Truncated Moments - a Way to Quantify Quark-Hadron Duality

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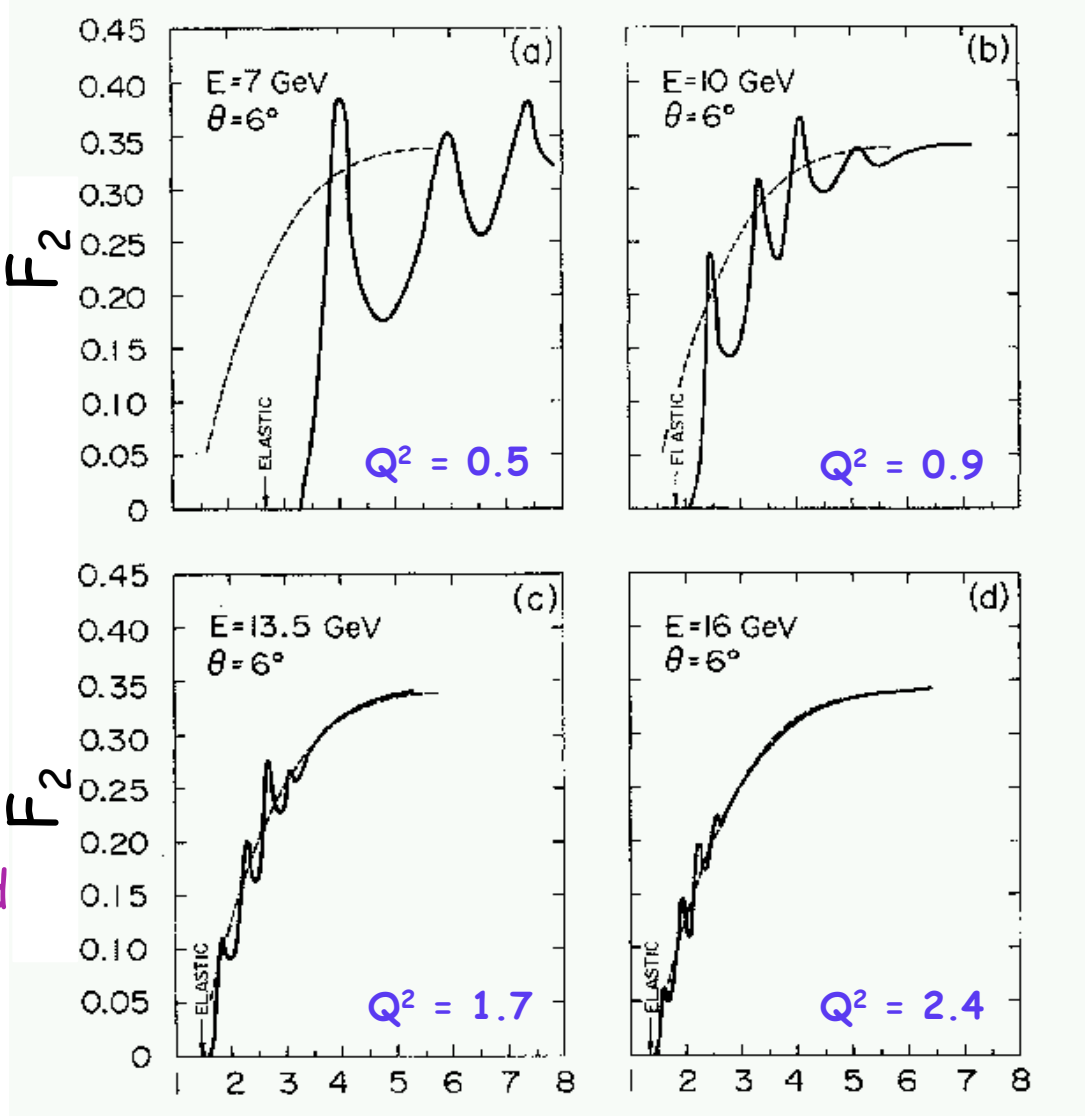
(work with Ales Psaker, Wally Melnitchouk, Eric Christy)

