Two-photon-exchange effects in electromagnetic N to Δ(1232) transition

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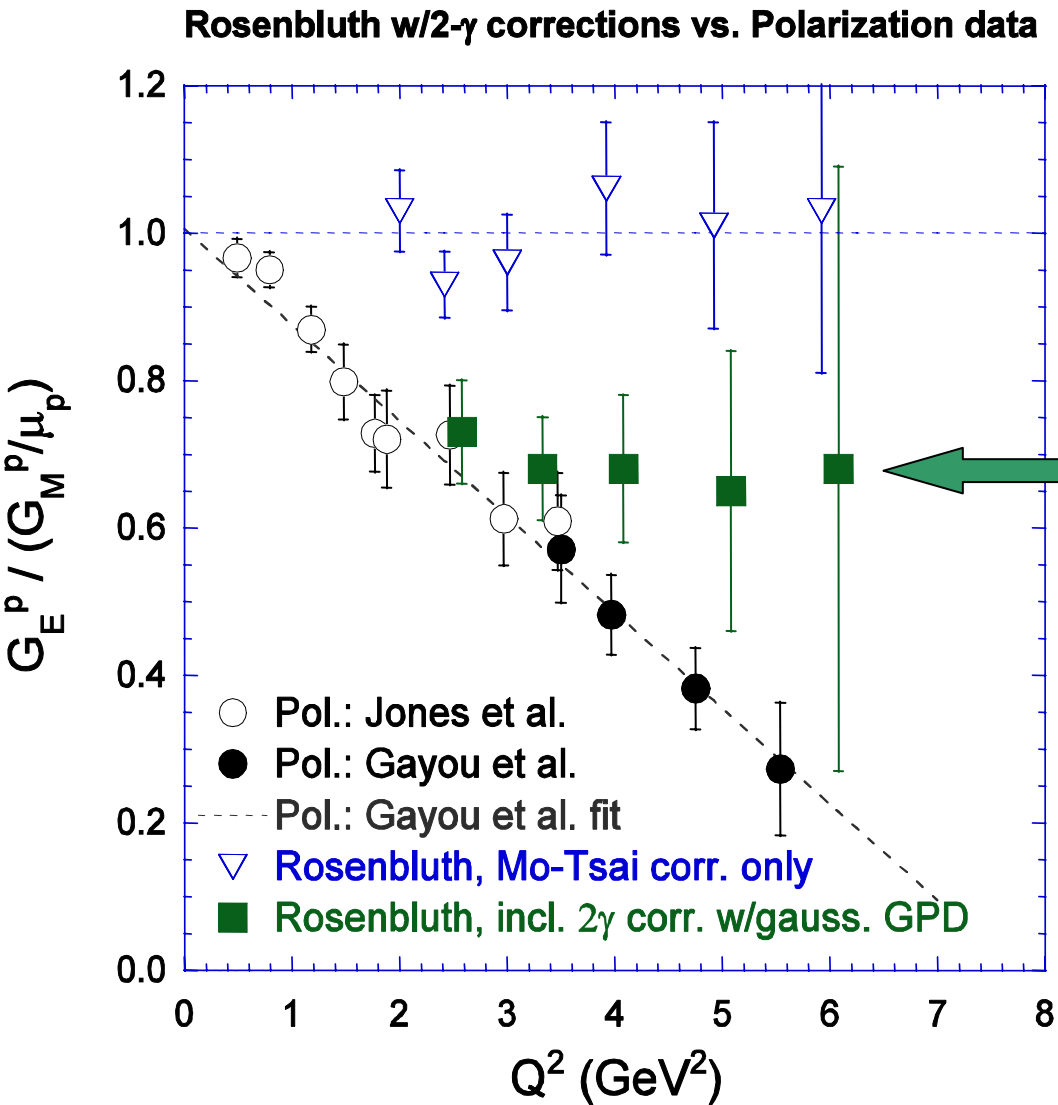
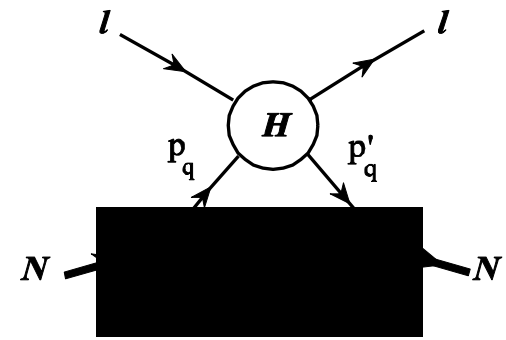
Two-photon exchange effect in nucleon form factors

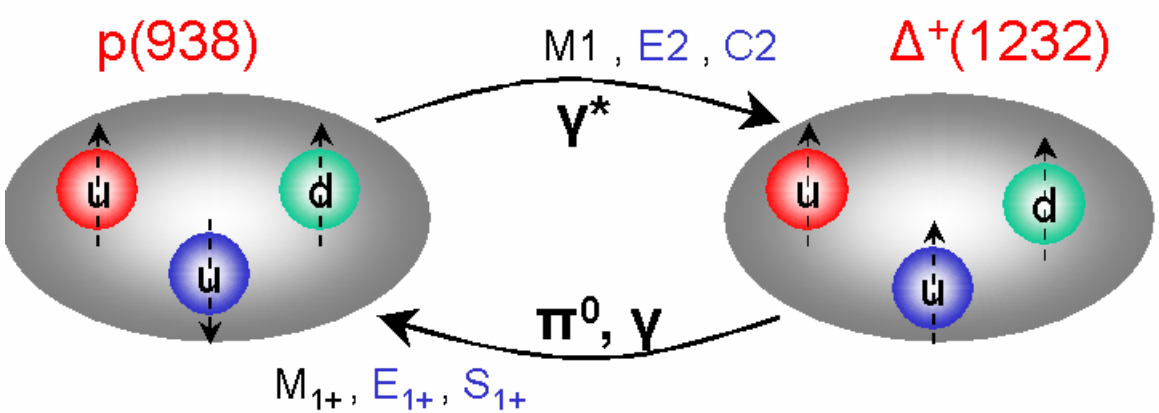




Partonic calculation

[Chen, Afanasev, Brodsky, Carlson, Vanderhaeghen, PRL (2004)]

