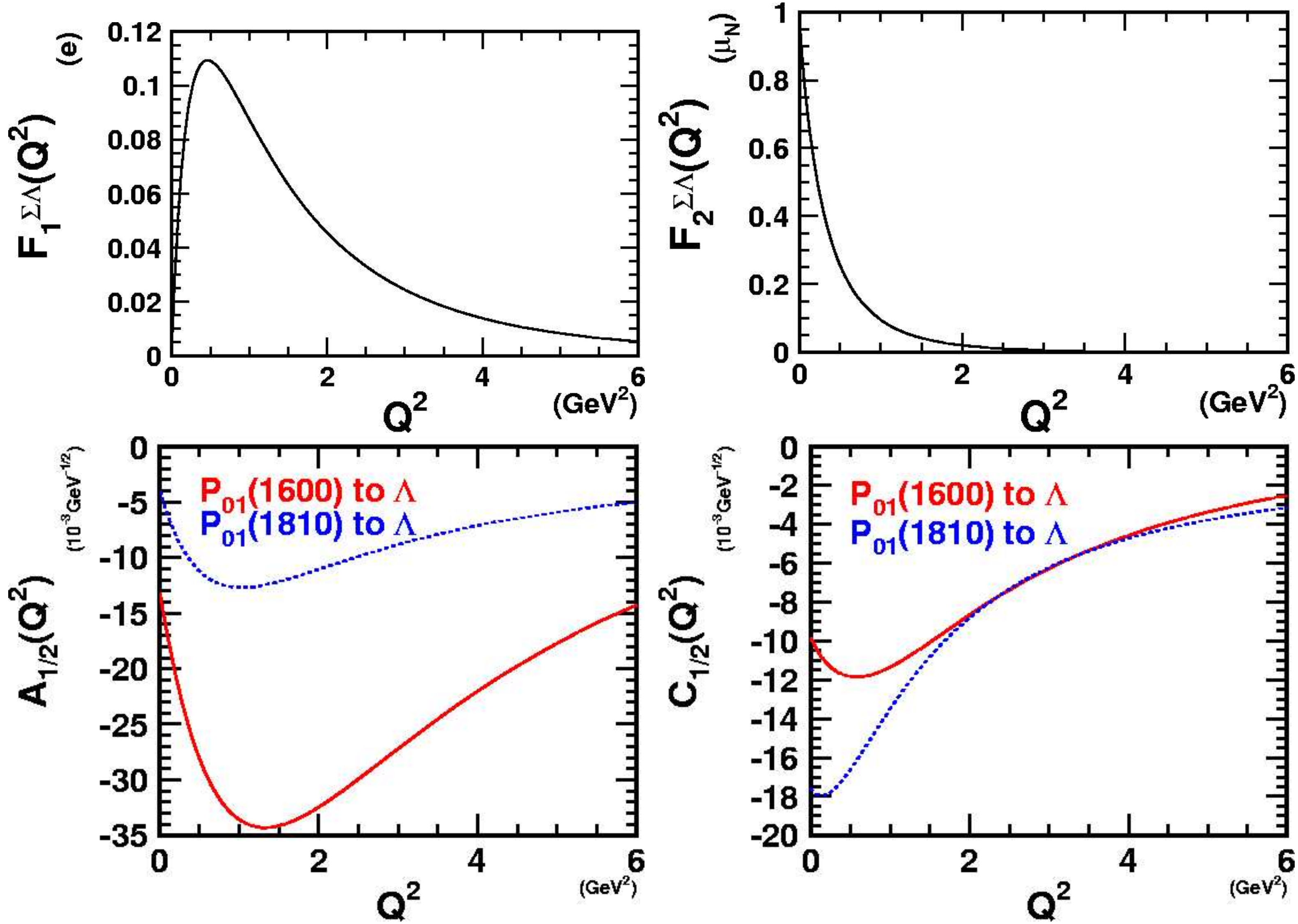
RCQM Ξ-hyperon form factors



BSE (solid): T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283