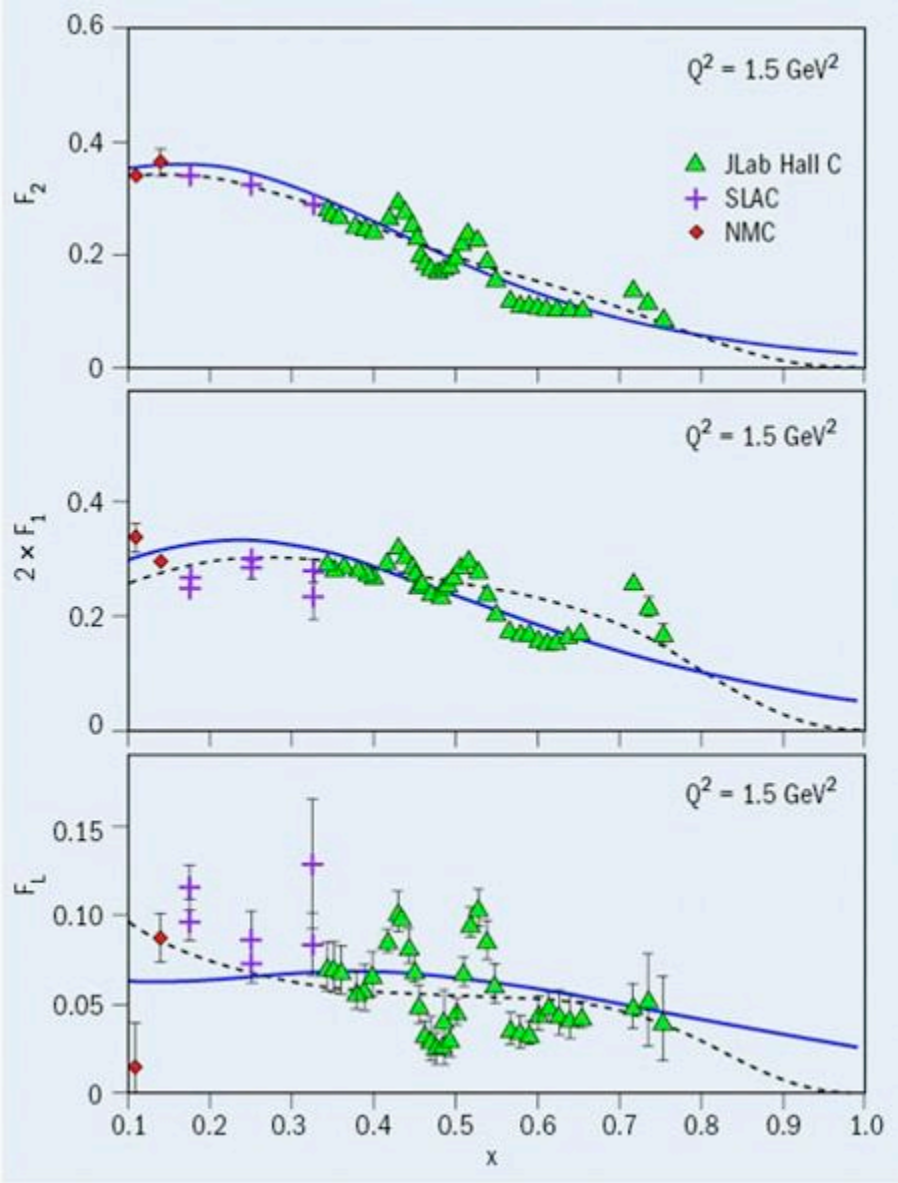
Quark-Hadron Duality - a reminder

- First observed ~1970 by Bloom and Gilman at SLAC
- Choose scaling variable ω' that relates high W^2,Q^2 data to low W^2,Q^2
- Integrated F_2 strength in nucleon resonance region (hadron) equals strength under scaling curve (quark)
- Resonances oscillate around curve *at all Q^2 - hadrons follow QCD scaling behavior*



$\omega' = 1+W^2/Q^2$

Quark-Hadron Duality - today



Observed now in nuclei, semi-inclusive scattering, spin structure functions, separated L/T channels, sought in neutron structure, neutrino scattering

Fascinating link between hadron and quark phenomenology- challenges our understanding of strong interaction dynamics

Tool to access large x regime

A wealth of high precision data now available from Jefferson Lab: at SPIRES TOP CITE papers and a really nice review article (!), CERN Courier feature, SURA Thesis Prize, 15 new experiments approved /run, dedicated workshops, global models developed based on duality

BUT……

Duality is *difficult* to quantify!

- Target mass corrections needed, but not axiomatic (see next talk!)

- Large x pdfs not well known - what to use for scaling curve?
- There is no fundamental prescription for averaging resonances
- The choice of regime for local testing can be arbitrary
- QCD Operator Product Expansion explanation only works for moments, i.e. full x regime
- Higher twist small, *or* averaging - can't untangle with moment analysis

New approach (truncated moments) mitigates all of this!!

Remember the original explanation….

- Moments of the Structure Function

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F(x,Q^2)$$

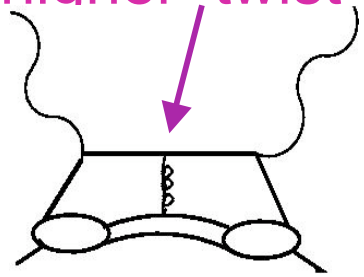
If n = 2, this is the Bloom-Gilman duality integral - *composed of resonances at low Q^2*

- Operator Product Expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} (nM_0^2/Q^2)^{k-1} B_{nk}(Q^2)$$

higher twist

logarithmic dependence (pQCD)



- Duality is described in the Operator Product Expansion *as higher twist effects being small or cancelling* **DeRujula, Georgi, Politzer (1977)**

Trucated Moments - the basic idea

Forte and Magnea, PLB 448, 295 (1999); Forte, Magnea, Piccione, and Ridolfi, NPB 594, 46 (2001); Piccione PLB 518, 207 (2001); Kotlorz and Kotlorz, PLB 644, 284 (2007)