Electromagnetic Nucleon to Delta transition

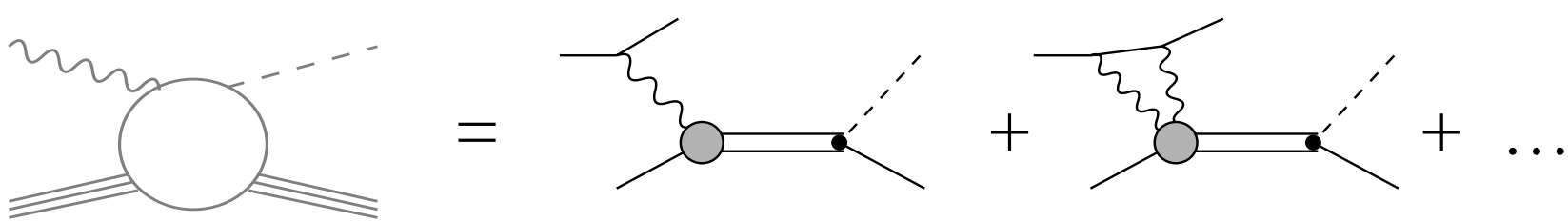


J^P=3/2+, M_Δ =1.232, Γ_Δ=0.2 GeV

N ! Δ transition:

π N ! Δ (99%), γ N ! Δ (<1%)

- γ¤NΔ is specified by three form-factors: G_M¤ (Q^2) [M1], G_E¤ (Q^2) [E2], G_C¤ (Q^2) [C2]
- studied in *pion electroproduction on the nucleon* at the resonance kinematics, s=M_Δ^2



EM nucleon to Delta transition: general formalism

❖ In general (any # of exchanged photons), the e N! e Δ transition can be described by 16 form factors, or, 16 independent helicity amplitudes: $T^{h',h}_{\lambda_\Delta,\lambda_N} \equiv \langle k',h';p_\Delta,\lambda_\Delta|T|k,h;p,\lambda_N\rangle$.

❖ m_e=0, cuts this number down to 8: $(\varepsilon_\pm = \sqrt{1\pm\varepsilon})$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*+G_E^*) \xrightarrow{\text{PQCD}} \frac{1}{Q^6}$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^* \xrightarrow{\text{PQCD}} \frac{1}{Q^5}$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*-3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*-3G_E^*) \longrightarrow \frac{1}{Q^4}$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^*+G_E^*)$$

$$R_{EM} = \frac{E2}{M1} = -\frac{G_E^*}{G_M^*} \xrightarrow{\text{PQCD}} 1$$

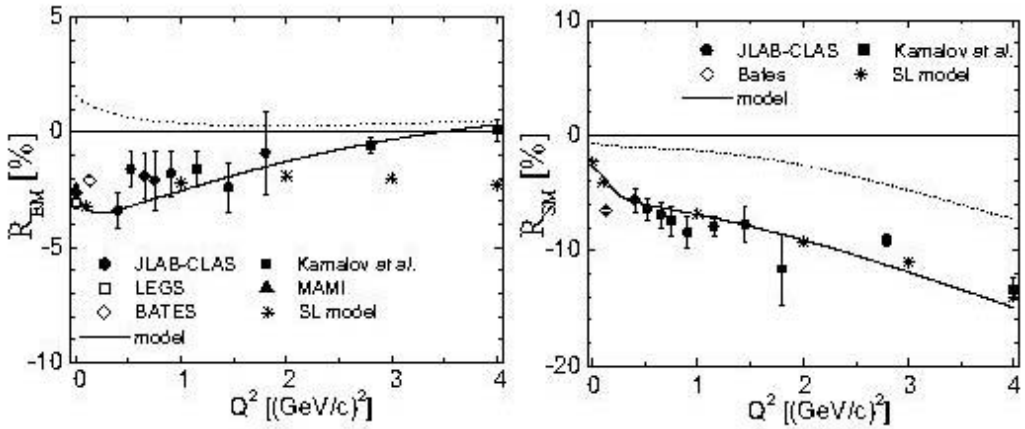
$$R_{SM} = \frac{C2}{M1} = -\frac{Q_+Q_-}{4\,M_\Delta^2}\frac{G_C^*}{G_M^*} \longrightarrow \text{const}$$

[Carlson, PRD (1986)]

one-photon exchange approximation

$R_{EM} = -G_E / G_M$

$R_{SM} = -(q/2M_\Delta)\, G_C / G_M$



Experimental status of E2/M1 and C2/M1 ratios

EM Nucleon to Delta transition: general formalism

- The unpolarized pion electroproduction cross-section is, *in general*, written as:

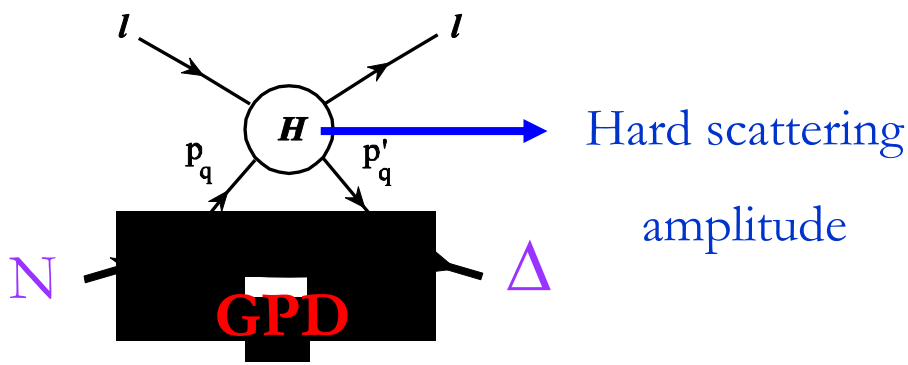
$$\frac{d\sigma}{(dE'_e\, d\Omega'_e)^{lab}\, d\Omega_\pi} \equiv \Gamma_v \frac{d\sigma}{d\Omega_\pi}\,, \qquad \text{Flux: } \Gamma_v = \frac{e^2}{(2\pi)^3}\left(\frac{E'_e}{E_e}\right)^{lab} \frac{(s_{\pi N} - M_N^2)/(2M_N)}{Q^2\,(1-\varepsilon)}.$$

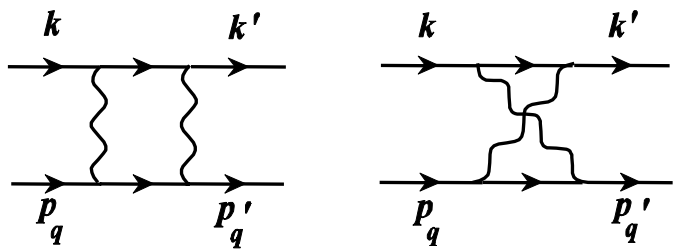
$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\,\cos(2\Phi)\,\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1+\varepsilon)}\,\cos\Phi\,\frac{d\sigma_{LT}}{d\Omega_\pi} + 2h\,\varepsilon\,\sin(2\Phi)\,\frac{d\sigma_{TTi}}{d\Omega_\pi} + 2h\sqrt{2\,\varepsilon\,(1-\varepsilon)}\,\sin\Phi\,\frac{d\sigma_{LTi}}{d\Omega_\pi}.$$