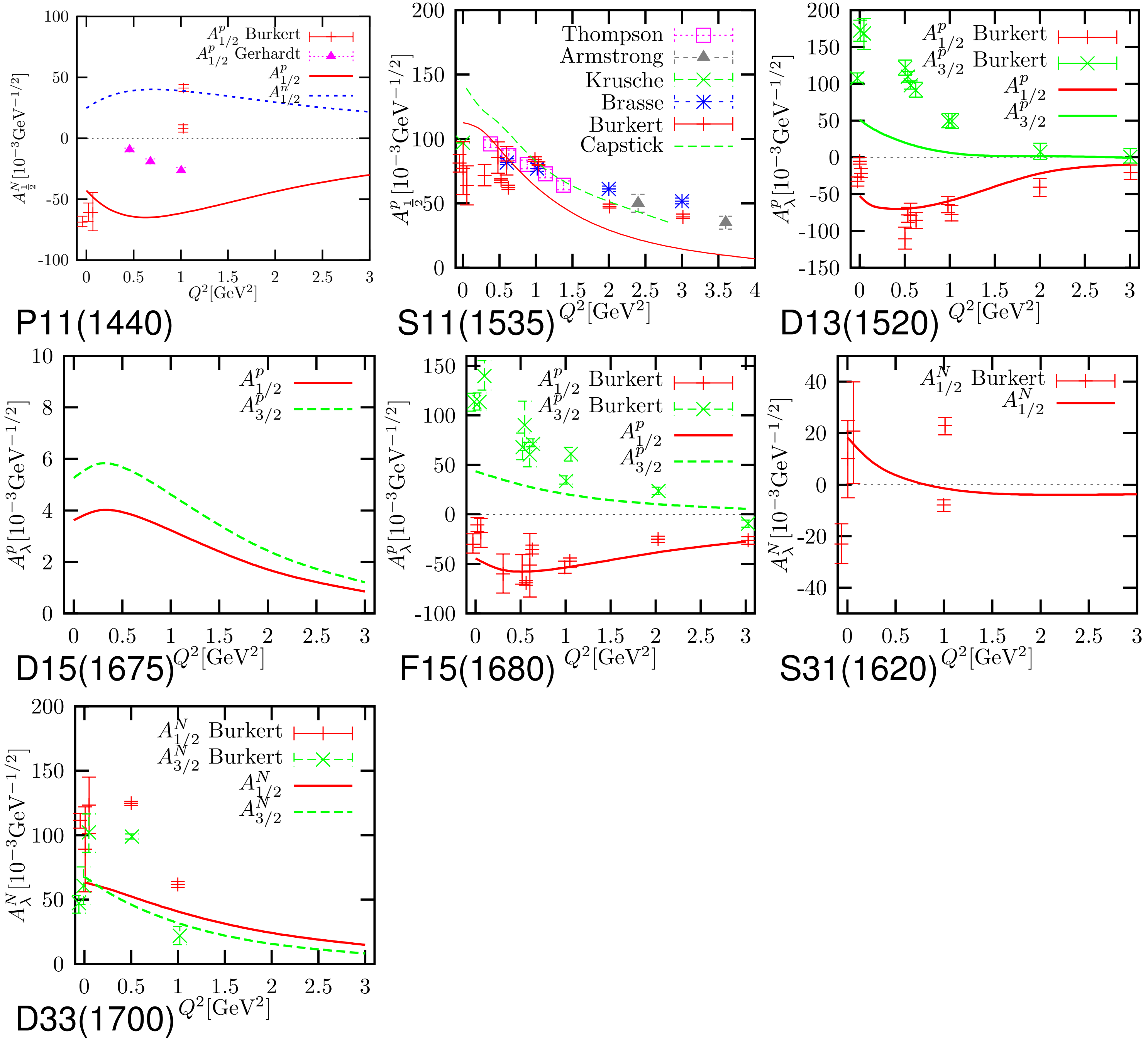
ChQSM (dashed-dotted): H. Ch. Kim *et al.*, Phys. Rev. **D53** (1996) 4013