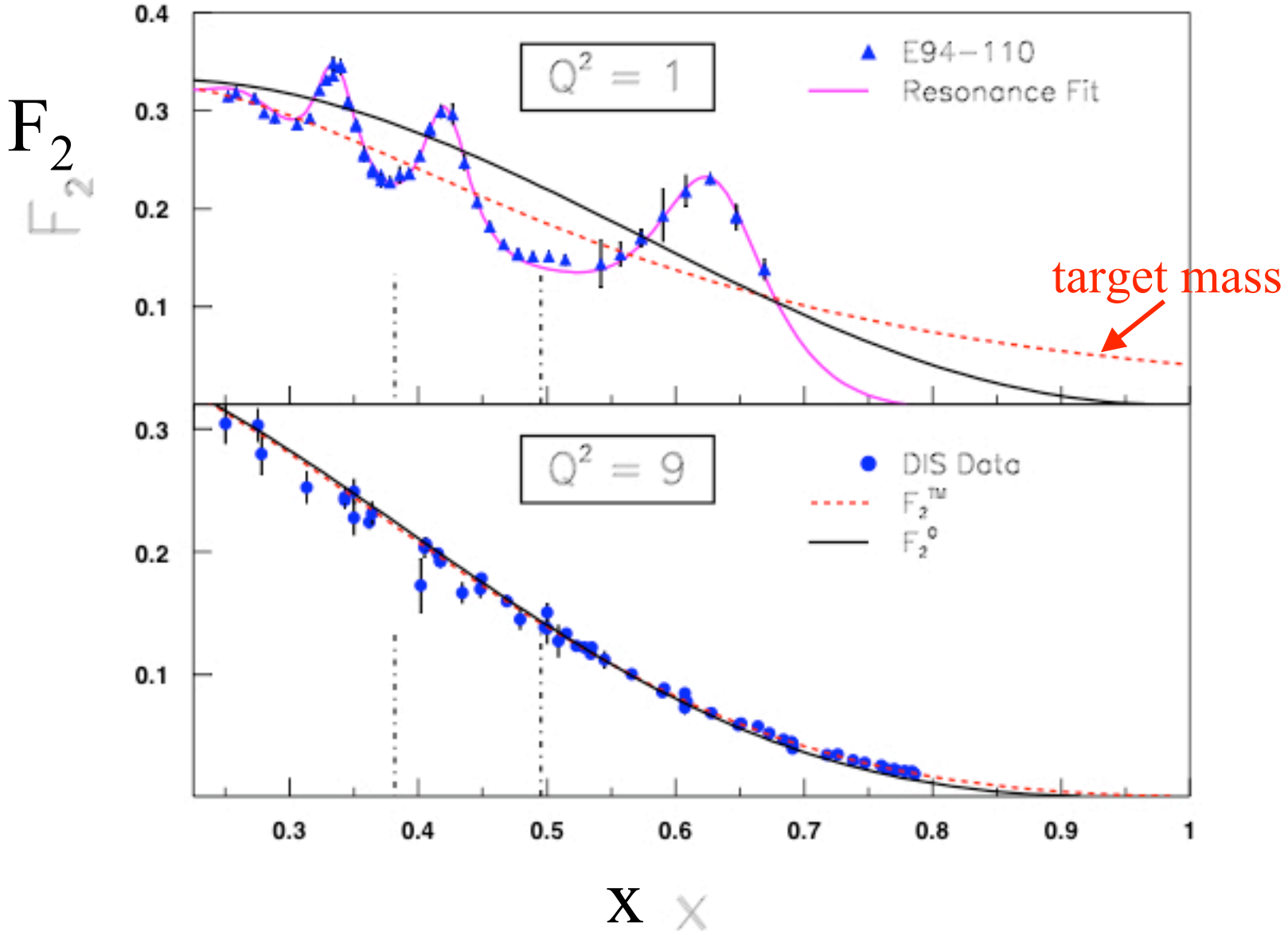
Truncated moments allow study of *restricted regions in x (or W)* within QCD in a well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

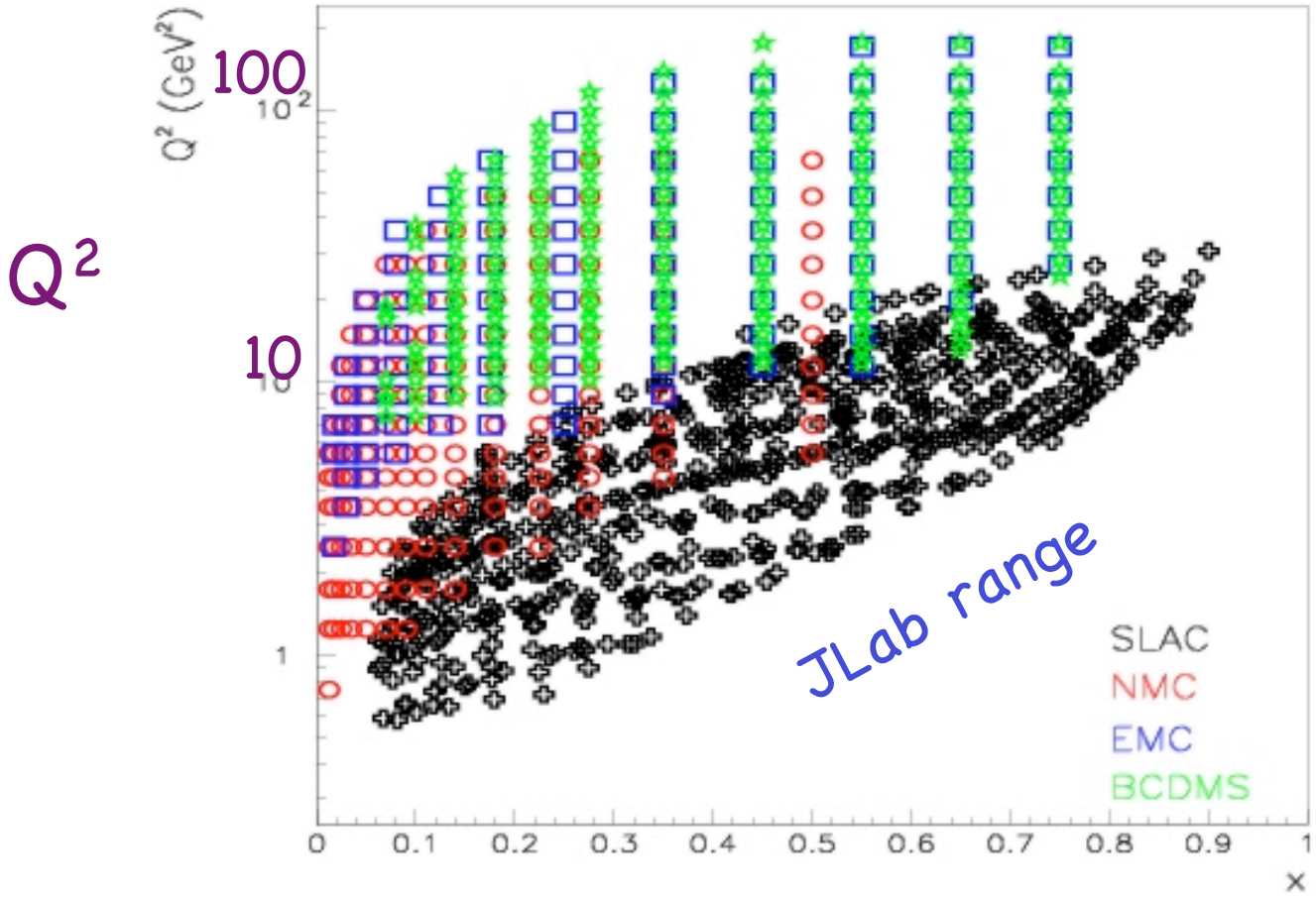
Truncated moments allow *DGLAP-like evolution* equations, similar to pdfs