- At the Δ-resonance, these cross-sections are expressed in terms of e N! e Δ helicity amplitudes

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\pi} &= \frac{1}{\pi}\,\frac{9\,Q^2\,(1-\varepsilon)}{16\,M_\Delta\,(M_\Delta^2 - M_N^2)\,\Gamma_\Delta} \\
&\times \Big\{ \frac{1}{2}\sin^2\theta_\pi\,\left[\,|T_1|^2 + |T_2|^2 + |T_7|^2 + |T_8|^2\,\right] + \frac{1}{6}(1 + 3\,\cos^2\theta_\pi)\,\left[\,|T_3|^2 + |T_4|^2 + |T_5|^2 + |T_6|^2\,\right] \\
&+ \cos\Phi\,\frac{1}{\sqrt{3}}\,\sin(2\theta_\pi)\,\Re\left[\,T_1\,T_3^* + T_2\,T_4^* - T_7\,T_5^* - T_8\,T_6^*\,\right] - \cos(2\Phi)\,\frac{1}{\sqrt{3}}\,\sin^2\theta_\pi\,\Re\left[\,T_1\,T_5^* + T_2\,T_6^* + T_7\,T_3^* + T_8\,T_4^*\,\right] \\
&+ \sin\Phi\,\frac{1}{\sqrt{3}}\,\sin(2\theta_\pi)\,\Im\left[\,T_1\,T_3^* + T_2\,T_4^* + T_7\,T_5^* + T_8\,T_6^*\,\right] - \sin(2\Phi)\,\frac{1}{\sqrt{3}}\,\sin^2\theta_\pi\,\Im\left[\,T_1\,T_5^* + T_2\,T_6^* - T_7\,T_3^* - T_8\,T_4^*\,\right] \Big\}.
\end{aligned}$$

Two-photon exchange : partonic calculation





$$A^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s}-\hat{u}}{Q^2} g_M^{hard} + g_A^{(2\gamma)} \right) \sqrt{\frac{2}{3}} \frac{1}{6} H_M^{(3)}$$ "magnetic" GPD

$$C^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s}-\hat{u}}{Q^2} g_A^{(2\gamma)} + g_M^{hard} \right) \mathrm{sgm}(x) \frac{1}{6} C_1^{(3)}$$ "axial" GPD

Large N_C

$$H_M^{(3)}(x,0,Q^2) = 2 \frac{G_M^*(0)}{\kappa_V} \left[E^u(x,0,Q^2) - E^d(x,0,Q^2) \right]$$

$$C_1^{(3)}(x,0,Q^2) = \sqrt{3} \left[\tilde{H}^u(x,0,Q^2) - \tilde{H}^d(x,0,Q^2) \right]$$

Nucleon GPD's

Nucleon GPDs

Modified Regge model [Guidal, Polyakov, Radyushkin, Vanderhaeghen, PRD (2005)]:

$$H^q(x,0,q^2) = q_v(x)\, x^{\alpha_1'(1-x)\,Q^2}$$

$$E^q(x,0,q^2) = \frac{\kappa_q}{N_q}(1-x)^{\eta_q}\, q_v(x)\, x^{\alpha_2'(1-x)\,Q^2}$$

➡ ***$q_v(x)$*** -- forward parton distributions at $\mu^2 = 1$ GeV^2

MRST2002 NNLO

$$\begin{cases} u_v = 0.262\, x^{-0.69}(1-x)^{3.50}\left(1+3.83\, x^{0.5}+37.65\, x\right) \\ d_v = 0.061\, x^{-0.65}(1-x)^{4.03}\left(1+49.05\, x^{0.5}+8.65\, x\right) \end{cases}$$

Leader, Sidorov, Stamenov (2002)

$$\begin{cases} \Delta u_v = 0.505 x^{-0.33}(1-x)^{3.428}\left(1+2.179 x^{0.5}+14.57 x\right) \\ \Delta d_v = -0.0185 x^{-0.73}(1-x)^{3.864}\left(1+35.47 x^{0.5}+28.97 x\right) \end{cases}$$

➡ Regge slopes : α_1' , α_2' determined from rms radii

$$r^2_{1,p} = -6\,\alpha_1' \int_0^1 dx \left\{ e_u\, u_v(x) + e_d\, d_v(x) \right\} \ln x$$

$$r^2_{1,n} = -6\,\alpha_1' \int_0^1 dx \left\{ e_u\, d_v(x) + e_d\, u_v(x) \right\} \ln x$$

$$\alpha_1' = 1.098\ \mathrm{GeV}^{-2}$$

$$\alpha_2' = 1.158\ \mathrm{GeV}^{-2}$$

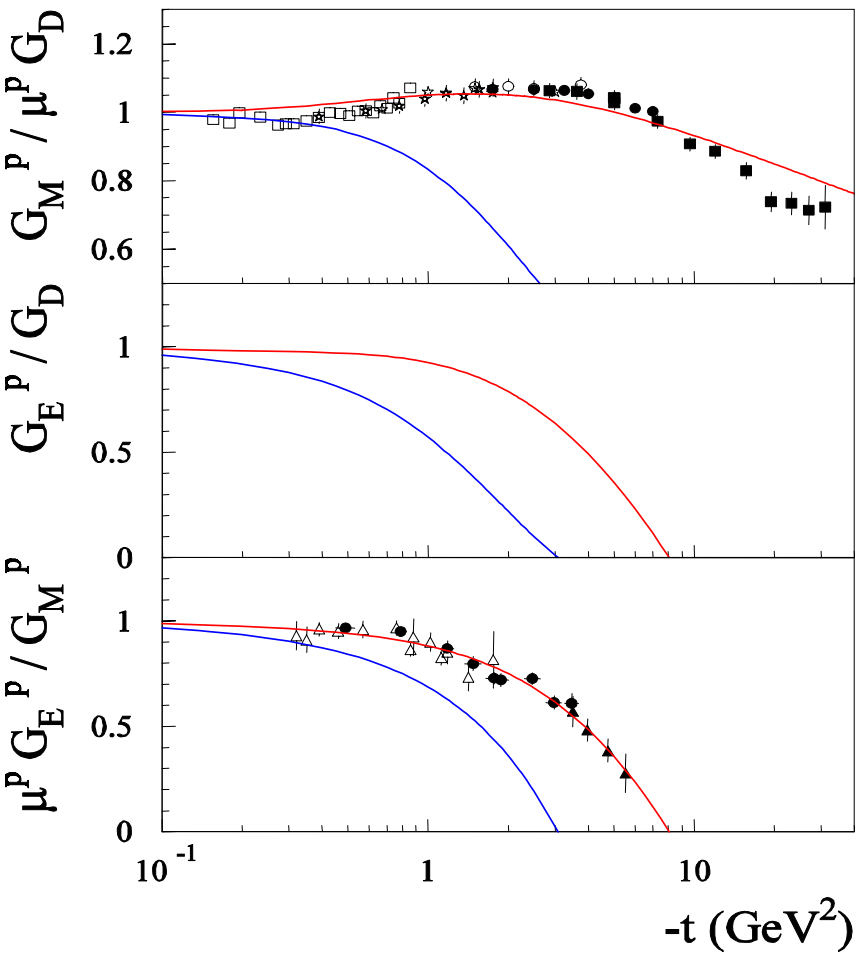
➡ η_u , η_d determined from F_2 / F_1 at large Q^2