RCQM hyperon transition form factors



BSE: T. van Cauteren *et al.*, Eur. Phys. J. **A20** (2004) 283;
T. van Cauteren *et al.*, nucl-th/0407017

Helicity amplitudes



Magnetic moment matrix element

$$\boxed{\mu = \frac{\langle \Phi^\Lambda_M \,|\, \hat\mu \,|\, \Phi^\Lambda_M \rangle}{2M}}$$

relativistic weight

single particle- ang. momenta

single particle- spins

$$\hat\mu := \boxed{\frac{\omega_1+\omega_2+\omega_3}{M}}\left\{\boxed{\sum_{\alpha=1}^{3} \frac{\hat e_\alpha}{2\omega_\alpha}\hat l^3_\alpha} + \boxed{\mathbb{1}\otimes\mathbb{1}\otimes\frac{\hat e_3}{2\omega_3}\Sigma^3 + \text{zykl. Perm.}}\right\}$$

$$\boxed{- \delta_{3i}\,\epsilon_{ijk}\frac{1}{M}\sum_{\alpha=1}^{3}\frac{\hat e_\alpha}{2\omega_\alpha}p^k_\alpha\sum_{\beta=1}^{3}\omega_\beta\frac{\partial}{\partial p^j_\beta}}$$

center-of-charge ang. momentum

Salpeter-Amplitude normalisation: $\langle \Phi^\Lambda_M | \Phi^\Lambda_M \rangle = 2M$