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

Truncated Moments - the basic idea

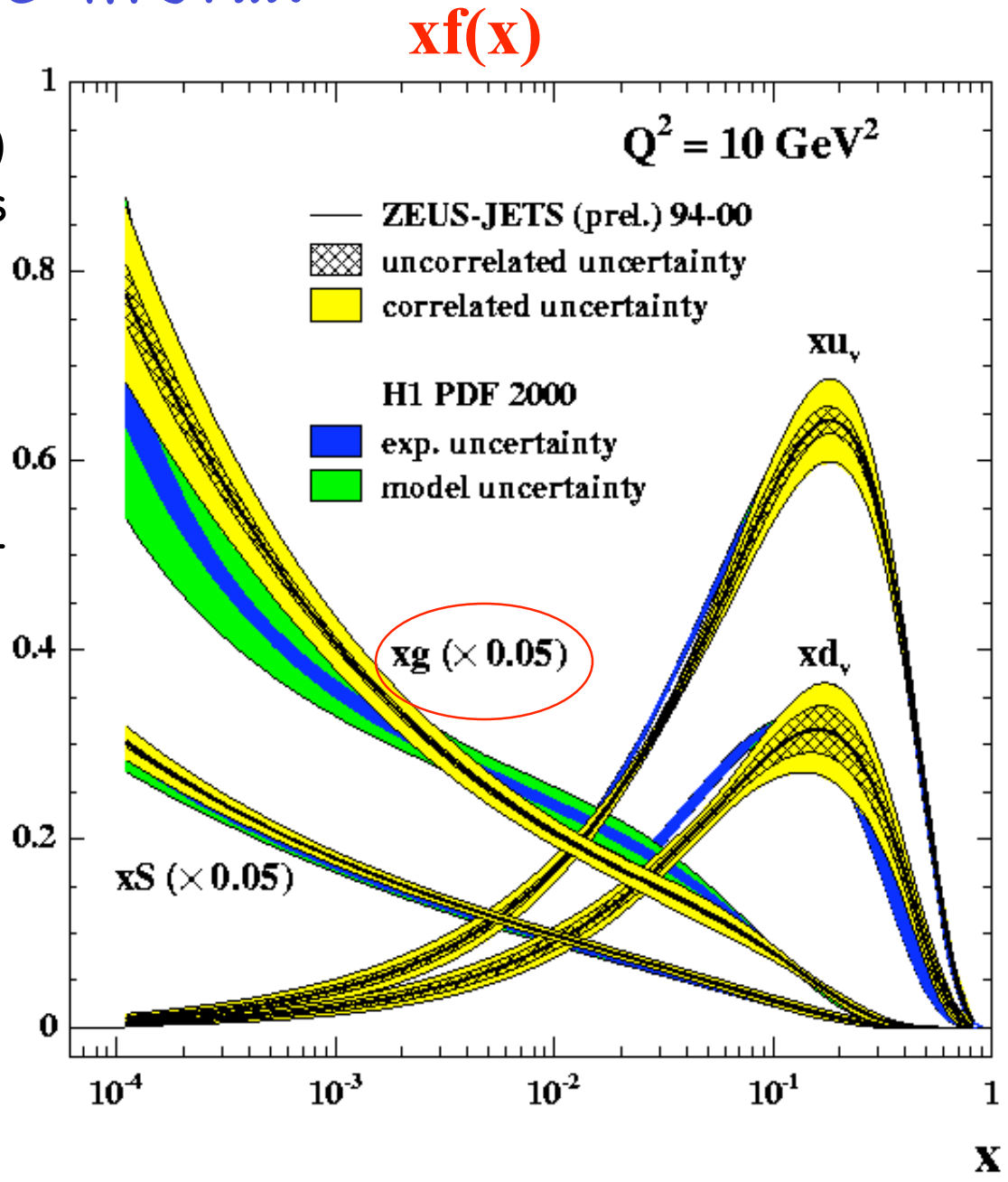


Global Data Set Kinematics Complement This Approach



One important issue first....

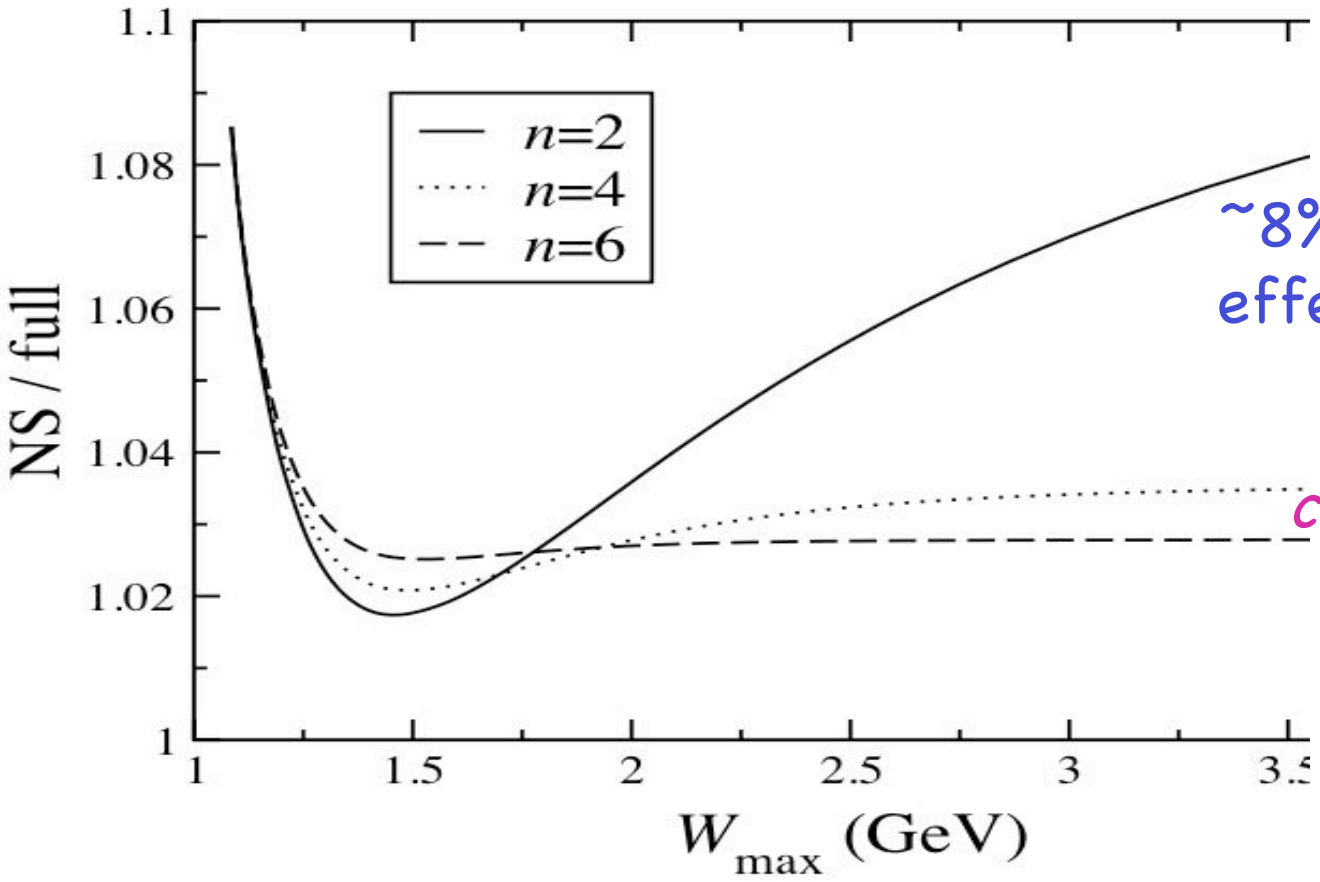
- Trucated moment evolution equations exist for singlet (s) and non-singlet (ns) equations separately
- Note: g(x) comparable to d(x) at large x - issue always existed
- For analysis of data, do not know how much of structure function is s, and how much is ns.
- Test by evolving trial structure function with known s, ns components
- Compare full evolution to ns alone to determine accuracy