$$\eta_u = 1.52$$

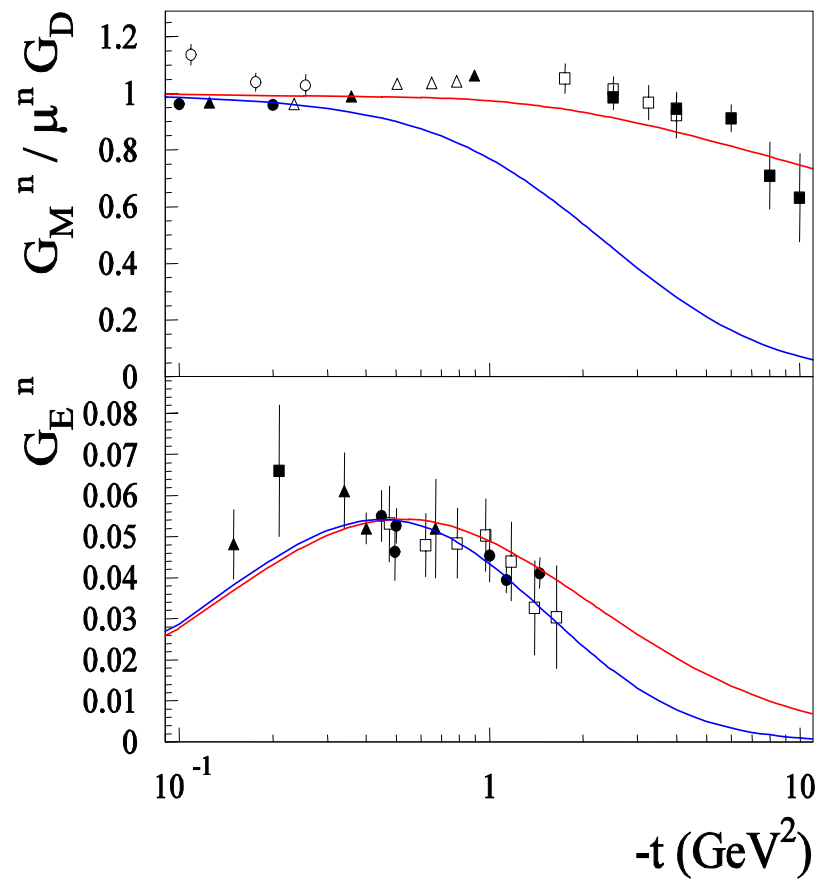
$$\eta_d = 0.31$$

Nucleon electromagnetic form factors

PROTON



NEUTRON



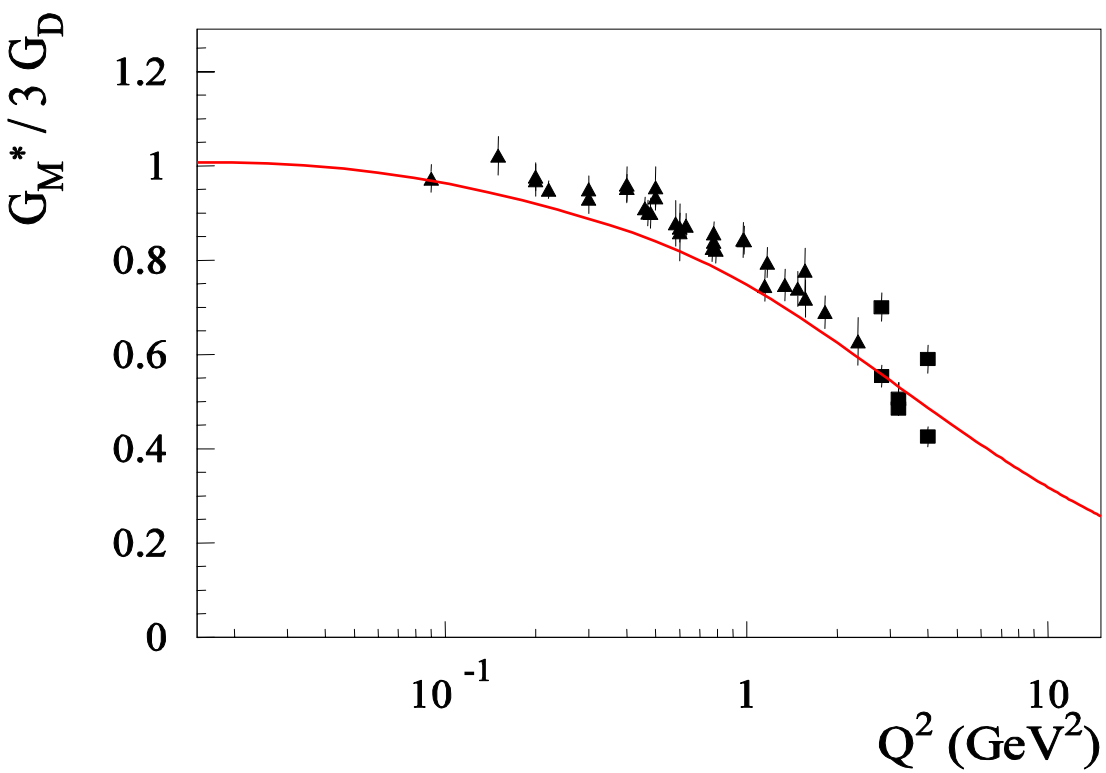
modified Regge parametrization

Regge parametrization

N -> Δ transition form factors from GPDs

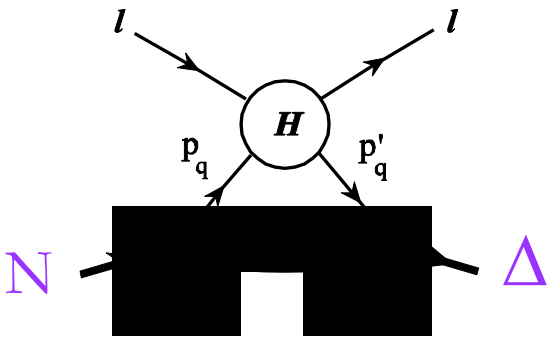
large N_c

$$G_M^*(t) = \frac{G_M^*(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G_M^*(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$