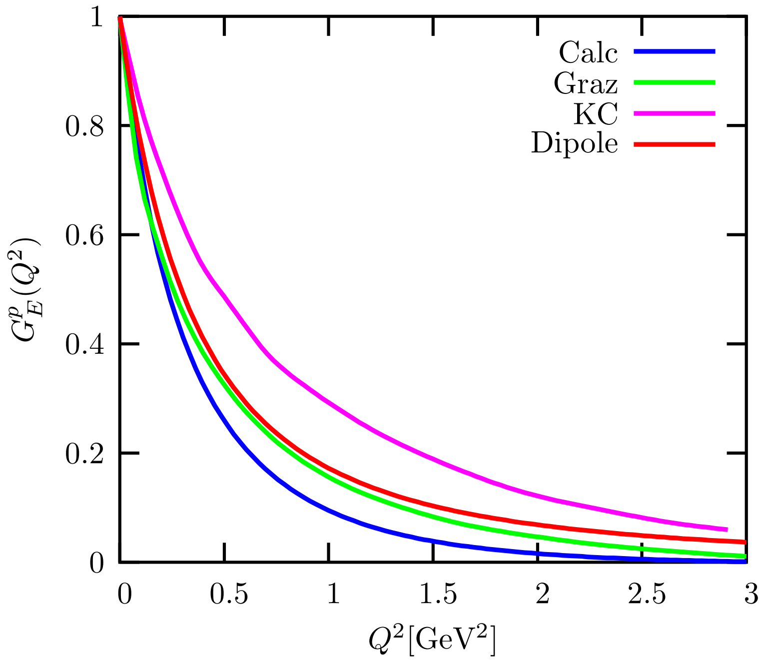
magnetic moments $[\mu_N]$

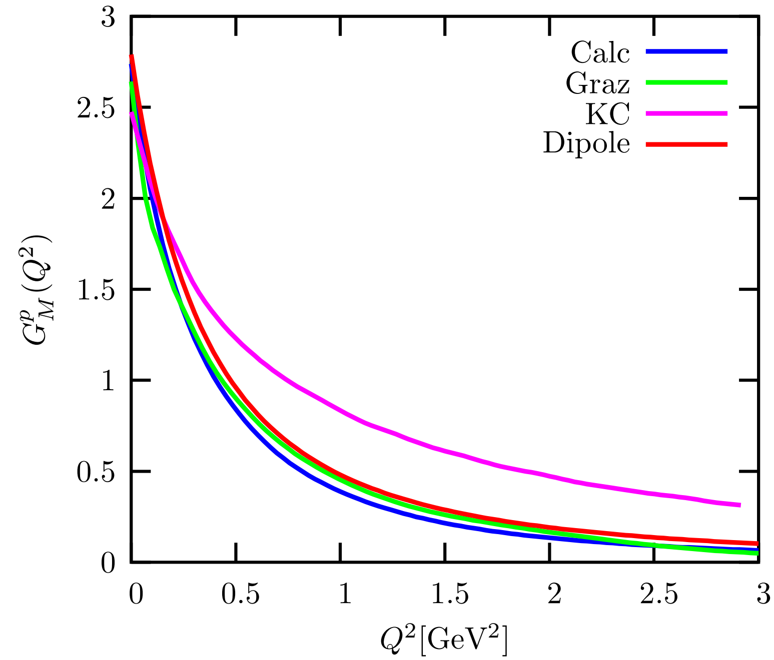
Baryon	BSE	Exp.	GBE
p	2.77	2.793	2.70
n	-1.71	-1.913	-1.70
Λ	-0.61	-0.613	-0.65
Σ^+	2.51	2.458	2.35
Σ^0	0.75	–	0.72
Σ^-	-1.02	-1.160	-0.92
Ξ^0	-1.33	-1.250	-1.24
Ξ^-	-0.56	-0.6507	-0.68
Δ^+	2.07	$2.7 \pm 1.5 \pm 1.3$	2.08
Δ^{++}	4.14	3.7 – 7.5	4.17
Ω^-	-1.66	-2.0200	-1.59

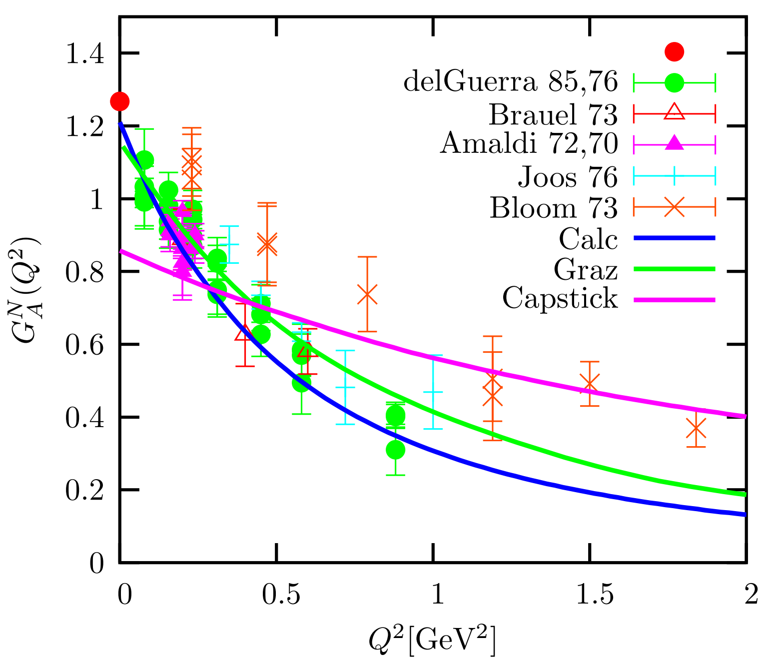
from: K. Berger, R.F. Wagenbrunn, W. Plessas, nucl-th/0407009

Tim van Cauteren, *et al.*: Eur. Phys. J. A**20** (2004) 283

Constituent Quark Model comparison







D. Merten *et al.*, Eur. Phys. J. A **14** (2002) 489;

R.F. Wagenbrunn *et al.*, Phys. Lett. B **511** (2001) 33; L.Ya. Glozmann *et al.*, Phys. Lett. B **516** (2001) 183;