Singlet / Non-singlet Evolution Comparison

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Evolve MRST from Q^2 = 9 to 1 GeV^2



~8% effect

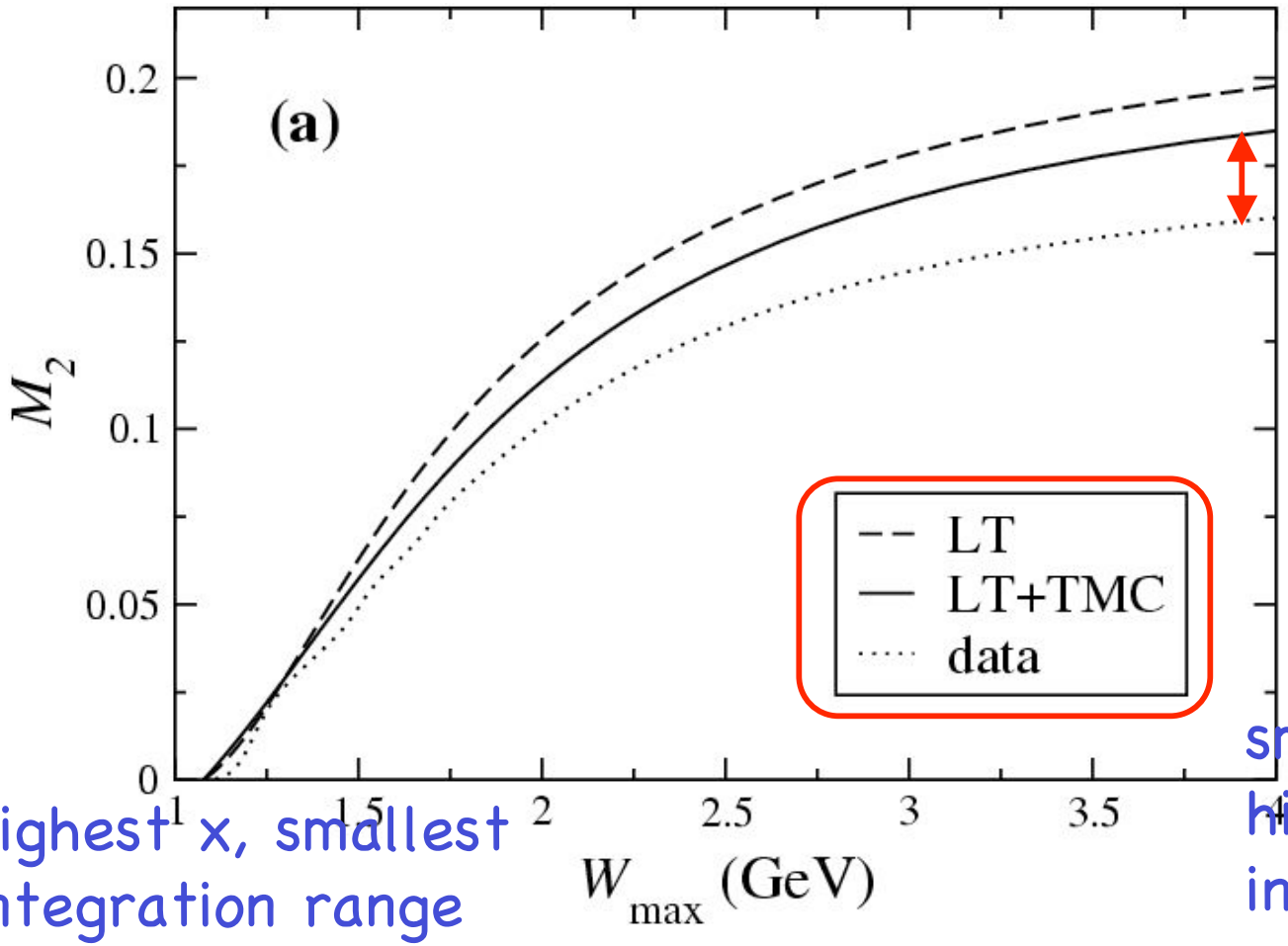
Higher order (higher n) moments dominated by larger x (smaller W) regime

Recall - high W corresponds to low x - glue increasingly more important.