→ ``modified Regge model''

Two-photon exchange contribution to helicity amplitudes



$$
\begin{aligned}
T_1^{2\gamma} &= \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_N Q_+} - C^*\frac{\sqrt{2}Q}{Q_-}\right\}, \\
T_2^{2\gamma} &= 0\,, \\
T_3^{2\gamma} &= -\frac{e^2}{2}\,C^*\sqrt{\frac{2}{3}}\,\frac{M_\Delta^2-M_N^2-Q^2}{M_\Delta Q_-} \\
T_4^{2\gamma} &= \sqrt{\frac{1}{3}}\,T_1^{2\gamma}\,, \\
T_5^{2\gamma} &= \sqrt{\frac{1}{3}}\,T_8^{2\gamma}\,, \\
T_6^{2\gamma} &= T_3^{2\gamma}\,, \\
T_7^{2\gamma} &= 0\,, \\
T_8^{2\gamma} &= \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_N Q_+} + C^*\frac{\sqrt{2}Q}{Q_-}\right\}.
\end{aligned}
$$

One + two-photon-exchange in observables

❖ The unpolarized pion electroproduction cross-section:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\cos(2\Phi)\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1+\varepsilon)}\cos\Phi\,\frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\sin(2\Phi)\frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\,\varepsilon\,(1-\varepsilon)}\sin\Phi\,\frac{d\sigma_{LTi}}{d\Omega_\pi} = \frac{1}{\pi}\,\frac{9\,Q^2\,(1-\varepsilon)}{16\,M_\Delta\,(M_\Delta^2 - M_N^2)\,\Gamma_\Delta}$$

$$\times\ \Big\{\frac{1}{2}\sin^2\theta_\pi\,\big[\,|T_1|^2 + |T_2|^2 + |T_7|^2 + |T_8|^2\,\big] + \frac{1}{6}(1+3\cos^2\theta_\pi)\,\big[\,|T_3|^2 + |T_4|^2 + |T_5|^2 + |T_6|^2\,\big]$$

$$+\cos\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Re\,[\,T_1\,T_3^* + T_2\,T_4^* - T_7\,T_5^* - T_8\,T_6^*\,] - \cos(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Re\,[\,T_1\,T_5^* + T_2\,T_6^* + T_7\,T_3^* + T_8\,T_4^*\,]$$

$$+\sin\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Im\,[\,T_1\,T_3^* + T_2\,T_4^* + T_7\,T_5^* + T_8\,T_6^*\,] - \sin(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Im\,[\,T_1\,T_5^* + T_2\,T_6^* - T_7\,T_3^* - T_8\,T_4^*\,]\Big\}\,.$$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* + G_E^*)$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N + M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^*$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* - 3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* - 3G_E^*)$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N + M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* + G_E^*)$$

$$T_1^{2\gamma} = \frac{e^2}{2}\left\{A^*\,\frac{3(M_\Delta + M_N)Q}{2\sqrt{2}M_N Q_+} - C^*\,\frac{\sqrt{2}Q}{Q_-}\right\},$$

$$T_2^{2\gamma} = 0\,,$$

$$T_3^{2\gamma} = -\frac{e^2}{2}\,C^*\sqrt{\frac{2}{3}}\,\frac{M_\Delta^2 - M_N^2 - Q^2}{M_\Delta Q_-}$$

$$T_4^{2\gamma} = \sqrt{\frac{1}{3}}\,T_1^{2\gamma}\,,$$

$$T_5^{2\gamma} = \sqrt{\frac{1}{3}}\,T_8^{2\gamma}\,,$$

$$T_6^{2\gamma} = T_3^{2\gamma}\,,$$

$$T_7^{2\gamma} = 0\,,$$

$$T_8^{2\gamma} = \frac{e^2}{2}\left\{A^*\,\frac{3(M_\Delta + M_N)Q}{2\sqrt{2}M_N Q_+} + C^*\,\frac{\sqrt{2}Q}{Q_-}\right\}.$$

Two-photon effects on observables

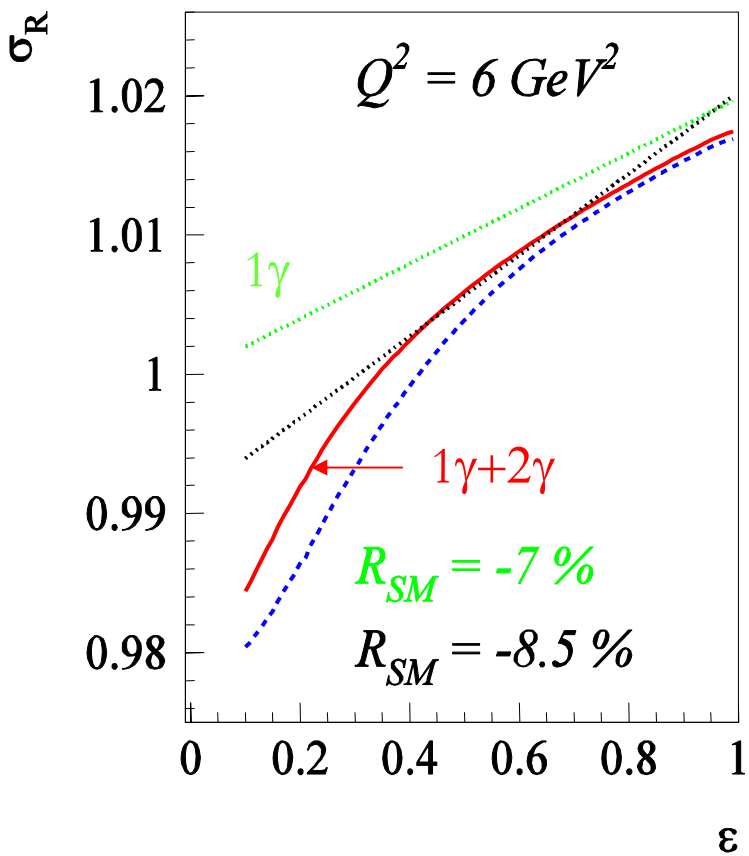
$$\sigma_R = 1 + 3\,(R^{1\gamma}_{EM})^2 + \varepsilon\,\frac{16\,M_\Delta^2\,Q^2}{Q_+^2\,Q_-^2}\,(R^{1\gamma}_{SM})^2 + \frac{1}{G_M^*}\sqrt{\frac{2}{3}}\left[\frac{1}{2}\,A^*\,\frac{Q^2}{Q_+\,Q_-}\,\varepsilon_+\,\varepsilon_- + 2\,C^*\,\frac{Q^2}{Q_-^2}\,\varepsilon_-^2\,\frac{M_N}{M_N + M_\Delta}\right].$$