B.D. Keister, S. Capstick, *N* Physics*, eds. T.-S. Lee, W. Roberts (World Scientific) (1997) 341

baryon form factors of RCQM

B. Juliá-Díaz, D.O. Riska, F. Coester Phys. Rev. C **69** (2004) 035212; hep-ph/0312169

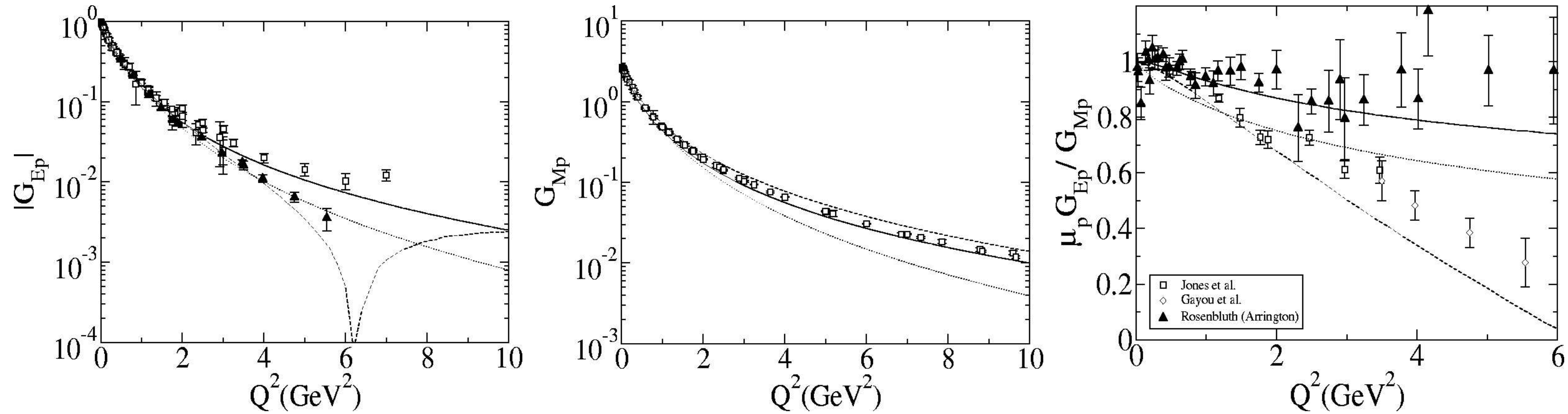
- Baryon states simultaneous eigenfunctions of the mass operator $\mathcal{M}$, spin J^2, J_3 and 3 kinematic operators corresponding to 3 choices of the kinematic subgroup of the Poincaré-group with generators, H, $\vec{P}$, $\vec{J}$, and boosts $\vec{K}$:

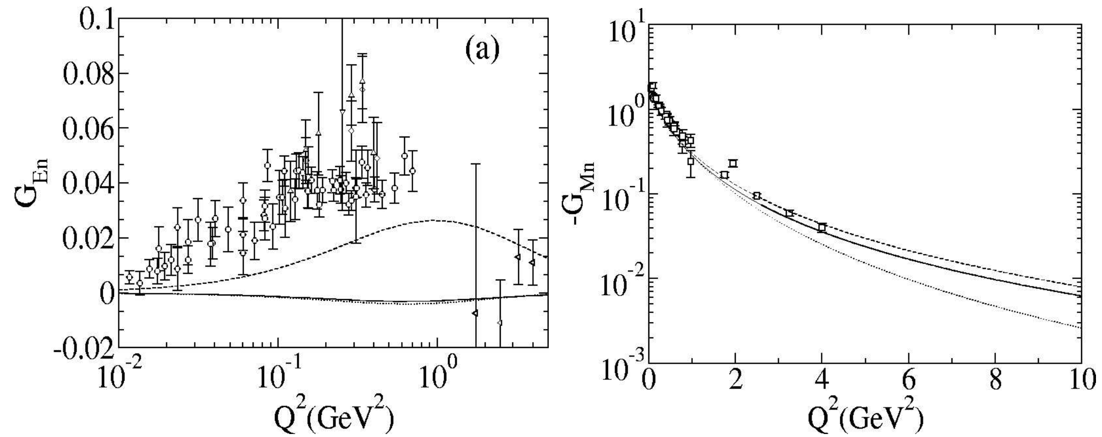
"form"	kinematic	dynamic	4-momentum
"point"	$\vec{J}, \vec{K}$	$\vec{P}$	$\underline{P} = \mathcal{M}(\sqrt{1+\lvert\vec{v}\rvert^2}, \vec{v})$
"instant"	$\vec{P}, \vec{J}$	$\vec{K}$	$\underline{P} = (\sqrt{\mathcal{M}^2 + \lvert\vec{P}\rvert^2}, \vec{P})$
"front"	$\vec{P}, \vec{K}$	$\vec{J}$	$\underline{P} = \left(\frac{\mathcal{M}^2+P_\perp^2}{P^+}, \vec{P}\right)$

- Poincaré covariant current densities from single-particle (free) currents accordingly
- simple hypercentric wave funtion $\propto \left(1 + \frac{\vec{k}^2+\vec{q}^2}{b^2}\right)^{-a}$
-

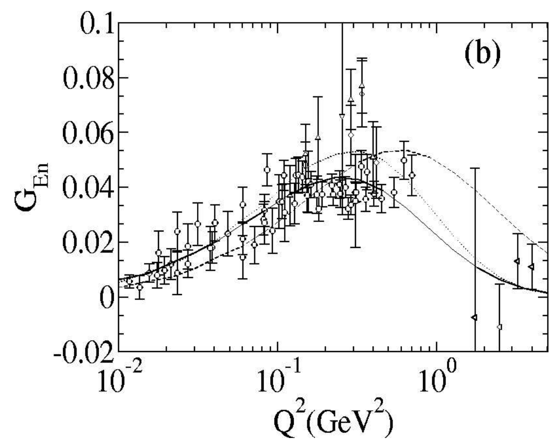
"form"	m_q[MeV]	b[MeV]	a	"size"[fm]
"point"	350	640	9/4	0.19
"front"	250	500	4	0.55
"instant"	140	600	6	0.63

Comparison of point-, instant- and front-form





"instant" (solid)
"point" (dotted)
"front" (dashed)



including some D-wave admixture

Finale

- model independent analyses:
  - dispersion theoretical analysis: updates in progress; "pion cloud" $\leftrightarrow$ accurate data at low Q^2
  - chiral perturbation theory: low $Q^2 < 4$ GeV^2; relevance of vector meson d.o.f
- lattice calculations of nucleon form factors:
  - chiral extrapolation from lattice data $M_\pi^2 \approx 0.3..1.2$ GeV^2 $\longrightarrow$ physical pion mass $M_\pi^2 \approx 0.02$ GeV^2; finite lattice spacing, finite volume effects
- model analyses of nucleon form factors:
  - emphasise the role of vector mesons and
  - "relativity" and "meson cloud" effects
- (constituent) quark model description:
  context: mass spectra $\rightarrow$ static properties $\rightarrow$ form factors, decays
  - field theoretically based (constituent) quark models
    - quark-diquark Dyson-Schwinger–Bethe-Salpeter approach
    - relativistically covariant CQM on the basis of the Salpeter equation

    really taking the "non-relativistic" out of the constituent quark model
  - Role of various Poincaré-invariant prescriptions for calculating currents in a (relativistic) quantum mechanical context.

thanks: H.-W. Hammer, U.-G. Meißner, H.R. Petry, U. Löring, D. Merten, Ch. Haupt, S. Migura