Becomes dominant uncertainty.

Truncated Moment Analysis (NLO) of Hall C F_2 Data

- *Assume* data at highest Q^2 (9 GeV^2 preliminary) is entirely leading twist
- Evolve (target mass corrected fit) as non-singlet, with uncertainty evaluated, from Q^2 = 9 GeV^2 down to lower Q^2

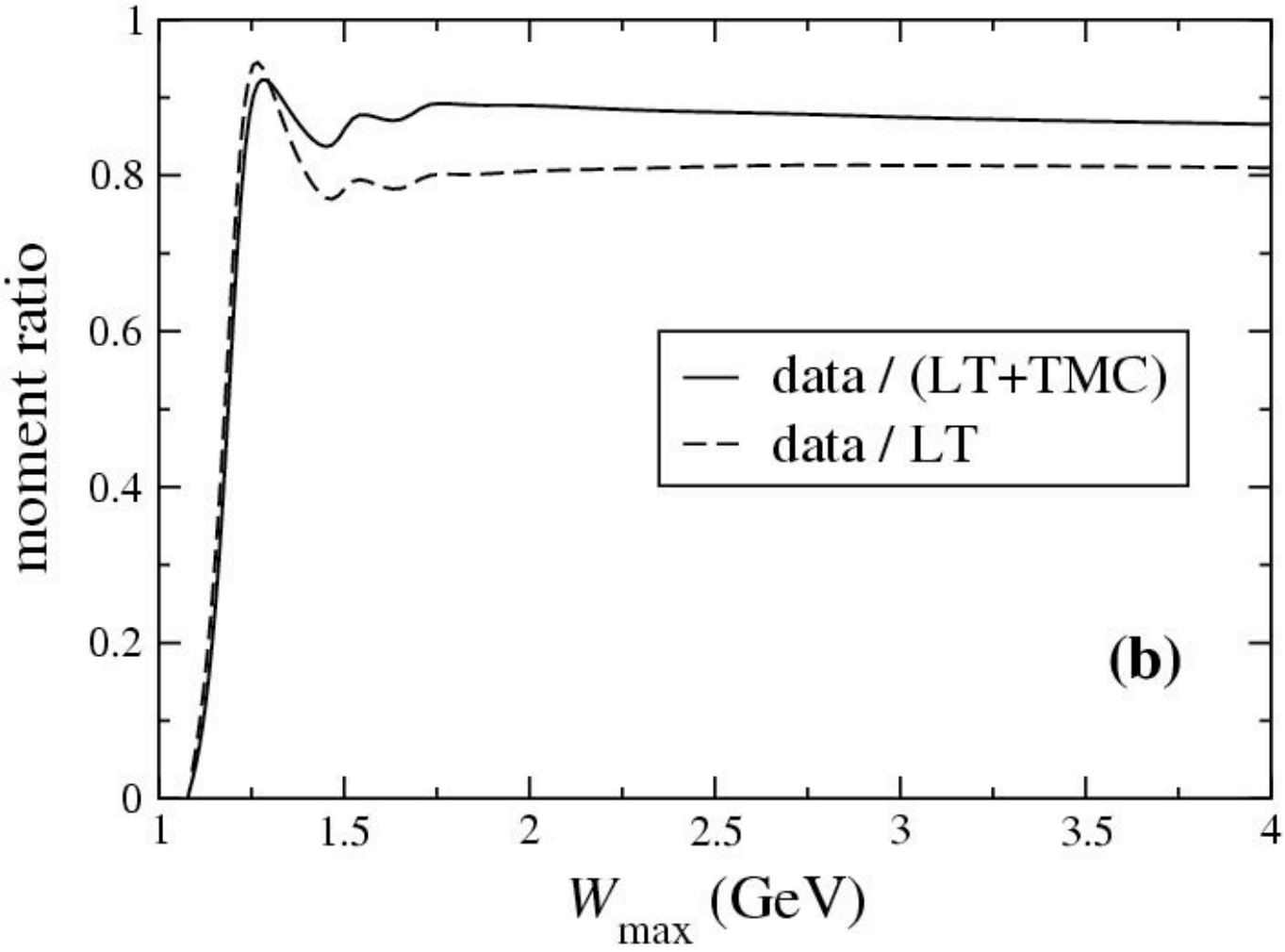


This difference quantifies the higher twist.