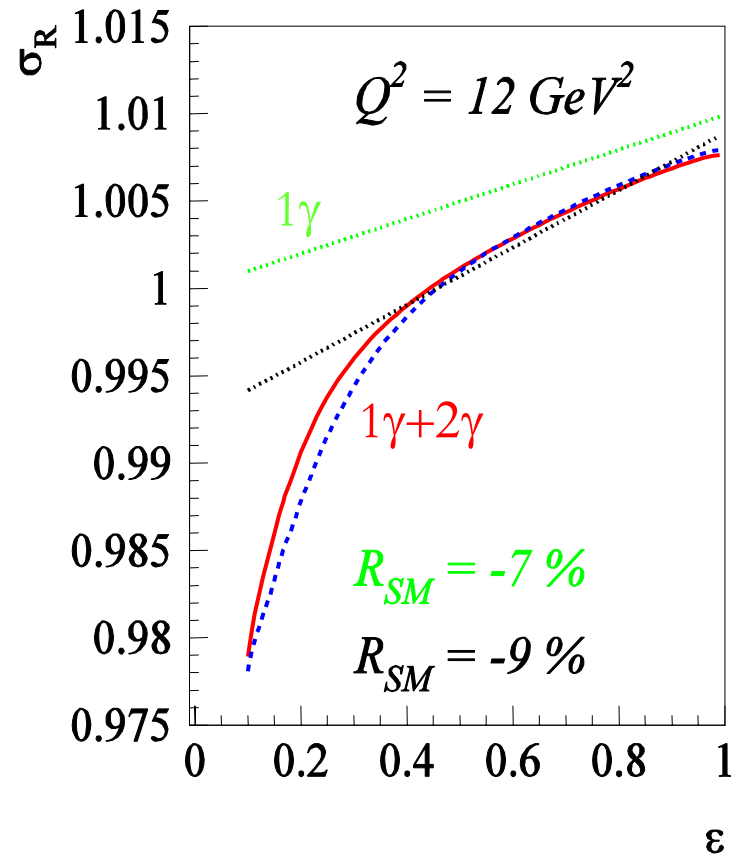
$$R_{EM} = R^{1\gamma}_{EM} + \frac{1}{8}\sqrt{\frac{3}{2}}\,\frac{Q^2}{Q_+\,Q_-}\,\frac{\varepsilon_-^3\,\varepsilon_+}{\varepsilon}\,\frac{1}{G_M^*}\,A^* - \frac{1}{4}\sqrt{\frac{2}{3}}\,\frac{Q^2}{Q_-^2}\,\frac{\varepsilon_-^2\,\varepsilon_+^2}{\varepsilon}\,\frac{M_N}{(M_N + M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

$$R_{SM} = R^{1\gamma}_{SM} - \sqrt{\frac{2}{3}}\,\frac{(Q^2 - M_\Delta^2 + M_N^2)}{4\,M_\Delta^2}\,\frac{Q_+}{Q_-}\,\frac{1}{\sqrt{2\,\varepsilon}}\,\frac{\varepsilon_-^2}{\varepsilon_+}\,\frac{M_N}{(M_N + M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

Two-photon effects on cross sections

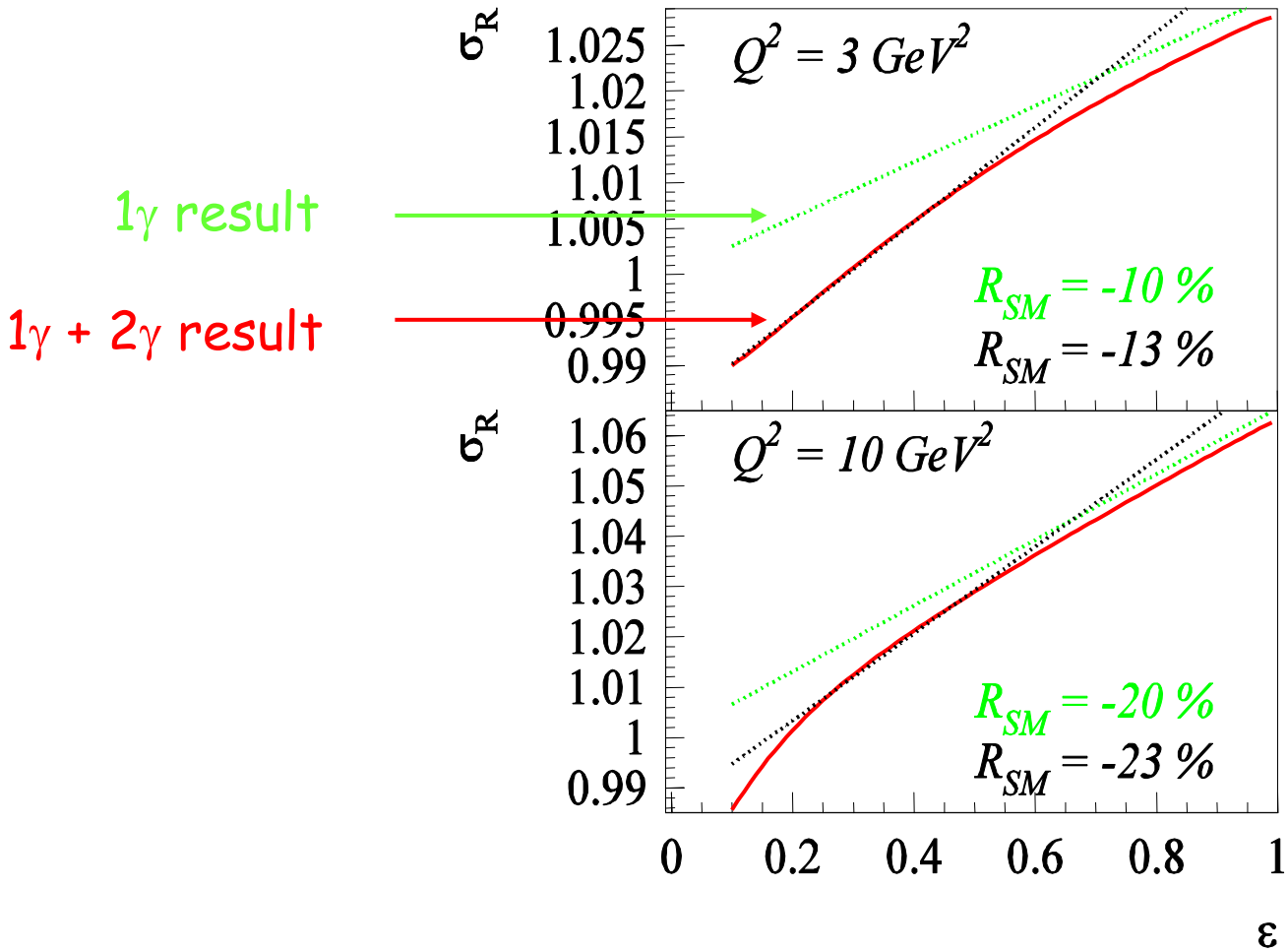
$$\sigma_R \sim \sigma_0 = \sigma_T + \varepsilon\,\sigma_L, \qquad \sigma_L \sim R^2_{SM}$$





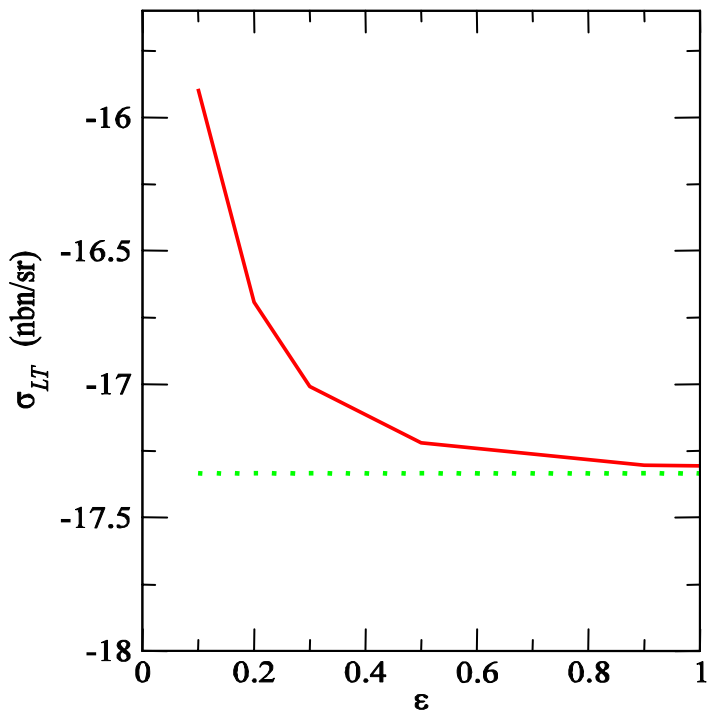
Two-photon effects on cross sections

$$\sigma_R \sim \sigma_0 = \sigma_T + \varepsilon\,\sigma_L, \qquad \sigma_L \sim R^2_{SM}$$



Two-photon effects on cross sections

$\sigma_{LT} \sim R_{SM}$, linear relation, in contrast to $\sigma_L \sim R_{SM}^2$.



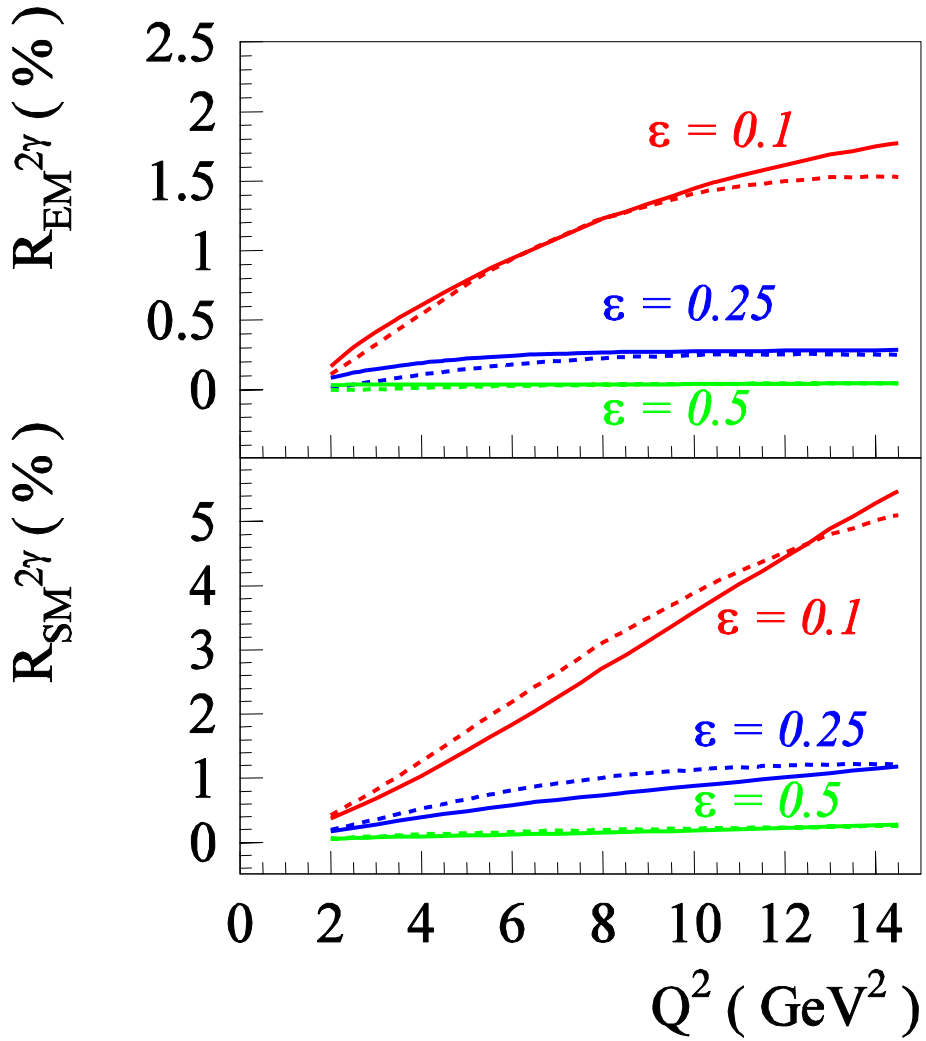
Q^2 = 6 GeV^2

1γ+2γ

1γ

σ_{TTi}, $\sigma_{LTi} \sim 1$ nbn/sr.