highest x, smallest integration range

smallest x (low x = high W), largest integration range

Quantified Higher Twist - ratio of curves on last plot

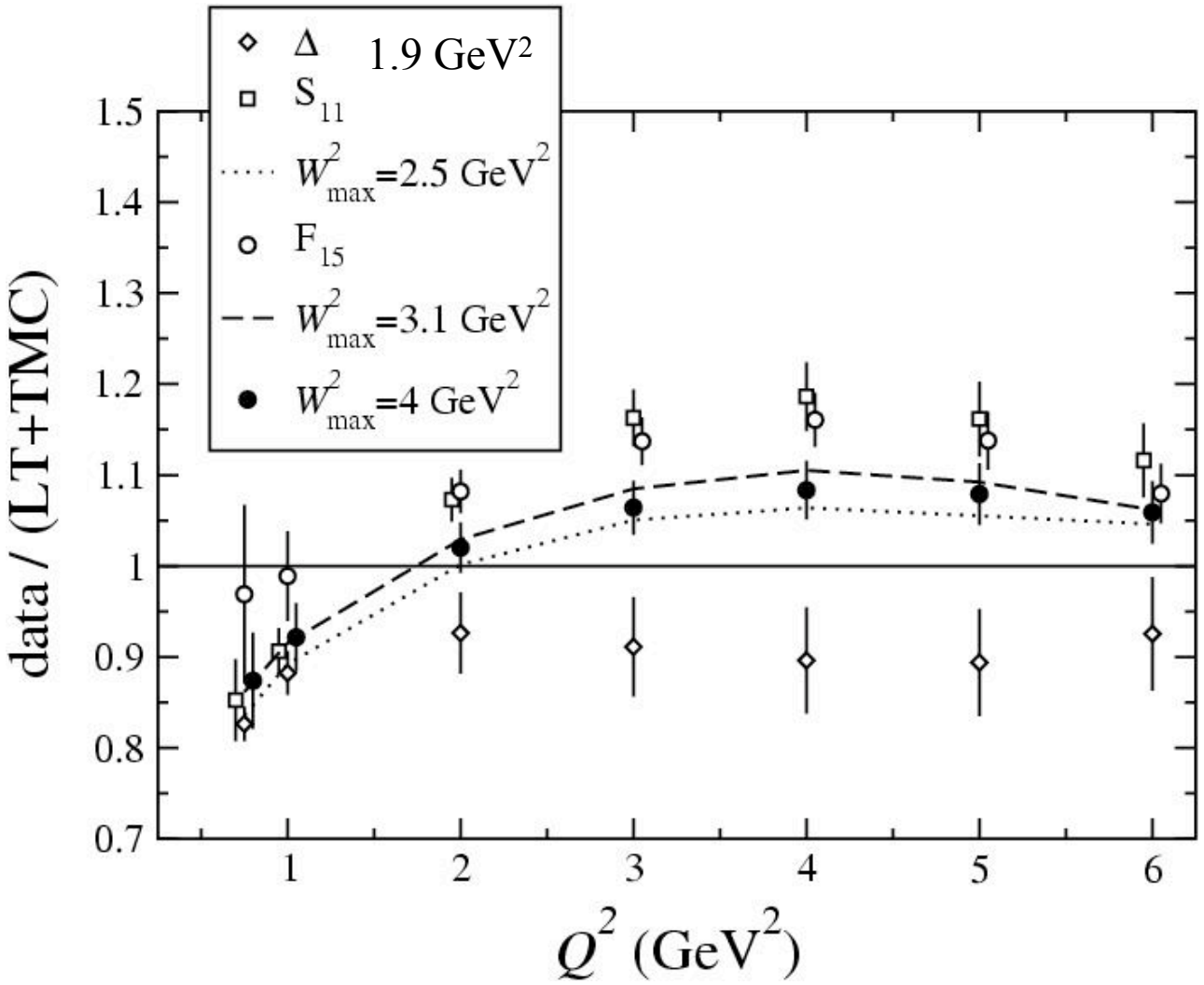


about 12% at $Q^2 = 1$ GeV^2

target mass corrections crucial

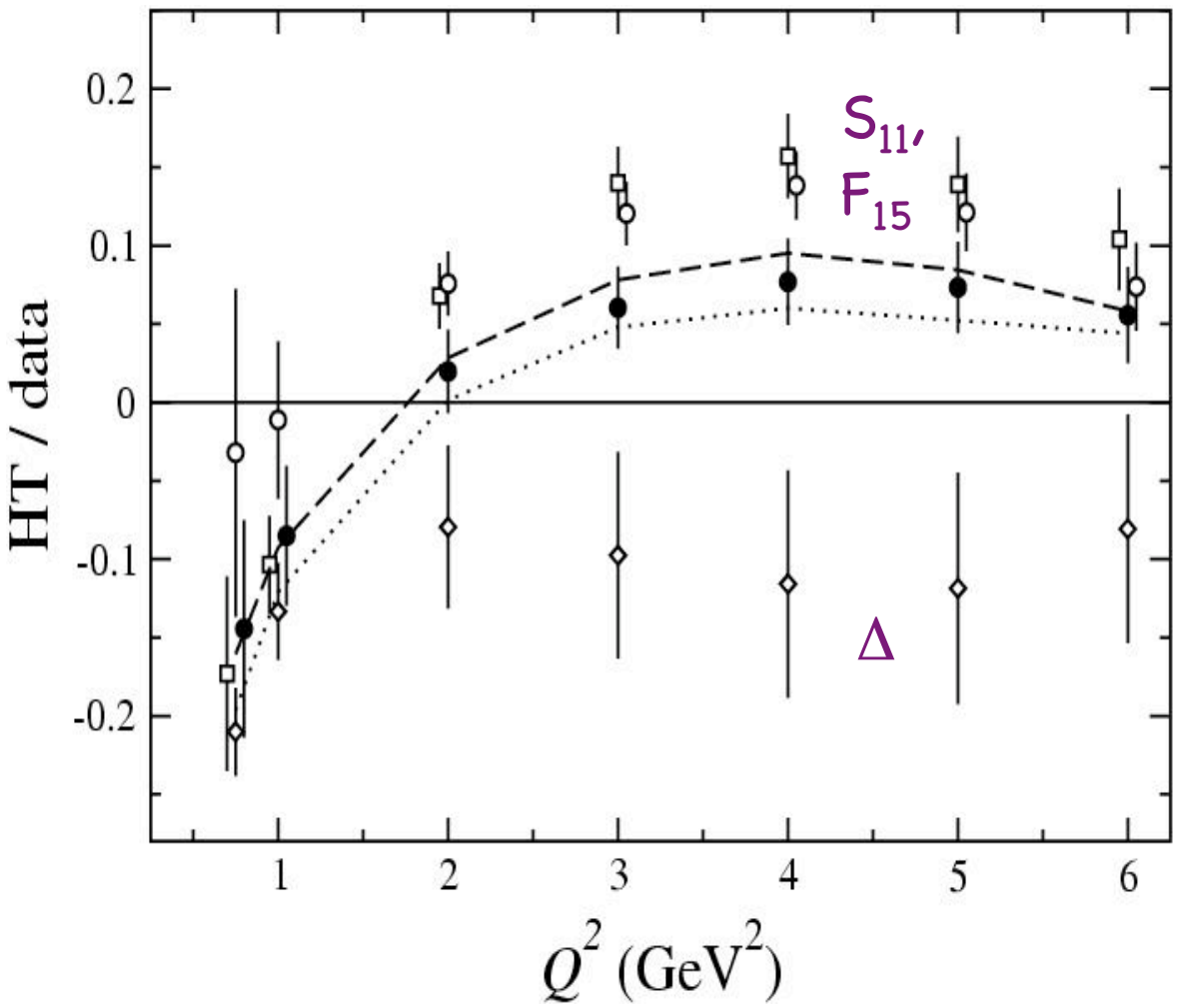
What about the Q^2 dependence?....