Two-photon effects on the ratios (Q^2 dependence)



$$\sigma_0 = A_0 + \frac{1}{2}(3\cos^2\theta_\pi - 1)\, A_2$$

$$\sigma_{TT} = \sin^2\theta_\pi\, C_0 \qquad \sigma_{LT} = \frac{1}{2}\sin(2\theta_\pi)\, D_1$$

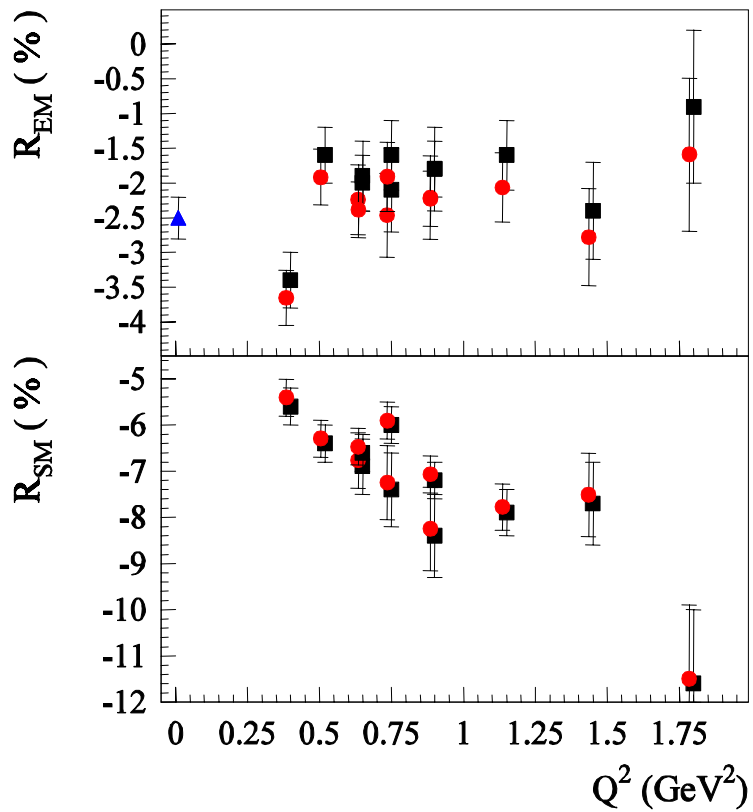
$$\sigma_{TTi} = \sin^2\theta_\pi\, C_{0i} \qquad \sigma_{LTi} = \frac{1}{2}\sin(2\theta_\pi)\, D_{1i}$$

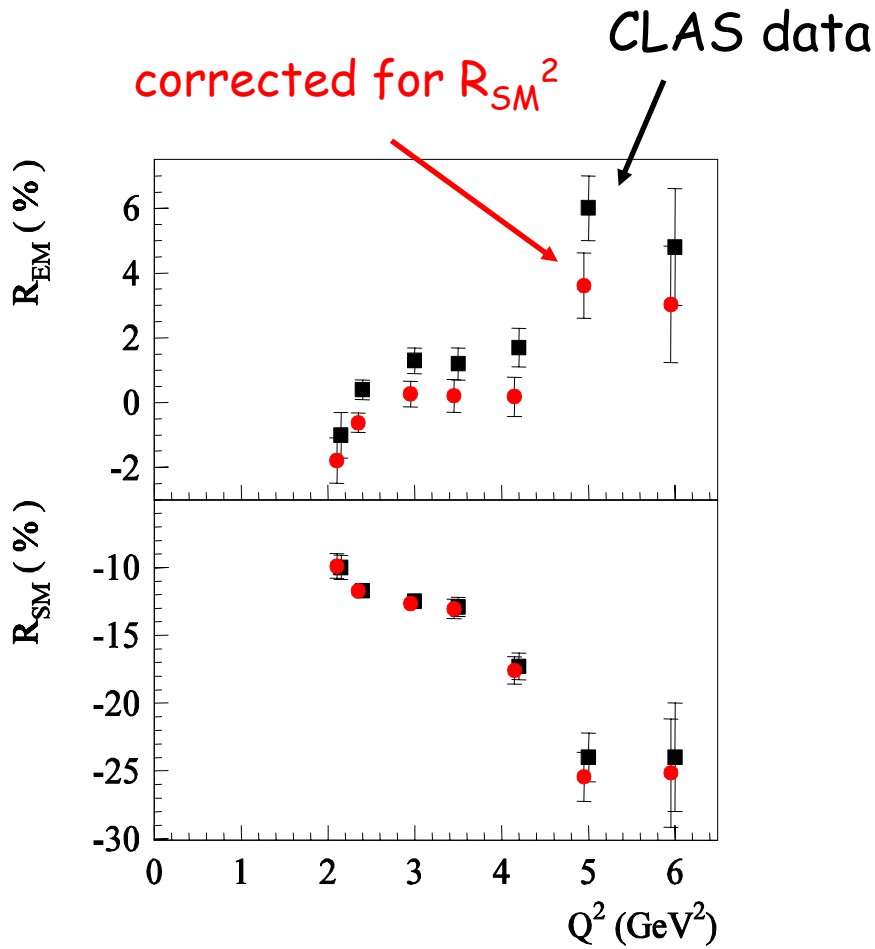
$$A_0 = \mathcal{I}\,\frac{e^2}{4\pi}\,\frac{Q_-^2}{4\,M_N^2}\,\frac{(M_\Delta + M_N)}{(M_\Delta - M_N)}\,\frac{1}{M_\Delta\,\Gamma_\Delta}\,(G_M^*)^2\,\sigma_R$$

$$R_{EM}^{exp,I} = \frac{3A_2 - 2C_0}{12A_0} \stackrel{1\gamma}{=} R_{EM} + \varepsilon\,\frac{4M_\Delta^2 Q^2}{Q_+^2\,Q_-^2} R_{SM}^2 + \ldots$$

$$R_{SM}^{exp} = \frac{Q_+\,Q_-}{Q\,M_\Delta}\,\frac{D_1}{6A_0} \stackrel{1\gamma}{=} R_{SM} - R_{SM}\,R_{EM} + \ldots$$

$$R_{EM}^{exp,II} = \frac{-(A_0 - A_2) - 2\,C_0}{3(A_0 - A_2) - 2\,C_0} \stackrel{1\gamma}{=} R_{EM}$$


R_{EM} (%)
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
R_{SM} (%)
-5
-6
-7
-8
-9
-10
-11
-12
0
0.25
0.5
0.75
1
1.25
1.5
1.75
Q^2 (GeV^2)


CLAS data
corrected for R_{SM}^2
R_{EM} (%)
6
4
2
0
-2
R_{SM} (%)
-10
-15
-20
-25
-30
0
1
2
3
4
5
6
Q^2 (GeV^2)

Conclusions

✓ General formalism for the unpolarized pion electroproduction cross-sections in terms of N!Δ helicity amplitudes.

✓New responses σ_{TTi} and σ_{LTi}.

✓Parton-model calculation of two-photon exchange N!Δ helicity amplitudes.

✓A first glance at two-photon effects on inelastic observables.

❖ Corrections on R_{EM} are small, analogous to corrections in the nucleon polarization-transfer observables.

❖ Substantial (up to 10%) corrections to the L responses (σ_L and σ_{LT}). Therefore, mainly R_{SM} is affected.

❖Effects increasing with increasing $1/\varepsilon$ and Q^2