Q^2 Dependence of Truncated Moments, x Regions Defined by Resonances



- Consider now individual and total resonance region
- Large Q^2 dependence below ~3 GeV² - decreases at higher Q^2
- Below Q^2 = 0.75 GeV² the applicability of pQCD analysis doubtful
- Facilitates careful Higher Twist analysis….

Q^2 Dependence of Truncated Moments, x Regions Defined by Resonances

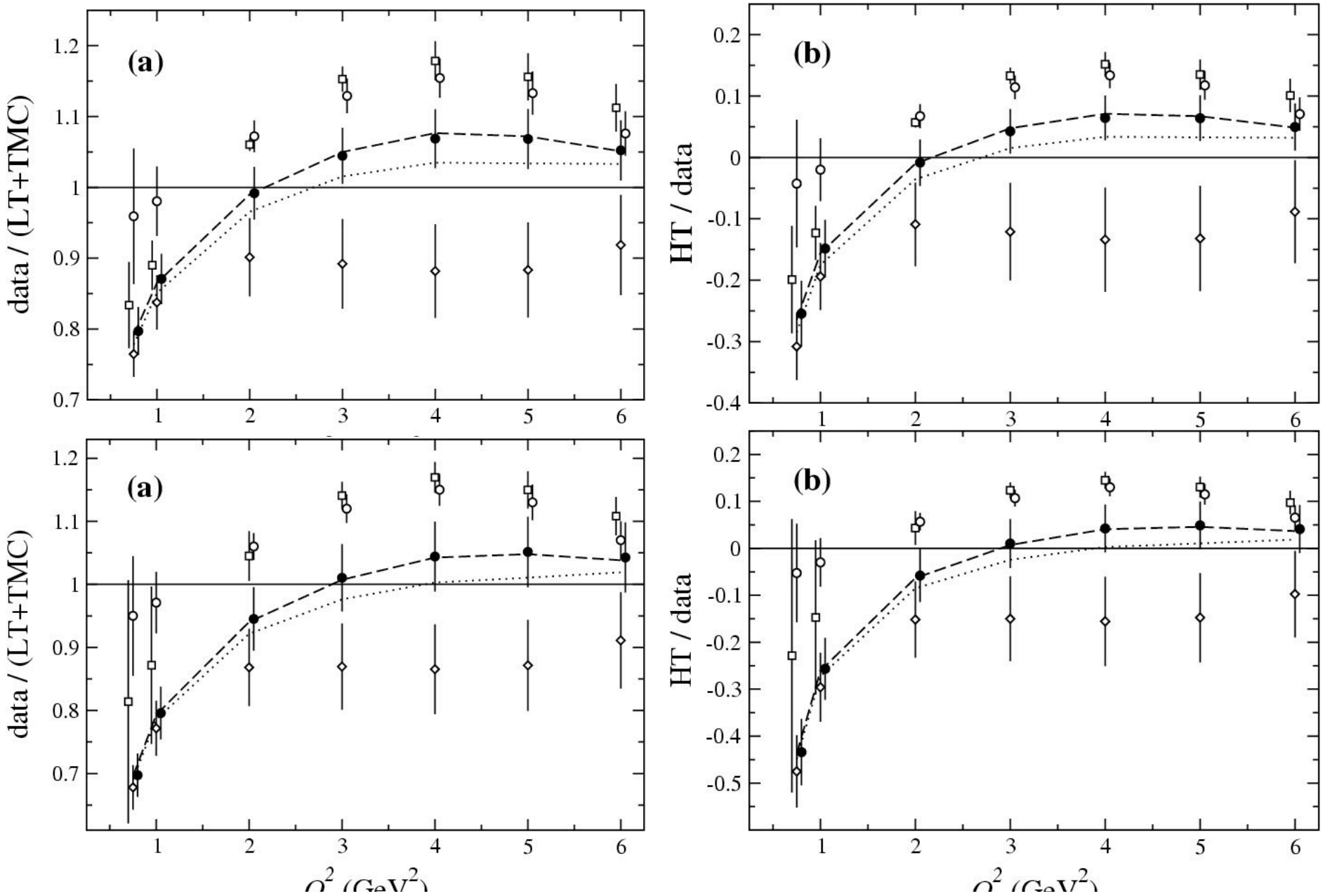


- Above Q^2 = 2 GeV^2, Δ about -10%, S_{11} and F_{15} less than +15% higher twist contribution
- First two resonances combined higher twist is about 5% (dotted line)
- All three resonances slightly higher (dashed line)
- *Less than 10%* for full region (black circles)
- Duality better with more resonances included - bears out quark model predictions

Similar for Higher Order Moments

n = 4

n = 6



Summary

- Truncated moments provide firm foundation for quantitative study of duality in QCD
- Higher twists both "small" and do tend to cancel on average
- This analysis also provides uncertainty on singlet evolution contribution
- Still to do

  Evolve from higher Q^2 (20 GeV^2 being prepared for publication)

  Quantify region dependence (choice of W,x range)

  Longitudinal structure function, spin structure functions,....

Duality is *difficult* to quantify - but getting easier...

- Large x pdfs not well known - what to use for scaling curve?
  - Use DIS data at high Q^2, minimize higher twist and large x pdf uncertainties
- There is no fundamental prescription for averaging resonances
  - Prescription for integration over arbitrarily small x regime
- The choice of regime for local testing can be arbitrary
  - True, but now testable *quantitatively*
- QCD Operator Product Expansion explanation only works for moments, i.e. full x regime
  - Low x no longer needed (uncertainty reduction)
  - Tool to reduce large x pdf uncertainties (evolve up)
- Higher twist small, *or* averaging - can't untangle with moment analysis
  - Reduction of averaging region facilitates test